SSA-based Optimization (Objectives)

- Given an CFG with ϕ-nodes, the student will be able to perform global common subexpression elimination (redundancy elimination) using a dominator-based approach.
- Given a CFG in SSA form, the student will be able to perform global constant propagation.
- Given a CFG in SSA form, the student will be able to perform strength reduction by finding loop, calculating loop invariants, finding induction variables and then applying the strength reduction transformation.
- Given a CFG in SSA form, the student will be able to perform dead-code elimination.
- Given a CFG in SSA form, the student will be able to perform global value numbering.

Dominator-based Global Common Subexpression Elimination

- A limited form of global CSE
  - used before dependence based optimization and other SSA-based optimizations
  - no code motion
  - redundancy found only along paths in the dominator tree
- In SSA all syntactically equivalent expression are semantically equivalent.
- Method:
  - keep a block structured table of available expression
    - StartBlock – add a scope in the expression table for this block.
    - EndBlock – remove the scope for the current block
  - perform CSE on the dominator tree while constructing SSA.

Algorithm

```
OPTRENAME(b) {
   for each T0 = φ(T1,...,Tn) ∈ Φ(b)
     push NewName() on NameStack(T0)
   StartBlock(b)
   for each I ∈ b in execution order {
     for each T ∈ Operand(I)
       replace T by Top(NameStack(T))
     if I.expr() ∈ AVAIL { // insert if ∉AVAIL
       T = I.lval()
       push GetTarget(AVAIL,i) on NameStack(T)
       DEAD ∪= {I}
     }
```

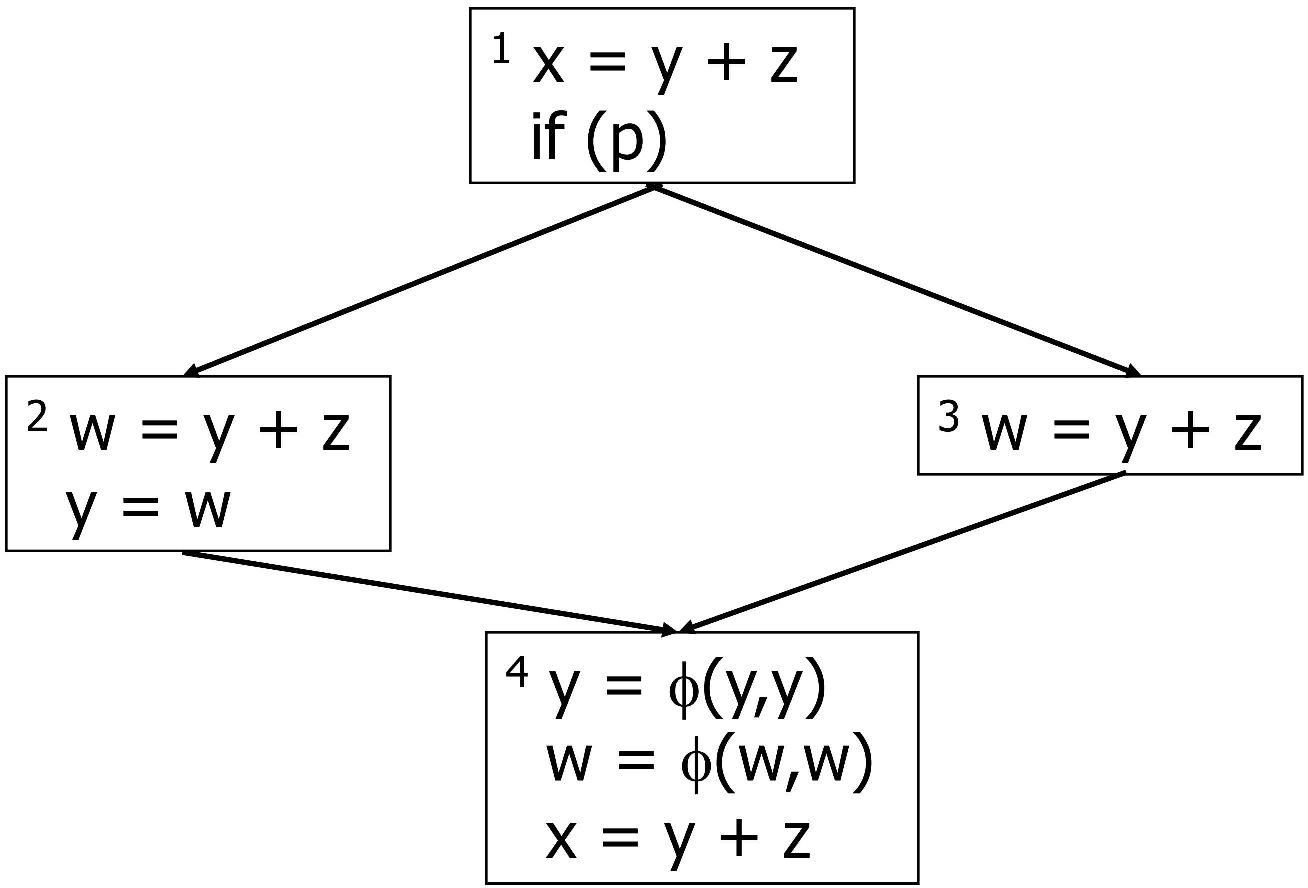
Algorithm

```
    else
      push NewName() on Top(NameStack(I.lval()))
  }
for each s ∈ succ(b) {
  j = WhichPredecessor(s,b)
  for each T0 = φ(T1,...,Tn) ∈ Φ(s)
    replace Tj with Top(NameStack(Tj))
}
for each c ∈ children(b)
  OPTRENAME(c)
```

Algorithm

```
for each I ∈ b in reverse order {
  X = Pop(NameStack(I.lval()))
  if I ∈ DEAD
    remove I
  else
    replace I.lval() with X
}
for each T0 = φ(T1,...,Tn) ∈ Φ(b)
  replace T0 by Pop(NameStack(T0)
EndBlock(b)
}
```

Example



Constant Propagation

- Propagate constants globally on a sparse representation
  - cheaper than previous algorithm
- Incorporate the effects of branch folding
  - if a block cannot be reached, it will be ignored
- Meet operations occur at ϕ-nodes

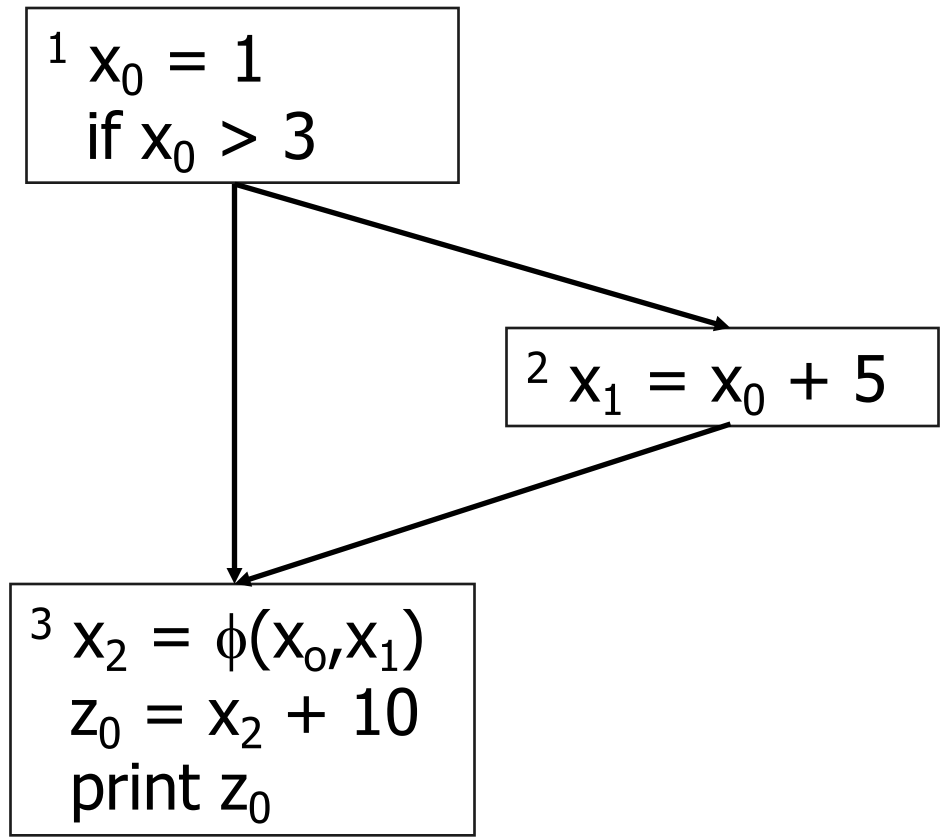
Algorithm

```
Procedure ConstProp {
    mark all edges in CFG not
          executable
    initialize all nodes in SSA
          Graph to unknown
    Work = ∅; Visited = ∅;
    Blocks = {ENTRY}
    while Work ≠∅ ∨ Blocks ≠ ∅ {
      while Work ≠ ∅ {
        take I from Work
        EvalInstruction(I)
      }
      while Blocks ≠ ∅ {
        take b from Blocks
        for each I ∈ Φ(b) {
          EvalInstruction(I)
        if b ∉ Visited {
          Visited ∪= {b}
          for each I ∈ b
            EvalInstruction(I)
        }
      }
    }
}
```

Algorithm

```
EvalInstruction(I) {
    if I is an arithmetic instruction or φ-node {
        evaluate I
        if result lowered
          for each j ∈ Uses(I.lval()) {
              propagate result
              if j.Block() ∈ Visited
                Work ∪= {j}
          }
      }
    else if I is a branch or the end of the block is reached
        for each possible destination, S
          if edge from I.block() to S is not executable {
            mark it as executable
              Blocks ∪= {S}
          }
}
```

Example



Strength Reduction

- Replace multiplication of a regularly varying variable by a constant in a loop with an addition.
- Example

```
i = 1
loop {
  j = 2*i
  i += 1
}
```

- Gets converted to

```
j = 0;
i = 1
loop {
  j += 2
  i += 1
}
```

- Useful for enabling opportunities for auto-increment mode
- cheaper instructions

Method

1. Find loops in CFG
2. Find the variables in a loop that are loop invariant.
3. Find loop induction variables (vary regularly)
4. Reshape expressions into canonical form
5. perform strength reduction

Step 1: Finding Loops

- Def^n: A loop is a set of basic blocks, L, such that if $b_0, b_1 \in L$ then there is a path from b_0 to b_1 and from b_1 to b_0. A block $b \in L$ is an entry block if b has a predecessor that is not in L. A block $b \in L$ is an exit block if b has a successor not in L.
  - We will look at natural loops where the entry block dominates all other blocks in the loop (single entry).
- Computing loops involves finding a block that has an incoming back edge (head dominates the tail).
- Modified from book, which does multiple entry loops (not natural)

Loop Tree

- Organize the loops in a function hierarchically.
  - A loop L1 is a child of loop L2 in the loop tree iff L1 $\subseteq$ L2
- The tree structure is recorded by (X is a loop or block)
  - LoopParent(X) – an attribute indicating which node in the tree of which this node is a child. It also indicates the loop in which a loop or block is contained. LoopParent(X) may be a special root node indicating that the loop is contained in no other loop.
  - LoopContains(X) – the set of children of a node in the loop tree. The blocks or loops contained in a loop.
  - LoopEntry(X) – the entry node of the loop.

Computing the Loop Tree

```
LoopTree() {
    compute post-order numbering for the CFG
    for each b ∈ G {
      LoopParent(B) = NIL
      LoopEntry(B) = B
      LoopContains(B) = B;
    }
    for each b ∈G in postorder
      FindLoop(b)
    Make all nodes w/o parents have a Root node as parent
}
```

Computing the Loop Tree

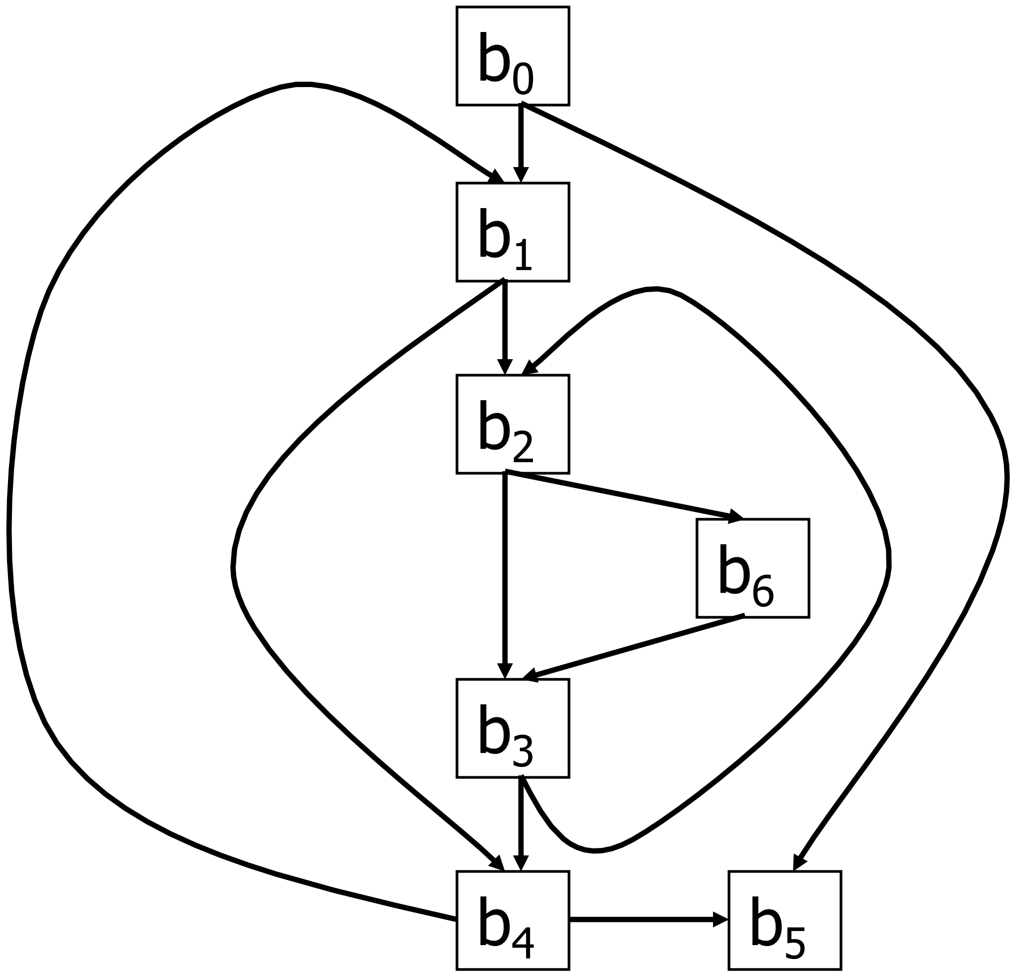
```
FindLoop(b) {
   Loop = ∅; Found = false
   for each p ∈ pred(b)
     if b ≫ p {
        Found = true;
        if p ∉ Loop ∧ p ≠ b {
         Loop ∪= {p}
         }
       }
   if Found
     FindBody(Loop,b)
}
```

Computing the Loop Tree

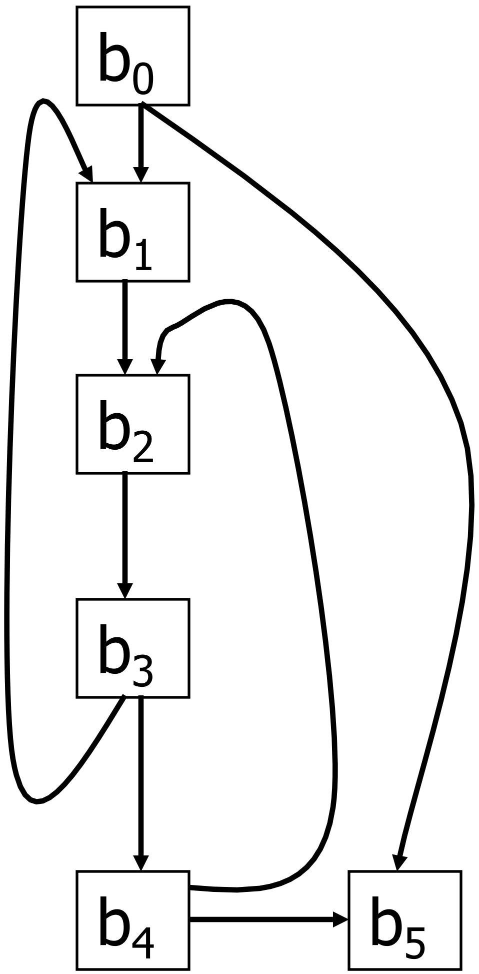
```
FindBody(Generators,H) {
    Loop = ∅; Queue = ∅
    for each b ∈ Generators {
      L = LoopAncestor(b)
      if L ∉ Loop then {
        Loop ∪= {L};Queue ∪= {L}
      }
    }
    while (Queue ≠ ∅) {
      b = Queue.Dequeue()
      Pred= pred(LoopEntry(b))
      for each p ∈ Pred
        if p ≠ H {
          L = LoopAncestor(p)
          if L ∉ Loop {
           Queue.Enqueue(L)
           Loop ∪= {L}
          }
        }
    }
    Loop ∪= {H}
    X = new Loop Tree node
    LoopContains(X) = Loop
    LoopEntry(X) = H
    LoopParent(X) = NIL
    for each b ∈ Loop
      LoopParent(b) = X
}

LoopAncestor(b) {
    while LoopParent(b) ≠ ∅
        b = LoopParent(b)
    return b
}
```

Example



Example



Step 2: Loop Invariants

- Def^n: A variable is loop invariant if it is either not computed in a loop or its operands are invariant.
- Compute variant(T), the innermost loop in which T is not invariant.
  - if T = ϕ(..), T is defined to be variant in the innermost loop containing it.
  - for pure functions like add, variant in the innermost loop that one of the operands in variant
  - for a LOAD, it varies in the innermost loop in which a store operation might modify the same location.
- Walk the dominator tree in preorder

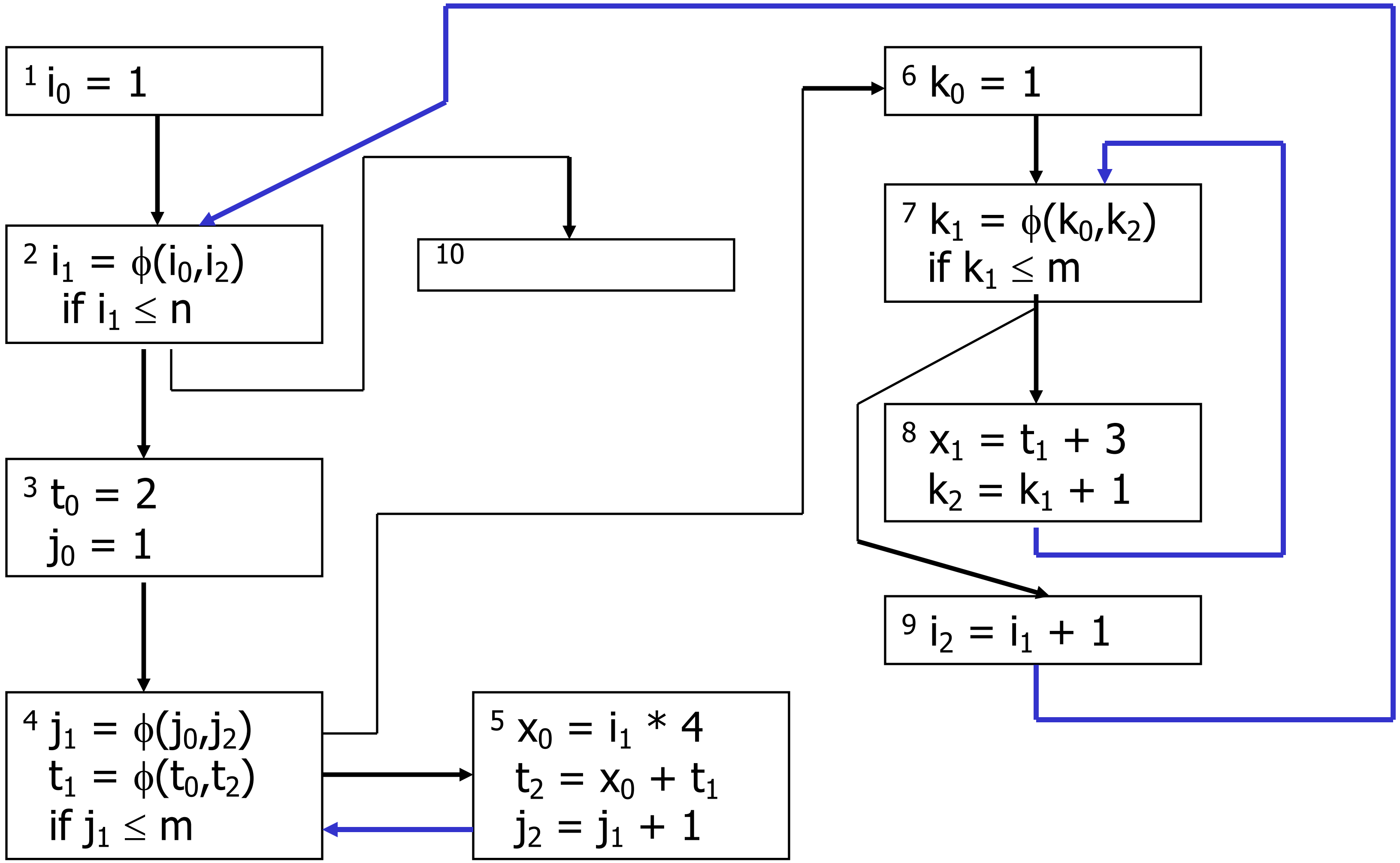
Finding Loop Invariants

```
CalcLoopInvariants(b) {
  for each T0 = φ(T1,...,Tn) ∈ Φ(b)
    variant(T0) = LoopParent(b)
  for each I ∈b in order {
    Varying = Root
    for each T ∈ Operands(I) {
      TVarying = LoopNearestAncestor(variant(T),b)
      if LoopNearestAncestsor(Varying,TVarying) == Varying
        Varying = TVarying
    }
    variant(I.lval()) = Varying
  }
}
```

Finding Loop Invariants

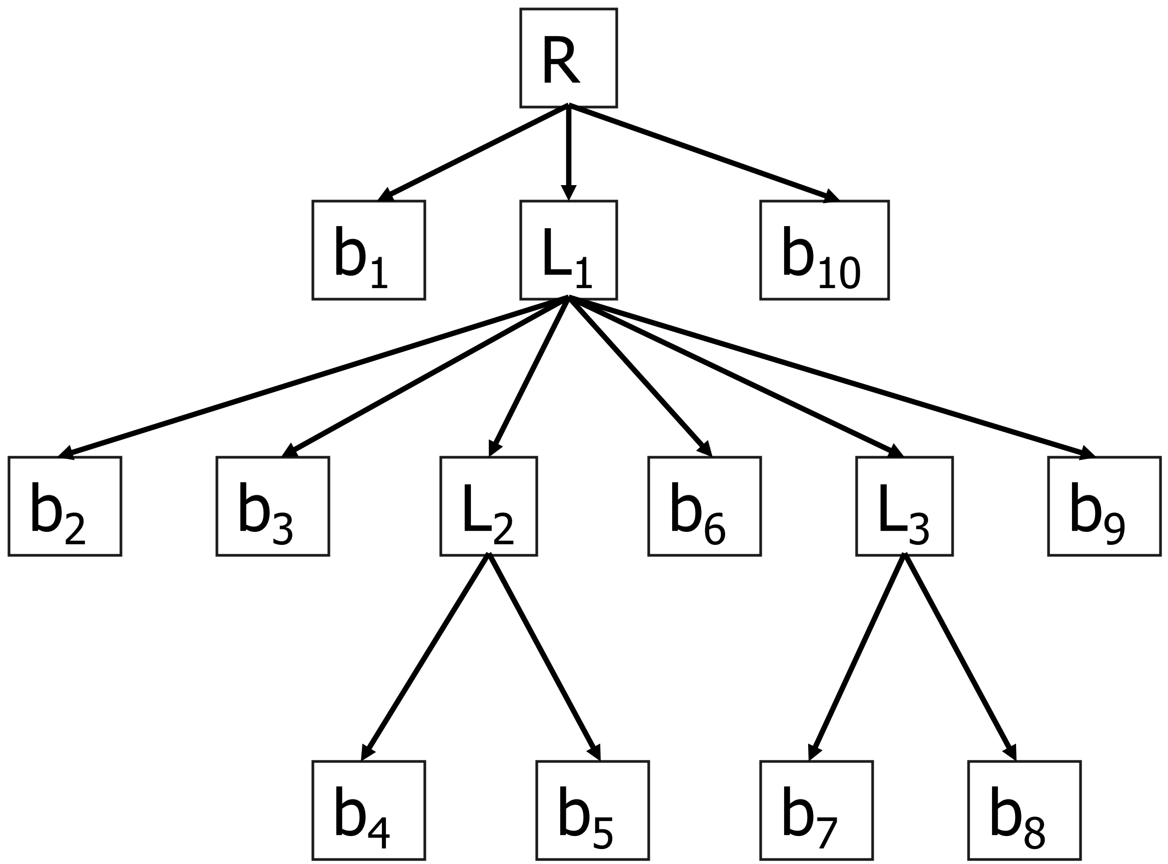
```
LoopNearestAncestor(L1,L2) {
    if is_ancestor(L2,L1)
      return L2
    L = L1
    while !is_ancestor(L,L2)
      L = LoopParent(L)
    return L
}
```

Example

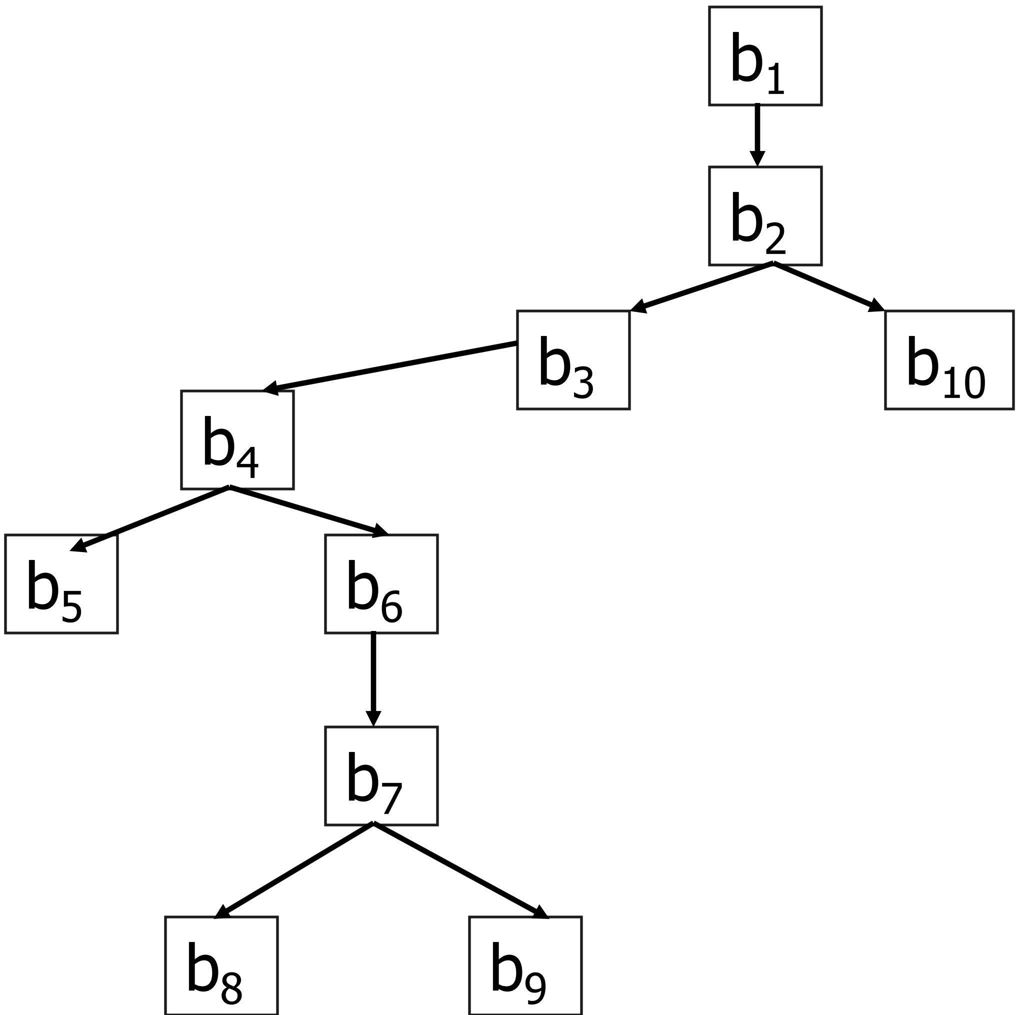


1 $i_0 = 1$
2 $i_1 = \phi(i_0,i_2)$
if $i_1 \leq n$
3 $t_0 = 2$
$j_0 = 1$
4 $j_1 = \phi(j_0,j_2)$
$t_1 = \phi(t_0,t_2)$
if $j_1 \leq m$
5 $x_0 = i_1 * 4$
$t_2 = x_0 + t_1$
$j_2 = j_1 + 1$
6 $k_0 = 1$
7 $k_1 = \phi(k_0,k_2)$
if $k_1 \leq m$
8 $x_1 = t_1 + 3$
$k_2 = k_1 + 1$
9 $i_2 = i_1 + 1$
10

Example: Loop Tree



Example: Dominator Tree



Step 3: Finding Induction Variables

- Def^n: A temporary T is a candidate temporary for loop L iff T is computed in L and the computation has one of the following forms:
  a) $T = T_i \pm T_j$ where one operand is a candidate in L and the other is loop invariant
  b) $T = \pm T_k$ where T_k is a candidate in L or is loop invariant in L
  c) $T = \phi(T_1,\ldots,T_n)$ where each of the operands is either a candidate in L or a loop invariant in L

Algorithm: Finding Induction Variables

```
CalcCandidates(L) {
  Candidates = ∅
  Work = ∅
  for each b ∈ L {
   for each I ∈ Φ(b) ∪ b of the form T = ...
    if Typeof(T) is integer
     if T has candidate syntax {
      Candidates ∪= {T}
      Work ∪= {T}
  }
```

Algorithm

```
while Work ≠ ∅ {
  take T from Work
  CandidatePrune(T)
  if T ∉ Candidates
    for each I ∈ Uses(T) where I ∈ L {
      U = I.lval()
      if (U ∈ Candidates ∧ U ∉ Work)
        Work ∪= {U}
    }
  }
}
```

Algorithm

```
CandidatePrune(T) {
   I = T.instruction()
   case on form of I {
     T = φ(T1,…,Tn):
              for i = 1, n
               if Ti ∉ Candidates ∧ !invariant(Ti,L) {
                 Candidates -= {T}
                 return
               }
```

Algorithm

```
T = Ti ± Tj:  if Ti ∈ Candidates ∧ invariant(Tj,L)
                return
              else
                if Tj ∈ Candidates ∧ invariant(Ti,L)
                  return
                else {
                  Candidates -= {T}
                  return
                }
```

Algorithm

```
T = ±Tk:  if Tk ∉ Candidates ∧ !invariant(Tk,L) {
              Candidates -= {T}
              return
          }
    }
}
```

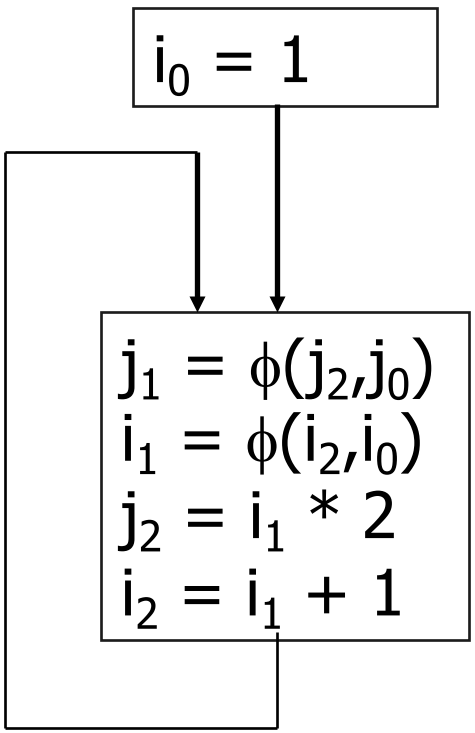
Example

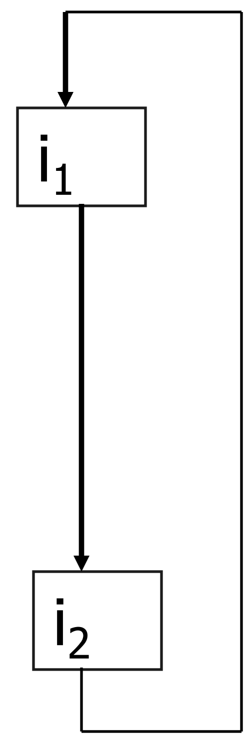
- Detect induction variables in previous example

Induction Sets

- Consider a graph where candidates are nodes and an edge is between two nodes, T and U, if T is used to compute U. And induction temporary is a temporary in a SCC in this graph. An induction set is the set of temporaries in the SCC.

Example





Algorithm

```
CalcInduction(L) {
   CalcCandidates(L)
   Construct candidate graph, G
   compute SCC(G)
   Anchors = {T | T is a target of a φ-node in
                       LoopEntry(L)}
   for each s ∈ SCC(G)
     if |s| > 1 ∧ Anchors ∩ s ≠ ∅
       add s to InductionSets
}
```

Example

- Compute the induction variables in the previous example.

Step 4: Reshape Expression

- Use commutative, associative, and distributive properties to reshape expressions contained in n loops as

$$E = E' + (LC_1 + (LC_2 + \ldots + LC_n))$$
$$E' = E'' + FD_1 * I_1 + FD_2 * I_2 + \ldots + FD_m * I_m$$

  where LC_i is invariant in L_i, I_i is the induction variable of L_i and FD_i is a loop invariant expression.
- LC_i can be moved outside of L_i
- Can cause an increase in cost (invariants into loops)

Strength Reduction

Consider an expression of the form $E = FD_i{*}I_i + LC_i$

Let IS_i be the induction set of I_i

Create temporaries $E_0,\ldots,E_q$, one for each element of IS_i plus any initial values coming in from outside the loop.

for all $T_j = T_k \pm c$ in the loop such that $T_j, T_k \in IS_i$
insert $E_j = E_k \pm FD_i{*}c$ after this point

for all $T_j = \pm T_k$ in the loop such that $T_j, T_k \in IS_i$
insert $E_j = \pm E_k$ after this point

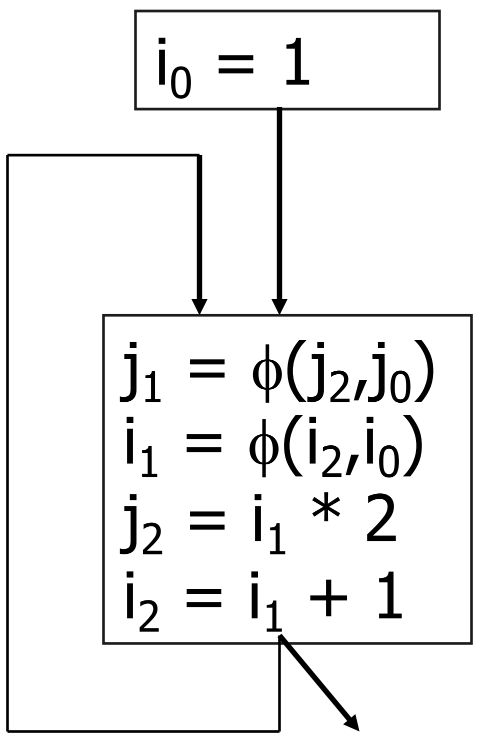
replace uses of E with the correspond E_j whose definition reaches the use

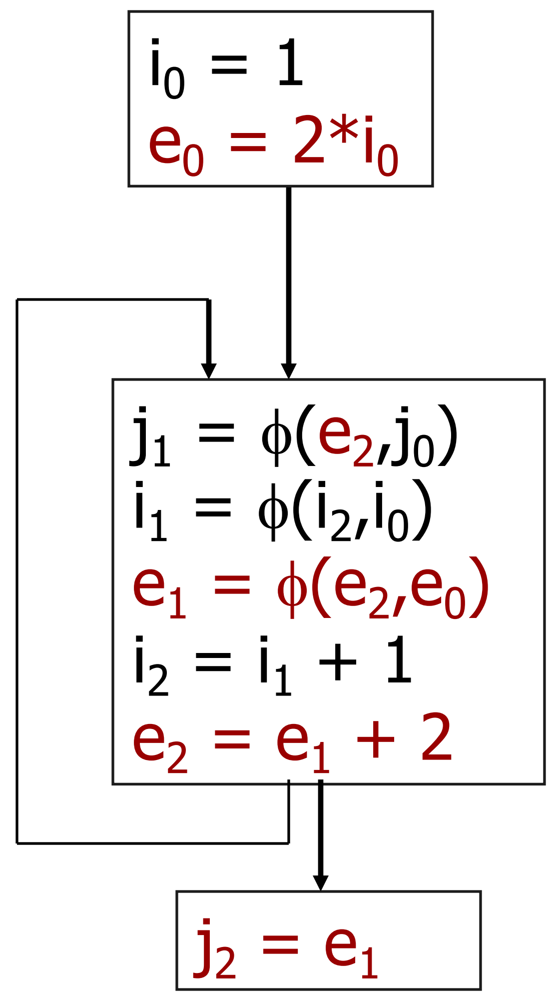
replace $E = FD_i{*}I_i + LC_i$ with the assignment $E = E_j$. If the block containing this assignment is executed on every path through the loop to a loop exit, it can be moved after the loop following each loop exit.

Handling ϕ-nodes

- Given $T_0 = \phi(T_1,\ldots T_n)$, $T_0 \in IS_i$, create a new ϕ-node $E' = \phi(\ldots)$
- for each predecessor block P_j
  - if the temporary T_j is in the induction set of T_0, put the temporary holding E at the end of P_j in the j^{th} position of the ϕ-node for E' (P_j must be in the loop because T_j is in the induction set).
  - if T_j is not in the induction set for T_0, insert the computation $E_j = FD_i * T_j + c$ at then end of P_j and place E_j into the corresponding entry in the ϕ-node for E' (P_j is not in the loop).
  - change E' to be the exposed use of a temporary for E.

Example


$i_0 = 1$
$j_1 = \phi(j_2, j_0)$
$i_1 = \phi(i_2, i_0)$
$j_2 = i_1 * 2$
$i_2 = i_1 + 1$


$i_0 = 1$
$e_0 = 2*i_0$
$j_1 = \phi(e_2, j_0)$
$i_1 = \phi(i_2, i_0)$
$e_1 = \phi(e_2, e_0)$
$i_2 = i_1 + 1$
$e_2 = e_1 + 2$
$j_2 = e_1$

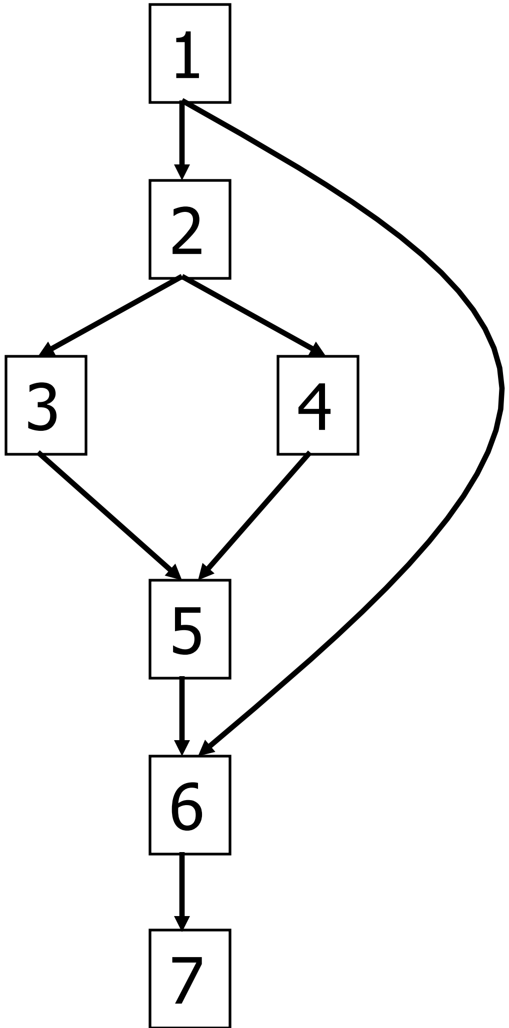
Dead-code Elimination

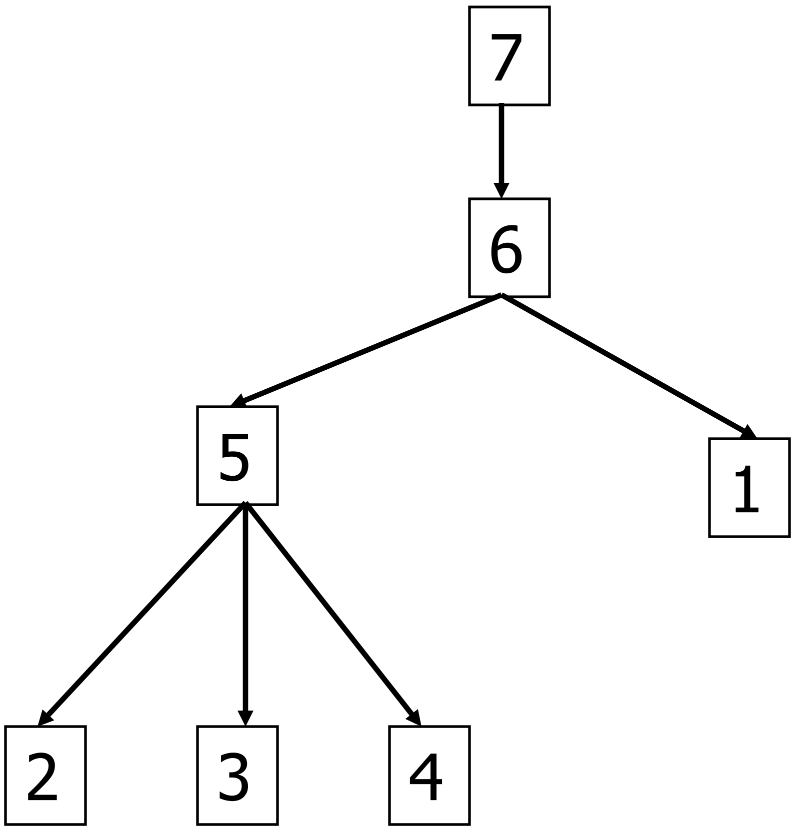
- Use the SSA graph (sparse) to detect dead code.
- Method
  - remove instructions that do not directly or indirectly use data that is observable outside the procedure.
  - allow for branches that are never taken (can eliminate loops this way)
    - uses control dependence

Control Dependence

- Use the idea of postdominators
- Def^n: A block X postdominates a block B iff every path from B to Exit contains X.
- Def^n: ipdom(B) represents the immediate postdominator of B and is the parent of B in the postdominator tree.
- Compute postdominators using the dominator relation on the reverse control flow graph.

Example





Control Dependence

- Consider two block B and X. When does B control the execution of X?
  1. If B has only one successor block, it does not control the execution of anything. B must have multiple successors.
  2. B must have some path leaving it that leads to the Exit block and avoids X. X cannot postdominate B
  3. B must have some path leaving it that leads to X.
  4. B should be the latest block that has these properties.

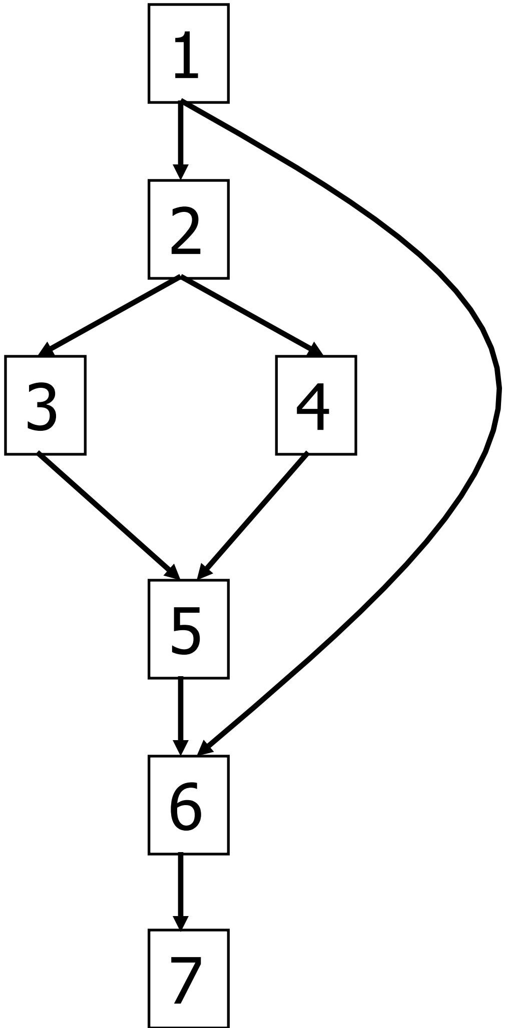
Control Dependence

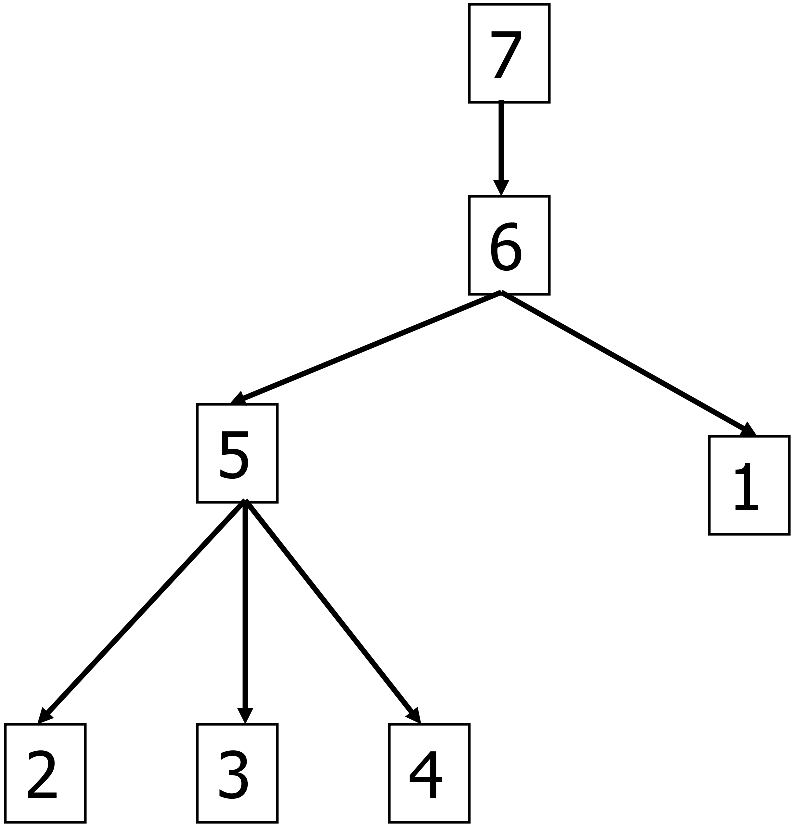
- A block X is control dependent on an edge (B,S) iff there is a non-empty path from B to X such that X postdominates each block on the path except B. And, X = B or X does not postdominate B.
- Compute control dependence by find the dominance frontier of every node in the reverse control-flow graph.

Computing Control Dependence

```
foreach n ∈ PDT in postorder{
    DF(n) = ∅
    for each c ∈ child(n)
      for each m ∈ DF(c)
        if !(n stricly postdominates m)
          DF(n) ∪= {m}
    for each m ∈ pred(n)
      if !(n strictly postdominates m)
        DF(n) ∪= {m}
}
```

Example





Algorithm

```
EliminateDeadCode()
    WorkList = ∅
    Necessary = ∅
    for each B ∈ N do
      for each I ∈ B do
        if (I stores into external data) ∨
            (I is an i/o instruction) ∨ (I is a call) {
          Necessary ∪= {I}
          WorkList ∪= {I}
        }
```

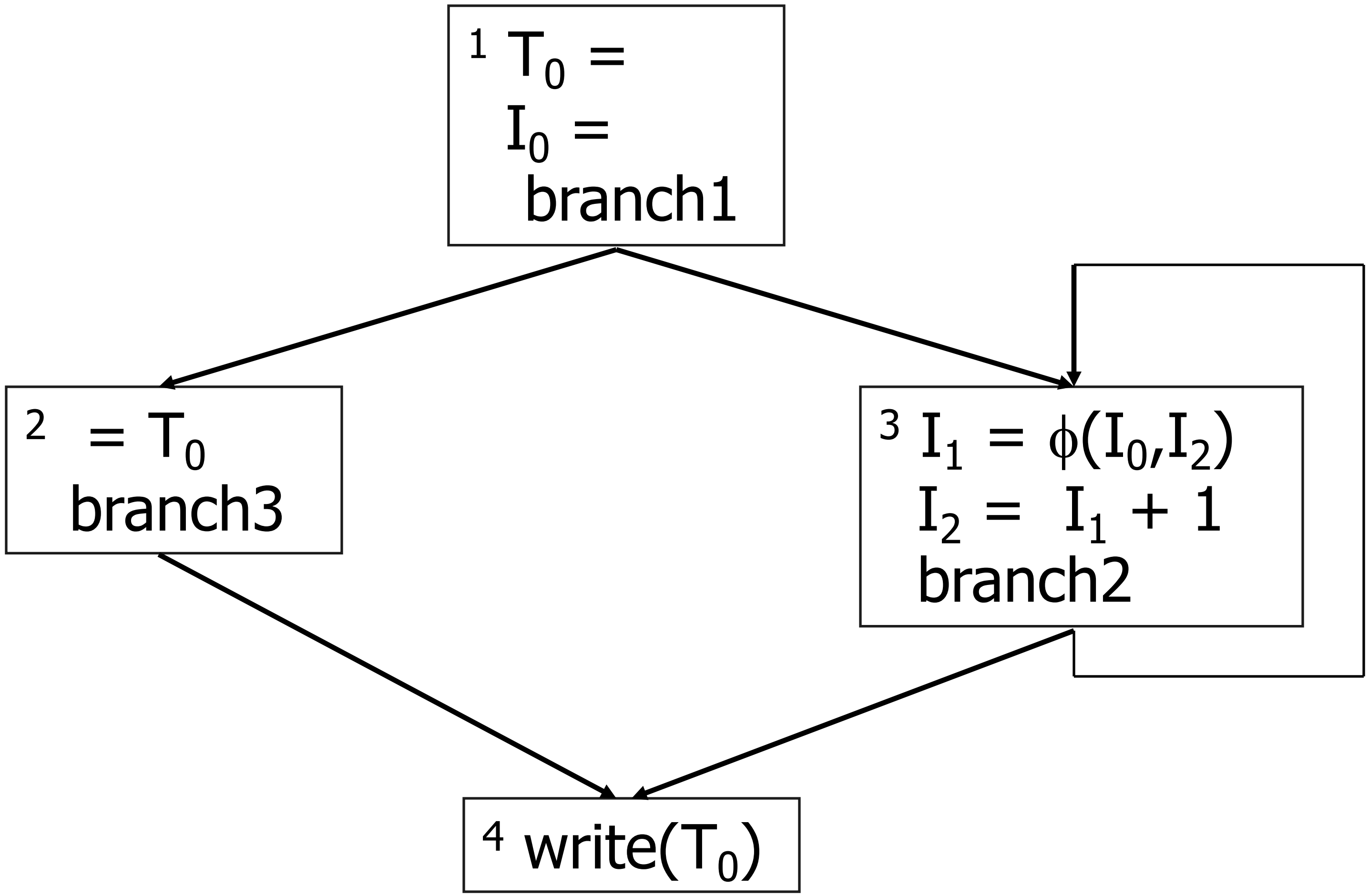
Algorithm

```
while WorkList ≠ ∅ {
  take I from WorkList
  b = I.ContainingBlock()
  for each C on which B is control dependent {
    J = branch in C
    if J ∉ Necessary {
      Necessary ∪= {J}
      WorkList ∪= {J}
    }
  }
```

Algorithm

```
    for each T ∈ Operand(I) {
      J = Definition(I)
      if J ∉ Necessary {
        Necessary ∪= {J}
        WorkList ∪= {J}
      }
    }
  }
  for each B ∈ N
    for each I ∈ B
      if I ∉ Necessary
        remove I
      else if I is a branch ∧ I ∉ Necessary
         change branch to immediate postdominator of block
}
```

Example



1 $T_0 =$
$I_0 =$
branch1
2 $= T_0$
branch3
3 $I_1 = \phi(I_0, I_2)$
$I_2 = I_1 + 1$
branch2
4 $\mathrm{write}(T_0)$

Global Value Numbering

- Apply value numbering to a global context for better redundancy elimination.
- Associate a field for each temporary to hold its value number
- If two temporaries have the same value number then they are equivalent.
- If there are no loops a reverse postorder walk of the CFG is sufficient (all operands defined before used)
- ϕ-nodes can only be equivalent in the same basic block
  - need control-flow information to compare ϕ-nodes from different blocks

Global Value Numbering

- What can we do about SCCs in the SSA graph?
  - The value number of some operands will not be known when trying to process an instruction.
  - This will happen at ϕ-nodes
  - Solution: assume the best case (an unknown value number does not affect the result) and iterate
  - Process nodes in an SCC in reverse postorder (as other nodes)

Processing ϕ-nodes

- There are 3 possibilities
  1. If a corresponding entry for the ϕ-node/block is already in the value table, then assign the target of this ϕ-node the same value_representative value.
  2. Consider the operands that do not have a value_representative value of NULL. If at least two of them have different values, assign the target a new value # and enter it into the value table
  3. Consider the operands that do not have a value_representative value of NULL. If all of them have the same value, then give the target the same value number and enter it into the table.

Efficiency Improvements

- When processing a SCC, use a temporary value table called a scratch table. Once the values in the scratch table have stabilized, move the results to the value table.

Algorithm

```
procedure CalcGlobalValue {
    compute the SCC of the SSA Graph: C_1,...,C_s ordered by SSA
       edges so that defs precede uses
    ValTab = ∅; ScratchTab = ∅;
    for each T ∈ Temporaries
      ValRep(T) = NULL;
    for i = 1, s
      if |C_i| > 1 {
        call CalcGlobalValueSCC(C_i)
        for each T ∈ C_i in reverse postorder {
          I = Definition(T); U = ValRep(T);
          apply algebraic simplification to I
          if ⟨opcode(I),ValRep(Operands(I))⟩ ∉ ValTab
            ValTab ∪= {⟨opcode(I),ValRep(Operand(i),U⟩}
      }
```

Algorithm

```
// let I be the single instruction in C_i
  else if I is a ϕ-node
    CalcϕValue(I,ValTab)
  else {
    apply algebraic simplification to I
    T = Target(I)
    if ⟨opcode(I),ValRep(Operands(I)⟩ ∉ ValTab {
        ValRep(T) = T;
        ValTab ∪= {⟨opcode(I),ValRep(Operands(I),ValRep(T)⟩}
    }
    else
      ValRep(T) = value from ValTab
  }
}
```

Algorithm

```
procedure CalcGlobalValueSCC(C) {
    change = false;
    repeat
      for each T ∈ C in reverse postorder {
        I = Definition(T)
        if I is a ϕ-node
          NewValue = CalcϕValue(I,ScratchTab)
        else {
            process algebraic simplification but don't change instructions
           if ⟨opcode(I),ValRep(Operands(I))⟩ ∈ ScratchTab
             NewValue = value in ScratchTab
            else {
             NewValue = T
             ScratchTab ∪= {⟨opcode(I),ValRep(Operands(I),T⟩}
            }
         }
        if NewValue ≠ ValRep(T) {
             change = true; ValRep(T) = NewValue;
         }
      }
    until not(change)
}
```

Algorithm

```
procedure CalcϕValue(I,Table) {
    Let I be T0 = ϕ(T1,…,Tn)
    if <ϕ,ValRep(Operands(I))> ∉ Table {
      if ∃Ti,Tj | ValRep(Ti) ≠ NULL ∧ ValRep(Tj) ≠ NULL ∧
                      ValRep(Ti) ≠ ValRep(Tj)
        NewValue = T0
      else
        NewValue = ValRep(Ti) where ValRep(Ti) ≠ NULL
      Table ∪= {⟨opcode(I),ValRep(Operands(I)),NewValue⟩}
    }
    else
      NewValue = ValRep from Table
    return NewValue
}
```

Example

1 $I_0 = 1$

$J_0 = 1$

2 $I_1 = \phi(I_0, I_2)$

$J_1 = \phi(J_0, J_2)$

$U = I_1 - J_1$

$I_2 = I_1 + 1$

$J_2 = J_1 + 1$

Now What?

- Give all temporaries with the same value # the same partition, and convert to normal form
- Apply common subexpression elimination
  - dominator-based
  - traditional AVAIL-based
  - partial redundancy elimination