

In optimization, two basic iterative approaches to find a local minimum x^* of an objective function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is **Line Search** and **Trust Region**

Line Search

Heuristics

The Line Search approach first finds a descent direction, along which the objective function will be reduced and then computes a step size that determines how far x should move along the direction. The descent direction can be computed by various methods, such as gradient, Newton's method and Quasi-Newton method. The step size can be determined either exactly or inexactly.

Example

- Set iteration counter $k=0$, and make an initial guess x_0 for the minimum
- Repeat
 - Compute a descent direction p_k
 - Choose α_k to loosely minimize $h(\alpha) = f(x_k + \alpha p_k)$ over $\alpha \in \mathbb{R}_+$
 - Update $x_{k+1} = x_k + \alpha_k p_k$ and $k = k + 1$
 - Until $\|\nabla f(x_k)\| < \text{Threshold}$

In the line search step of choosing the α_k , the algorithm can choose two heuristics.

- To exactly minimize h , by solving $h'(\alpha_k) = 0$
- Ask for a sufficient decrease in h

Conjugate Gradient method is an example of the former heuristic. The latter is called **Inexact Line Search**, the examples are **Backtracking Line Search** or by using **Wolfe Conditions**

Trust Region

Trust Region denotes the subset of the region of the objective function that is approximated using a model function (often a quadratic). If an adequate model of the objective function is found within the trust region, the region is expanded. Conversely, if the approximation is poor, the region is contracted. Trust Region is also known as **Restricted-Step Method**.

The fit is evaluated by comparing the ratio of expected improvement from the model approximation with the actual improvement observed in the objective function. Simple thresholding of the ratio is used as the criterion for expansion and contraction-a model function is "trusted" only in the region

where it provides a reasonable approximation.

For detailed example, please review [Trust Region Example](#)