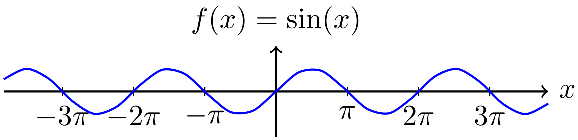
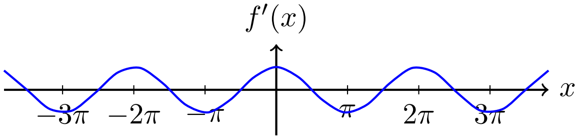
Sine



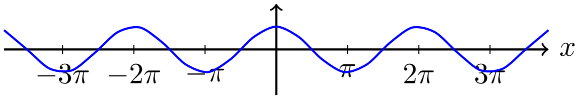
First we might ask where the derivative of $f(x) = \sin(x)$ is zero. Then ask, where is it positive or negative? If we graph this information, we get something like:



What does this look like?

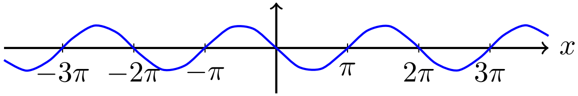
Cosine

$f(x) = \cos(x)$



Again, first we might ask where the derivative of $f(x) = \cos(x)$ is zero. Then ask, where is it positive or negative? If we graph this information, we get something like:

$f'(x)$



What does this look like?

Derivative of Sine and Cosine

For x in radians,

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

and

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

Clicker Question

Consider the function $H(x) = 5\sin(x) + 6\cos(x)$. What is $H'(x)$?

a) $H'(x) = \cos(x) + \sin(x)$

b) $H'(x) = 5\cos(x) - 6\sin(x)$

c) $H'(x) = 5\cos(x) + 6\sin(x)$

d) $H'(x) = \cos(x) - \sin(x)$

Derivative of Tangent

For x in radians,

$$\frac{d}{dx}(\tan(x)) = \frac{1}{\cos^2(x)} = \sec^2(x)$$

Clicker Question

Consider the function $g(x) = \cos(x)\,(3 + \tan(x)) + 4\tan(x)$. What is $g'(x)$?

1. $g'(x) = -3\sin(x) + \cos(x) + 4\sec^2(x)$
2. $g'(x) = 3\cos(x) + \sin(x) + 4\sec^2(x)$
3. $g'(x) = -\sin(x)(\sec^2(x)) + 4\sec^2(x)$
4. $g'(x) = -\sin(x) + 5\sec^2(x)$

Why we need a new rule

We will now consider functions of the form $F(x) = f(x)g(x)$.

Let's consider the derivative of $f(x)g(x) = (x^2+1)(x^3)$.

$$\frac{d}{dx}\left((x^2+1)(x^3)\right) = \frac{d}{dx}\left(x^5+x^3\right) = 5x^4+3x^2$$

What if we tried to just take the derivative of each?

$$\frac{d}{dx}(x^2+1)\frac{d}{dx}(x^3) = (2x)(3x^2) = 6x^3 \neq 5x^4+3x^2$$

Finding a rule

Let's consider the definition of the derivative:

$$\frac{d}{dx}(f(x)g(x)) = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

For the next step we will add 0, that is: $f(x+h)g(x) - f(x+h)g(x)$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) + f(x+h)g(x) - f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \left[f(x+h)\frac{g(x+h)-g(x)}{h} + g(x)\frac{f(x+h)g(x)-f(x)}{h} \right]$$

$$= \lim_{h\to 0} f(x+h)\frac{g(x+h)-g(x)}{h} + \lim_{h\to 0} g(x)\frac{f(x+h)g(x)-f(x)}{h}$$

Product Rule

Product Rule

If $u = f(x)$ and $v = g(x)$ are differentiable functions, then:

$$(fg)' = f'g + fg'.$$

The product rule can also be written

$$\frac{d(uv)}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}.$$

In words:
The derivative of a product is the derivative of the first times the second plus the first times the derivative of the second.

Example

Consider the function $F(x) = 4e^x \cos(x)$. Find $F'(x)$.

In this example, $F(x) = f(x)g(x)$ where $f(x) = 4e^x$ and $g(x) = \cos(x)$. Using the product rule,

$$F'(x) = f'g + fg' = 4e^x \cos(x) + 4e^x(-\sin(x))$$

$$= 4e^x(\cos(x) - \sin(x)).$$

Consider the function $H(x) = \sqrt{x}(x^3 + 2x + 1)$. What is $H'(x)$?

a) $H'(x) = \dfrac{1}{2\sqrt{x}}(3x^2 + 2)$

b) $H'(x) = x^{-1/2}(x^3 + 2x + 1) + \sqrt{x}(3x^2 + 2)$

c) $H'(x) = x^{2.5} + 2x^{1.5} + 2x^{0.5}$

d) $H'(x) = \dfrac{1}{2\sqrt{x}}(x^3 + 2x + 1) + \sqrt{x}(3x^2 + 2)$

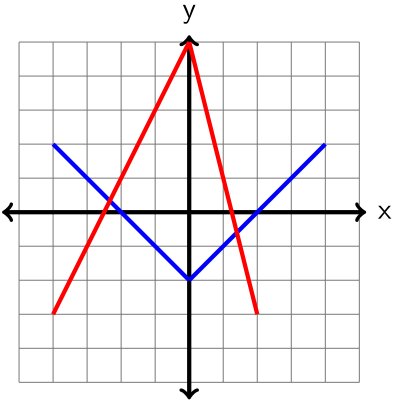
Clicker Question

Consider the function $G(t) = t^2 \tan(t) + 2^t \sin(t)$. What is $G'(t)$?

1. $G'(t) = 2t\tan(t) + t^2\sec^2(t) + \ln(2)2^t\sin(t) + 2^t\cos(t)$
2. $G'(t) = 2t\sec^2(t) + \ln(2)2^t\sin(t) + 2^t\cos(t)$
3. $G'(t) = 2t\tan(t) + t^2\sec^2(t) + \ln(2)2^t\cos(t)$
4. $G'(t) = 2t\sec^2(t) + \ln(2)2^t\cos(t)$

Clicker Question

If $h(x) = f(x)g(x)$, what is $h'(1)$? In the graph $f(x)$ is blue and $g(x)$ is red.



a) Not enough information

b) $h'(1) = 5$

c) $h'(1) = 4$

d) $h'(1) = -4$

Examples

For each of the following calculate the derivative.

1. $x^3(4x^2 + 10x)$
2. $\sqrt{x}e^x$
3. $\sin(x)\cos(x)$