Math 107-Lecture 2

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Warm-up. Review of Lecture 1.

Last time we reviewed definitions and interpretations for the definite and indefinite integral, and also some integration formulas. The connection between differentiation and integration is done through

Theorem (The Fundamental Theorem of Calculus)

Part I *If f is continuous on $[a, b]$ and F is an antiderivative for f on $[a, b]$, then* $\displaystyle\int_a^b f(t)dt = F(b) - F(a)$.

Part II *If f is continuous on $[a, b]$, then* $F(x) = \displaystyle\int_a^x f(t)dt$ *is continuous on* $[a, b]$*, differentiable on* (a, b) *and*

$$F'(x) = \frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$$

Remark: The two statements are equivalent; they basically state that differentiation and integration are processes inverse to each other.

Clicker Question 1 (Review from last time)

Which of the following can be computed directly, i.e. without any special integration techniques?

1. $\displaystyle\int \frac{x^5 - 7}{\sqrt{x}}\ dx$
2. $\displaystyle\int \frac{e^{2x}}{e^{2x} + 6}\ dx$
3. $\displaystyle\int (x^4 - 3x)^2\ dx$
4. $\displaystyle\int \frac{e^{2x} + 6}{e^{2x}}\ dx$

(A) 1 and 3 only; (B) 1 only; (C) 1, 3, and 4 only; (D) all of the above; (E) none of the above.

The Substitution Method

Integration Rule #2: If you can't easily simplify an integral to basic pre-memorized cases, try SUBSTITUTION next.

Key Idea: The Substitution Method is the result equivalent to differentiation with the Chain Rule, but in the integration world.

Recall the Chain Rule:

$$\frac{d}{dx} f\big(u(x)\big) = f'(u(x))u'(x)$$

We can read the above as

$$\int f'(u(x))u'(x)\;dx = f\big(u(x)\big) + C.$$

Typical choices for substitution

$$(\overbrace{\cdots}^{u})^p, \qquad \sqrt{\overbrace{\cdots}^{u}}, \qquad \ln\left(\overbrace{\cdots}^{u}\right), \qquad e^{\left(\overbrace{\cdots}^{u}\right)}, \qquad \sin\left(\overbrace{\cdots}^{u}\right), \qquad \text{etc.}$$

Remarks:

- Make sure after substitution there is only **ONE** variable u (no original "x" or "t", etc., left).
- For an indefinite integral you need to go back to the original variable.

Simplest case: "affine" substitution

Let

$$u = mx + b \quad \text{for } \textbf{constants } m, b.$$

Example 1:

- $\displaystyle\int \cos(3x - 5)dx =$

Example 2:

- $\displaystyle\int \frac{2}{1 + (1 - 4x)^2} dx =$

More examples

Example 3:

$$\int e^{3\sin x}\cos x \; dx$$

Example 4:

$$\int_0^1 (1 + x^{3/2})^{2018}\sqrt{x} \; dx$$

Example 5:

$$\int \frac{\ln(2x)}{x} \; dx.$$

Example 6:

$$\int_{0.1}^{0.5} \frac{e^{1/x}}{x^2} dx$$

Clicker Question 2 (Substitution)

Which of the following can be computed by a substitution?

1: $\displaystyle\int x^{16}(x^{17}+7x)^4\,dx$; 2: $\displaystyle\int x^{16}(x^{17}+16)^4\,dx$;

3: $\displaystyle\int \frac{e^{2x}}{e^{2x}+6}\,dx$; 4: $\displaystyle\int \frac{x}{1+6x^3}\,dx$

(A) 2, 3, and 4 only

(B) 1, 2, and 3 only

(C) 3 and 4 only

(D) 2 and 3 only

(E) All of them

Conclusions

- Today we covered several examples of varying difficulty for integration by substitution (7.1); you worked on 2 clicker problems.
- For next time finish working on the suggested problems from section 7.1.
- Webwork 1 is due on Sunday, 01/13 at 11:59 pm CST.
- For next Tuesday read section 7.2 (Integration by parts) - new material.