Math 107-Lecture 27

Dr. Adam Larios

University of Nebraska-Lincoln

Taylor Polynomials

The Taylor polynomial of degree n for the function $f(x)$ near $x = a$ is the n-th degree polynomial

$$P_n(x) = f(a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)^2}{2!} + f'''(a)\frac{(x-a)^3}{3!} + \cdots + f^{(n)}(a)\frac{(x-a)^n}{n!}.$$

Question: How can we find a Taylor series from a known one? (taking so many derivatives can be overwhelming ...)

Some simple examples (hint: substitution will mostly do it)

Example 1. Find the Taylor series near 0 for the function f below and determine for what values of x it converges.

$$f(x) = \frac{x}{1 - (x/2)}$$

Example 2. Same for g

$$g(x) = \frac{x^2}{2 + x^4}$$

Example 3. How about for h?

$$h(x) = x^3 e^{x^2}$$

Clicker question

What is the Taylor series for the function $f(x) = \dfrac{1}{2-x}$ near $x = 1$?

- $\displaystyle\sum_{n=0}^{\infty}(x-1)^n$
- $\displaystyle\sum_{n=0}^{\infty}(2-x)^n$
- $\displaystyle\sum_{n=0}^{\infty}(1-x)^n$
- $\displaystyle\sum_{n=0}^{\infty}(1+x)^n$
- $\displaystyle\sum_{n=0}^{\infty}(2+x)^n$

We can do more. A lot more!

Recall that inside the interval of convergence we can also multiply, integrate, and differentiate a power series term by term. Using this approach we can find more easily the Taylor series around the origin for many functions.

Examples 3 & 4:

- $f(x) = \ln(1 - x)$
- $g(x) = \arctan x$

Example 5: Find the first four terms of the Taylor series near the origin for

$$\ln(1 - x^2)\arctan(x^3).$$

Using series for approximation of integrals

Recall that

$$\int e^{x^2}\,dx$$

can not be expressed in terms of elementary functions, hence we can not apply the FTC to compute

$$\int_0^1 e^{x^2}\,dx.$$

However we can get a really good approximation (how good?) for this integral by using series.

Clicker question

What are the first four nonzero terms of the Taylor approximation for $x^2 e^{x^4} - x^6$ about $a = 0$?

- $x^2 + x^6 + \frac{x^{10}}{2} + \frac{x^{14}}{6}$
- $1 + x^4 + \frac{x^8}{2} + \frac{x^{12}}{6}$
- $x^2 + \frac{x^{10}}{2} + \frac{x^{14}}{6} + \frac{x^{18}}{24}$
- $1 + x^2 + x^6 + \frac{x^{10}}{2}$
- $x^2 - x^6 + \frac{x^{10}}{2} + \frac{x^{14}}{6}$

Comparisons of functions

Decide which function is larger near 0

$$f(x) = x\sin(x^2)$$

or

$$e^{x^3} - 1.$$