Math 107-Lecture 19

Dr. Adam Larios

University of Nebraska-Lincoln

General Series

Definition [Infinite series] Let (a_n) be a sequence beginning with the index n_0. Then we define

$$\sum_{n=n_0}^{\infty} a_n = \lim_{\mathbf{N}\to\infty} \sum_{n=n_0}^{\mathbf{N}} a_n$$

Examples:

- $$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \ldots$$
- $$\sum_{n=0}^{\infty} \left(\frac{4}{3}\right)^n$$

Convergence/divergence issues

Convergence means that we have a finite sum.
Can we tell if a series converges or diverges without evaluating it?

1. First few terms of a series do not affect convergence or divergence
2. Geometric series $\sum_{n=0}^{\infty} ar^n$ converges if $|r| < 1$ and diverges otherwise.
3. **Divergence test**: If $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=n_0}^{\infty} a_n$ diverges. But if $\lim_{n\to\infty} a_n = 0$ **we can not say anything.**

Clicker question #1

Find

$$\sum_{n=1}^{\infty} (0.1)^n$$

- $\dfrac{10}{9}$
- $\dfrac{1}{9}$
- 0.2
- diverges to ∞
- diverges (neither to $+\infty$, nor to $-\infty$)

More examples

What can you say about the following two series?

$$\sum_{n=0}^{\infty}(-1)^n$$

$$\sum_{n=1}^{\infty}\frac{n+2}{n+3}$$

The Harmonic Series

$$\sum_{N=1}^{1} \frac{1}{N} = 1 \qquad \sum_{N=1}^{2} \frac{1}{N} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\sum_{N=1}^{10} \frac{1}{N} \approx 2.9290 \qquad \sum_{N=1}^{100} \frac{1}{N} \approx 5.1874 \qquad \sum_{N=1}^{1000} \frac{1}{N} \approx 7.4855$$

$$\sum_{N=1}^{10000} \frac{1}{N} \approx 9.7876 \qquad \sum_{N=1}^{100000} \frac{1}{N} \approx 12.090 \qquad \sum_{N=1}^{10^6} \frac{1}{N} \approx 14.393$$

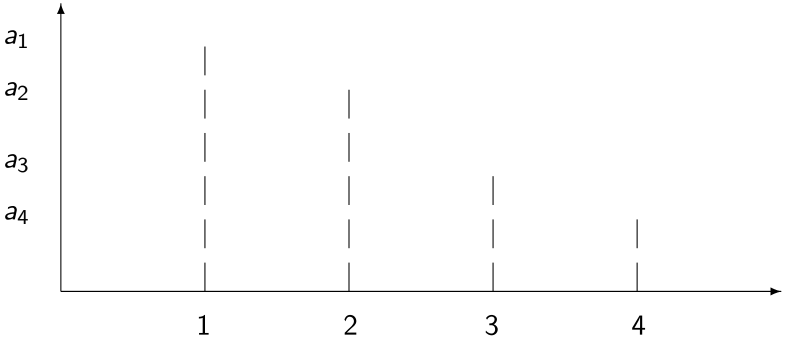
The integral test

Let $\displaystyle\sum_{n=n_0}^{\infty} a_n$ be a series of **non-negative terms**, such that for $f(x)$: a **non-increasing continuous** function s.t. $f(n) = a_n$ for $n \geq c$. Then

- $\displaystyle\int_c^{\infty} f(x)dx$ converges $\Rightarrow \displaystyle\sum_{n=n_0}^{\infty} a_n$ converges.
- $\displaystyle\int_c^{\infty} f(x)dx = \infty$ (diverges to ∞) $\Rightarrow \displaystyle\sum_{n=n_0}^{\infty} a_n$ diverges (to ∞)

In other words, the series will converge only if the integral converges!

How things look geometrically



The integral test in action

Use the integral test to find out if the following series converge or diverge.

- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

Clicker question #2

What would you use to calculate the integral

$$\int \frac{1}{\sqrt{x}\,(\sqrt{x}+1)}dx$$

- Elementary integral
- Regular substitution $u = \ldots$
- Trigonometric subtitution
- Partial fractions
- Integration by parts