Announcements

- The Alternate Online Request Form for Exam 2 closes tomorrow, Tuesday, March 5th at 5pm;
- Today we will cover section 9.1 - Sequences, and possibly start on 9.2 - Geometric Series.
- Thursday's the quiz will cover sections 8.5 and 9.1.
- Exam 2 is next week, Wednesday, 03/13, 6:30–8:00 pm.

Sequences

Definition A **sequence** with parameter n, written (a_n), is a function from the index set $I = \{n_0, n_0+1, n_0+2, n_0+3, \ldots\}$, beginning with some integer n_0, into $\mathbb{R}$.

Examples of sequences:

- with direct definition: for $n \geq 1$ define $a_n = \begin{cases} \frac{1}{n} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$

  So $a_1 = 0,\ a_2 = 1/2,\ a_3 = 0,\ a_4 = 1/4, a_5 = 0, \ldots$
- Recursive definition: $a_0 = 0,\ a_1 = 1,\quad$ and for $n \geq 2$ take

  $$a_n = a_{n-1} + a_{n-2}$$

  This is the Fibonacci sequence with terms

  $$a_0 = 0,\ a_1 = 1,\ a_2 = 1,\ a_3 = 2,\ a_4 = 3, a_5 = 5, a_6 = 8, a_7 = 13, \ldots$$

Limits of sequences

- The number that the sequence a_n approaches as n becomes unbounded (i.e. very, very large) is called **the limit of a_n** and is denoted by $\lim_{n\to\infty} a_n$.
- If $\lim_{n\to\infty} a_n$ exists (is a real number) we say a sequence **converges**. Otherwise we say it **diverges**.

Limits of sequences - Examples

Determine if the following sequences converge or diverge. If they converge, find the limit

- $\lim\limits_{n\to\infty} \dfrac{1}{n}$
- $\lim\limits_{n\to\infty} (-1)^n$
- $\lim\limits_{n\to\infty} (-1)^n \dfrac{1}{n}$
- $\lim\limits_{n\to\infty} n!$
- $\lim\limits_{n\to\infty} \sin(n)$

Clicker question #1

$$\lim_{n\to\infty} (-1)^n\, n =$$

- $+\infty$ (diverges)
- $-\infty$ (diverges)
- 0
- 1
- it does not exist (diverges)

Properties for limits of sequences

1. Limit of sum/multiple/product/ratio = sum/multiple/product/ratio of the limits (no division by 0).
2. If g is continuous at L and $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} g(a_n) = g(L)$.
3. Case: **"Bounded $\times$ convergent to 0"**
   If sequence (b_n) is **<u>bounded</u>** and $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty}(b_n a_n) = 0$
   Example: $\lim_{n\to\infty} \sin(n)\dfrac{1}{n^2} = 0$ **since** $|\sin(n)| \leq 1$ **and** $\lim_{n\to\infty} \frac{1}{n^2} = 0$
4. If there is $f(x)$ such that $f(n) = a_n$ and $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} a_n = L$

Properties for limits of sequences

(V) The "sandwich theorem" applies: If $a_n \leq b_n \leq c_n$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$ then

$$\lim_{n\to\infty} b_n = L$$

(VI) Important limits **(here x is fixed)**

$$(i)\ \lim_{n\to\infty} \sqrt[n]{n} = 1,$$

$$(ii)\ \lim_{n\to\infty} x^{1/n} = 1, \quad (x > 0),$$

$$(iii)\ \lim_{n\to\infty} x^n = 0 \ \text{ if } |x| < 1$$

$$(iv)\ \lim_{n\to\infty} \frac{x^n}{n^n} = 0, \quad x > 0$$

$$(V)\ \lim_{n\to\infty} \frac{n!}{n^n} = 0$$

Clicker question #2

$$\lim_{n\to\infty} \frac{(n-1)! + 3^n}{n!} =$$

- $+\infty$ (diverges)
- 0
- 3
- 1
- Not sure how to justify the answer

More examples of limits

$$\lim_{n\to\infty} \frac{2^n - 1}{3^n}$$

$$\lim_{n\to\infty} \frac{n + 7n^3 + 1}{2n^3 - 9}$$

$$\lim_{n\to\infty} \frac{(n-1)! + 3^n}{n!}$$

Wrapping up:

- For next time read section 9.2; solve the problems from section 9.1.
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