Calculus 1
Measuring Speed

Dr. Adam Larios

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Clicker Question

Suppose we want to find how the volume, V, of a balloon changes as it is filled with air. We know $V(r) = 4/3\pi r^3$, where r is the radius in inches and $V(r)$ is in cubic inches. The expression $\frac{V(3)-V(1)}{3-1}$ represents

1. The average rate of change of the radius with respect to the volume when the radius changes from 1 inch to 3 inches.
2. The average rate of change of the radius with respect to the volume when the volume changes from 1 cubic inch to 3 cubic inches.
3. The average rate of change of the volume with respect to the radius when the radius changes from 1 inch to 3 inches.
4. The average rate of change of the volume with respect to the radius when the volume changes from 1 cubic inch to 3 cubic inches.

Velocity Verses Speed

We distinguish between velocity and speed. Suppose an object moves along a line. One direction is designated as positive and the other negative. Speed is the magnitude of the velocity and so is always positive or zero.

Average Velocity

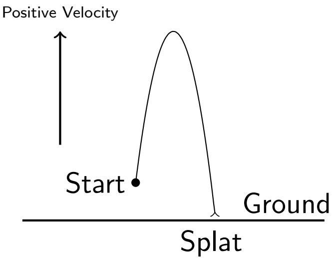
If $s(t)$ is the position of an object at time t, then the average velocity of the object over the interval $a \leq t \leq b$ is

$$\text{Average Velocity} = \frac{\text{Change in position}}{\text{change in time}} = \frac{s(b) - s(a)}{b - a}$$

In words, the average velocity of an object over an interval is the net change in position during the interval divided by the change in time.

A Rising and Falling Object

Consider the speed of a small object (say, a grapefruit) that is thrown straight upward into the air at $t = 0$ seconds. It leaves the thrower's hand at high speed, slows down until it reaches its maximum height, and then speeds up in the downward direction and finally, "Splat!"



t (sec)	0	1	2	3	4	5	6
$y = s(t)$ (feet)	6	90	142	162	150	106	30

A Rising and Falling Object

Average velocity over intervals from the previous table gives valuable information but does not give the velocity of the grapefruit at exactly $t = 1$ second. To get closer to an answer to that question, we look near $t = 1$ in more detail.

Instantaneous velocity

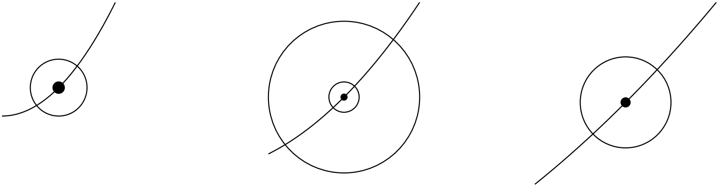
Let $s(t)$ be the position at time t. Then the instantaneous velocity at $t = a$ is defined as

$$\lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$

In words, the instantaneous velocity of an object at time $t = a$ is given by the limit of the average velocity over an interval, as the interval shrinks around a.

How can I visualize this?

Imagine taking the graph of a function near a point and "zooming in" to get a close-up view. The more we zoom in, the more the curve appears to be a straight line. We call the slope of this line the slope of the curve at the point.



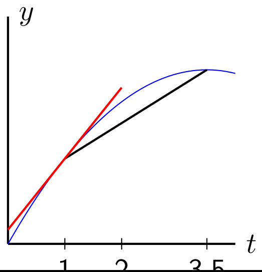
Instantaneous Velocity

Instantaneous velocity

The instantaneous velocity is the slope of the curve at a point.

Average velocity

The average velocity over any time interval $a \leq t \leq b$ is the slope of the line joining the points on the graph of $s(t)$ corresponding to $t = a$ and $t = b$.



Limits and Instantaneous Velocity

Using limits, compute the instantaneous velocity.

In a time of t seconds, a particle moves a distance of s meters from its starting point, where $s = \sin(2t)$. (a) Find the average velocity between $t = 1$ and $t = 1 + h$ if:

$(i) h = 0.1,$

$(ii) h = 0.01,$

$(iii) h = 0.001.$

(b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t = 1$.

Limits and Instantaneous Velocity

h	$\sin(2(1+h))$	$\frac{\sin(2(1+h))-\sin(2))}{h}$
0	0.90930	
0.1	0.80850	-1.00801
0.01	0.90079	-0.85042
0.001	0.90846	-0.83411

An estimate for the instantaneous velocity at $t = 1$ would be -0.83 .