Calculus 1
Exam 3 Review

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Which formula gives the average value of f from a to b for any function f?

1. $\frac{f(b)-f(a)}{b-a}$
2. $\frac{1}{b-a} f(x)$
3. $\frac{1}{b-a} \int_a^b f(x)dx$
4. $\int_a^b f(x)dx$
5. $\frac{f'(b)-f'(a)}{b-a}$

L'Hopitals Rule

Determine whether the limit exists, and where possible evaluate it.

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7. $\lim_{x\to 0} \left(\frac{1}{x^2} - \frac{\cos x}{x^2}\right)$

Parametric Curves

Consider the curve given parametrically by $x(t) = t^2 - 1$, $y(t) = 3t + 1$ for t from $-\infty$ to ∞.

a) Find all the points (x, y) where the graph has either a vertical or a horizontal tangent line.

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a) Find all the points (x, y) where the graph has either a vertical or a horizontal tangent line.
b) Find the slope of the curve $\frac{dy}{dx}$ as a function of t.
c) Find the parametric equation of the tangent line to the curve at the point $(8, 10)$.

Geometry and Integration

Draw an appropriate sketch for

$$\int_{-4}^{0}(x+4)dx+\int_{0}^{4}(-x+4)dx$$

and then evaluate this integral.

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Which of the following is equal to $\frac{d}{dx}\int_2^{x^3} \sin(t^2)dt$?

a) $\sin(x^6)$

b) $\sin(9x^4)$

c) $\sin(t^2)$

d) $\sin(t^2)x^3$

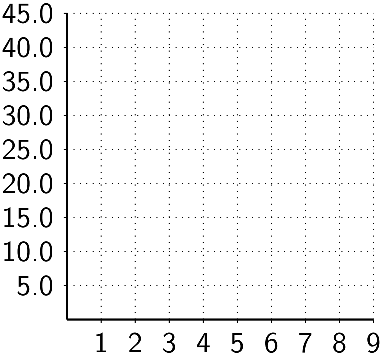
e) $\sin(x^6)3x^2$

Riemann Sums

At time t, in minutes, the velocity v of an object, in feet per minute, is given by the following data:

t	0	1.5	3	4.5	6	7.5	9
$v(t)$	42	38	33	30	25	18	9

Draw the graph of $v(t)$ and draw a left-hand sum to estimate the distance traveled from $t = 0$ to $t = 9$ using $n = 3$ subdivisions. Write down an expression for the sum. Explain if you estimate is an underestimate or an over estimate.



FTC

Compute the following derivative:

$$\frac{d}{dy}\int_3^y (\ln(1+x^2) + cos(3-x))dx,$$

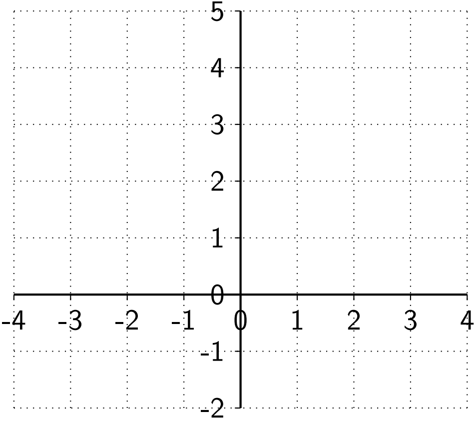
briefly justifying your answer.

Differential Equations

A water balloon is tossed upward from the top of Oldfather hall, at a velocity of 5ft/s. Given that Oldfather Hall is about 150 feet high, when does the balloon hit the group? Recall that $g = -32ft/s^2$. (Ignore air resistance)

Integrals

Sketch the curves $y = 1 + x^2$ and $y = 3 + x$ and find the exact area between them.



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Which parametrization is that of a line?

a) $x = 3t^3$, $y = t^3$

b) $x = \sin t$, $y = -t$

c) $x = |3t|$, $y = t$

d) $x = \cos t$, $y = \sin t$

e) $x = |3t|$, $y = 3t$

Integrals

Compute each of the following, given that $\int_1^6 f(x)dx = 15$, $\int_6^{10} f(x)dx = -3$, and $\int_1^6 g(x)dx = 6$.

a) $\int_1^{10} f(x)dx$

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e) The average value of $f(x)$ in the interval $[1, 10]$

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Integrals

Compute each of the following exactly showing the details of your work.

a) $\int_1^3 (x^2 + \frac{1}{x}) dx$

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b) $\int (\frac{2}{1+x^2} + 5e^x + \sin(x)) dx$

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b) $\int (\frac{2}{1+x^2} + 5e^x + \sin(x)) dx$

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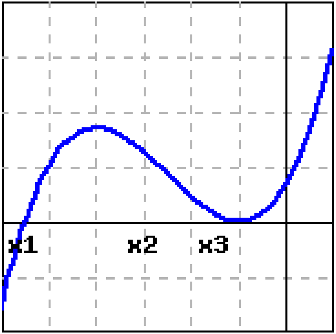
For the graph of $f(x)$ shown below, sketch two functions F with $F'(x) = f(x)$. In one let $F(0) = 0$; in the other, let $F(0) = 1$. Mark x_1, x_2 and x_3 on the x-axis of your graph. Identify local maxima, minima and inflection points of $F(x)$.



(a) At which point does $F(x)$ achieve its largest value?

- A. x_1
- B. x_2
- C. x_3

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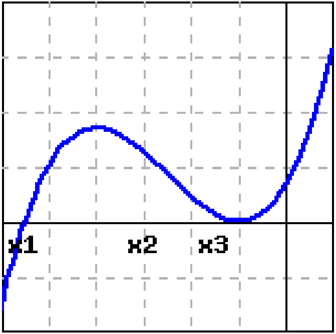

(b) At which point does $F(x)$ achieve its smallest value?

- A. x_1
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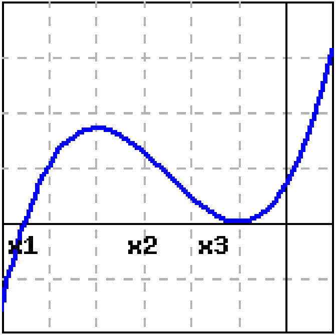

For the following questions, **consider only the interior points on the domain** on which $f(x)$ is shown.

(c) How many critical points does $F(x)$ have?

- A. 0
- B. 1
- C. 2
- D. 3
- E. more than 3

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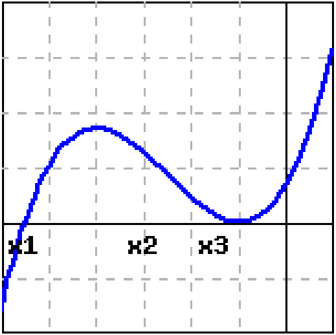

For the following questions, **consider only the interior points on the domain** on which $f(x)$ is shown.

(d) How many local maxima does $F(x)$ have?

- A. 0
- B. 1
- C. 2
- D. 3
- E. more than 3

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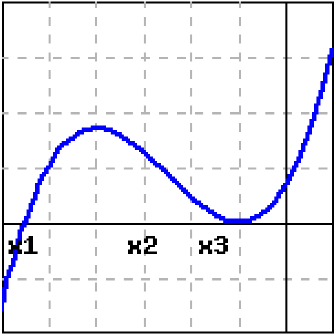

For the following questions, **consider only the interior points on the domain** on which $f(x)$ is shown.

(e) How many local minima does $F(x)$ have?

- A. 0
- B. 1
- C. 2
- D. 3
- E. more than 3

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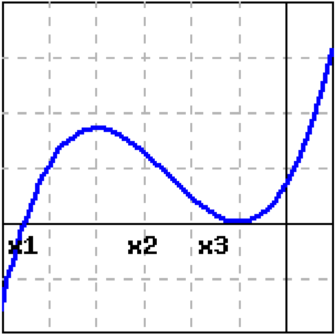

For the following questions, **consider only the interior points on the domain** on which $f(x)$ is shown.

(f) How many inflection points does $F(x)$ have?

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

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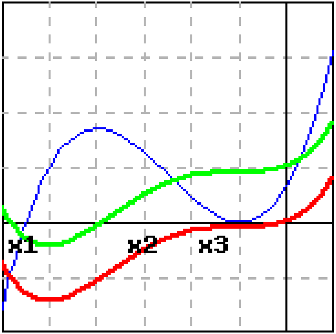

The Correct Answers Are
C,A,C,A,B,C

Solution: The graphs of the two antiderivatives are shown in red and green in the graph below. We can find the critical points on the graph of $F(x)$ by finding where the function $f(x)$ is zero. $F(x)$ has a local maximum or minimum at those critical points where $f(x)$ changes sign. Where $f(x)$ is positive, the derivative of $F(x)$ is positive, so $F(x)$ is increasing (and vice versa, for $f(x)$ negative). We can find inflection points by looking for local maxima and minima of $f(x)$.