Math 107-Lecture 12

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Announcements

- Today we will cover section 8.2 - Area, Volume, Arc length
- The deadline for passing the Gateway exam is Friday, February 22 (2 days away). The Gateway counts for 7% of the grade.
- Make sure you check your grades: clicker, webwork, Gateway, exam scores.

Clarification on WeBWork notation

For next WeBWork homework, to enter the Riemann sum for an area use the Leibnitz notation and write

$$\sum f(y)Dy \quad \text{or} \quad \sum g(x)Dx$$

where $f(y)$ or $g(x)$ are exactly what would appear in the integral for the area. No indices, just the differential notation.
E.g. To approximate the area under the circle of radius 6 centered at the origin above the Ox axis write

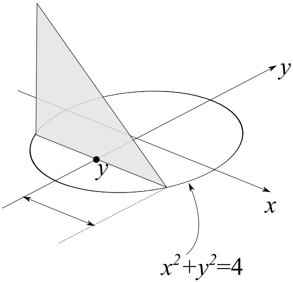
$$\sum \sqrt{6 - x^2}Dx.$$

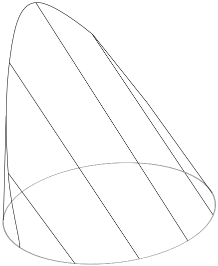
The integral that gives this area would be

$$\int_{-\sqrt{6}}^{\sqrt{6}} \sqrt{6 - x^2}\, dx.$$

Example 1

Set up the integral to find the volume of the shape with circular base of radius 2, whose perpendicular cross-sections are **isosceles** right-triangles, as shown on the picture.





$V = \sum A_y$.

For a cross-section through y : $A_y \approx \dfrac{l(y)^2}{2}$, where $l(y) =$ side of triangle.

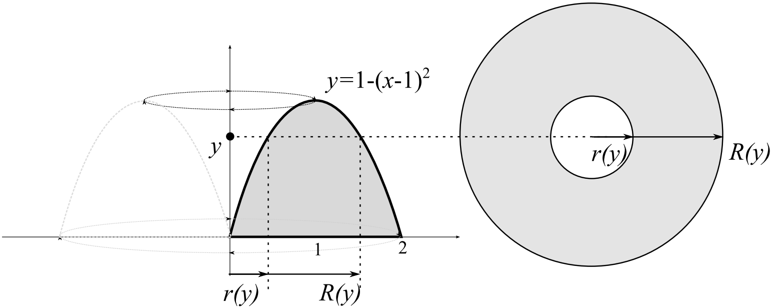
Clicker question #1

What is the volume of the body described in Example 1 above?

A) $\sqrt{4-y^2}$
B) 8
C) 16
D) $8/3$
E) $64/3$.

Computing volumes: Example 2

Find the volume of the “Bundt cake” obtained by rotating the graph of $y = 1 - (x-1)^2$ on $[0, 2]$ about the y-axis



$$V = \text{ stack of annuli } = \int (\text{ area annuli A(y)})dy = \int \pi([R(y)]^2 - [r(y)]^2)dy$$

Arclength

For a short curve, the length ds is approximated through the Pythagorean theorem by

$$(ds)^2 = (dx)^2 + (dy)^2$$

where dx = horizontal change and dy = vertical change.

- Curve given parametrically by $(x(t), y(t))$ from $t = t_0$ to $t = t_1$

$$L = \int_{t_0}^{t_1} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt$$

- Curve given by $y = f(t)$ (i.e. x=t, y=f(t)) from $t = a$ to $t = b$:

$$L = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt = \int_a^b \sqrt{1 + (f'(t))^2}\, dt.$$

Examples for computing the arclength

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Example 3. Compute the length of

$$x(t) = \cos t, \quad y(t) = \sin t$$

for $0 \leq t \leq \pi$.
[Can you guess what this is?]

Example 4

Set up the arclength integral for computing the length of the parabola

$$y = (x-2)^2 - 4,$$

between the points $(4, 0)$ and $(2, -4)$.

Use both options:

- parametrization formula
- curve as a graph of a function

Example 5 and Clicker question #2

To compute the length of the curve that is parametrically given by

$$x(t) = e^t \cos(t), \quad y(t) = e^t \sin(t)$$

inside the integral we should have:

- $\sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2}$
- $\sqrt{e^{2t} \cos^2 t + e^{2t} \sin^2 t} = \sqrt{e^{2t}}$
- $\sqrt{1 + (e^t \sin t + e^t \cos t)^2}$
- $\sqrt{(e^t \cos t)^2 - (e^t \sin t)^2}$
- Not sure...

Wrapping up:

- For next time read section 8.3.
- Solve the problems from section 8.2.
- The deadline for passing the Gateway is this Friday, February 22. The Gateway counts for 7% of the grade.