Math 107-Lecture 15

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Lecture and course feedback

- Recall the significance of the numbers: 0 (extremely poor), 1 (poor), 2 (fair), 3 (good), 4 (very good)
- In the Math department we view everything below 3 as "not going well"
- If you are considering a 0 or 1 on any of the questions, please see one of us **immediately**: me (instructor and Undergraduate Advisor), Allan Donsig (Vice-chair), or Tom Marley (Chair)
- Please be specific in your feedback!

Announcements

- The Alternate Online Request Form for Exam 2 closes on Tuesday, March 5th at 5pm;
- Today we will cover section 8.5 - Applications in Physics; Work.

Varying density in a plate

Example 1. Find the mass of the triangular lamina with vertices $(0, 0)$, $(0, 3)$, $(2, 3)$ given that the density at (x, y) is

$$\rho(x, y) = 2e^x.$$

Solution: Recall that the triangular region is described by $0 \leq x \leq 2, \quad \frac{2}{3}x \leq y \leq 3$. Hence

$$M = \int_0^2 2e^x(3 - \frac{3x}{2})\, dx = 6e^x|_0^2 - 3\int_0^3 xe^x\, dx,$$

which after simplifying yields

$$M = 6e^2 - 6 - (xe^x - e^x)|_0^2 = 3e^2 - 9.$$

Clicker question #1

What are the bounds for (x, y) that describe the triangular region with vertices $(1, 1)$, $(3, 1)$, $(2, 3)$?

A) $1 \leq x \leq 3, \quad 1 \leq y \leq 3$

B) $1 \leq x \leq 3, \quad 1 \leq y \leq 2x - 1$

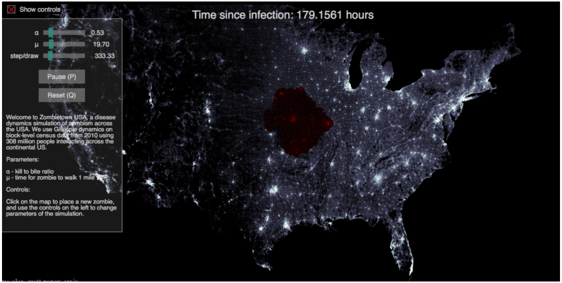
C) $1 \leq x \leq 3, \quad 1 \leq y \leq 7 - 2x$

D) $1 \leq y \leq 3, \quad 1 \leq x \leq \dfrac{7 - y}{2}$

E) $1 \leq y \leq 3, \quad \dfrac{y + 1}{2} \leq x \leq \dfrac{7 - y}{2}$

Integrating on circular areas using rings

Example 2. Something happens in a circular region: e.g. pollutant dispersing, population placed in a city, water burst. Or, we have a zombie infestation starting in a location and spreading in (almost) circular fashion.



Simulation run at: `https://mattbierbaum.github.io/zombies-usa/`

How many zombies do we have?

We have a density of the quantity (pollutant, population, water, zombies) as a function of the distance from the origin, let's say

$$\rho(r) = \frac{1}{r^2+1}, \quad 0 \le r \le R = 5.$$

Find the total amount of the quantity (pollutant, population, water, zombies) contained inside the circle of radius $R(= 5$ here).
Solution. Idea: see what happens on every ring of radius r, $0 \le r \le R$.

$$\begin{aligned} \text{Quantity}_r &= (\text{density on ring} r) \times (\text{“area” of ring } r) \\ &= \rho(r) \times 2\pi r \, \Delta r. \end{aligned}$$

Hence, the total quantity Q is:

$$Q = \int_0^R \rho(r) 2\pi r \, dr.$$

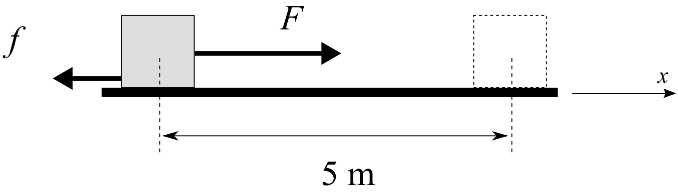
Clicker question #2

In the above example, how many (thousands of) zombies would be in a radius of $R = 5$ (in hundreds of miles) when the density of zombies (in thousands) is

$$\rho(r) = \frac{1}{r^2+1}, \quad 0 \leq r \leq R = 5?$$

- A) $2\pi \arctan 5$
- B) $2\pi(\arctan 5 - \arctan 1)$
- C) $\pi \ln 26$
- D) $2\pi \ln 26$
- E) $2\pi \ln 5$

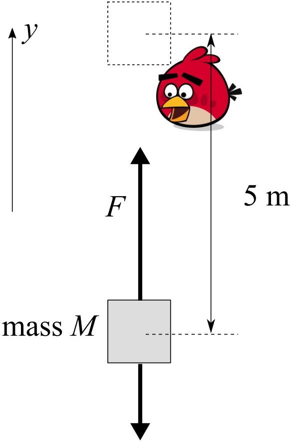
Example 1. A block is pushed by an external force of $3N$ to the right and the friction force it is experiencing is $1N$. Indicate the work performed by: (1) external force F, (2) by the friction force f, and (3) by the net force, over a distance of 5 m.



Recall that for W = work, F = force, and d = distance we have:

$$W = F \cdot d$$

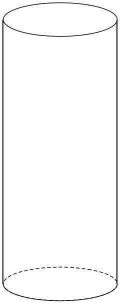
Example 2. Let the force F be 3N directed upward. Indicate the work performed by: (1) the external force F, (2) by the gravity, and (3) by the net force.



Force that varies with the distance

Example 3. How much work is needed to pump the water over the top of a cylindrical tank, 2 meters in radius and 10 meters tall? The density of water is about 1000 kg/m^3.

Note: that less work is needed to pump out the liquid that's near the top of the tank...



Steps for constructing the integral that gives the total work

- Approximate element: disc of height Δx
- Its volume $\pi 2^2 \Delta x = 4\pi \Delta x$ m^3
- Its mass $m = (4\pi\Delta x)\rho = 4,000\pi\Delta x$ Kg
- Force needed to pump water of element at height x is $F(x) = mg = 4,000\pi g \Delta x \approx 39,200\pi\Delta x$ N
- The (approximate) work we perform on the element at height x is $W(x) = F \cdot (10 - x) = 4,000\pi g (10 - x)\Delta x$ J
- Our total work

$$W = \int_0^{10} W(x)dx = 4,000\pi g \int_0^{10} (10 - x)\, dx = 200\pi g \text{ KJ}$$

Above N=Newton, J=Joule, g = gravitational acceleration (≈ 9.8 m/s^2).

A cosmic view

The gravitational force on a 1 kg object at distance r meters from the center of the earth is given by the formula

$$\frac{4 \cdot 10^{14}}{r^2}\ N \quad \text{(towards the earth)}$$

Find the work done in moving the object from the surface of the earth to a height of 10^6 meters above the surface. The radius of the earth is about $6.4 \cdot 10^6$ meters.

Wrapping up:

- For next time read section 8.5; solve the problems from section 8.4.