Math 107-Lecture 8

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Announcements

- Exam 1 is next Wednesday, Feb. 6, 6:30-8:00 pm. All requests for alternate exams have been handled (if you did not receive a reply, this means that your request was granted and you can take the alternate exam).

Plan for today

1. Review improper integrals.
2. Comparison tests for improper integrals (Section 7.7).

Improper Integrals

Type I Integrals:

$$\int_a^\infty f(x)dx = \lim_{N\to\infty} \int_a^N f(x)dx$$

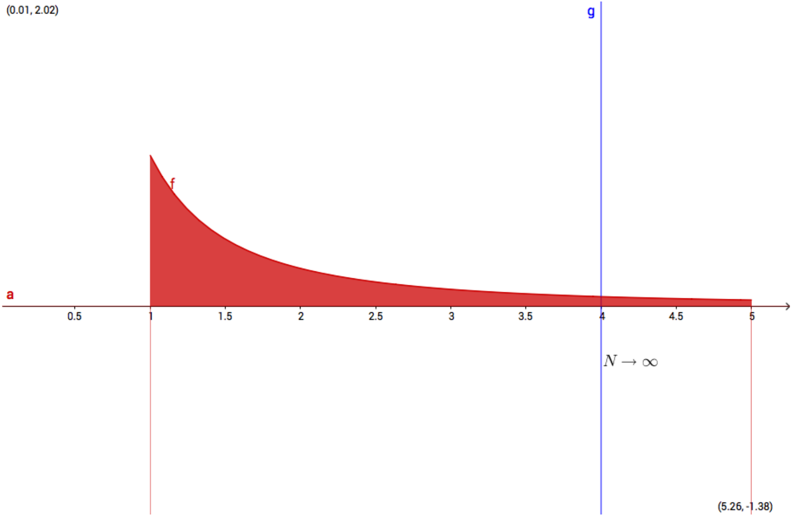
Type II Integrals: for f such that $\lim_{x\to a} f(x) = \pm\infty$ we define

$$\int_a^b f(x)dx = \lim_{t\to a+} \int_t^b f(x)dx$$

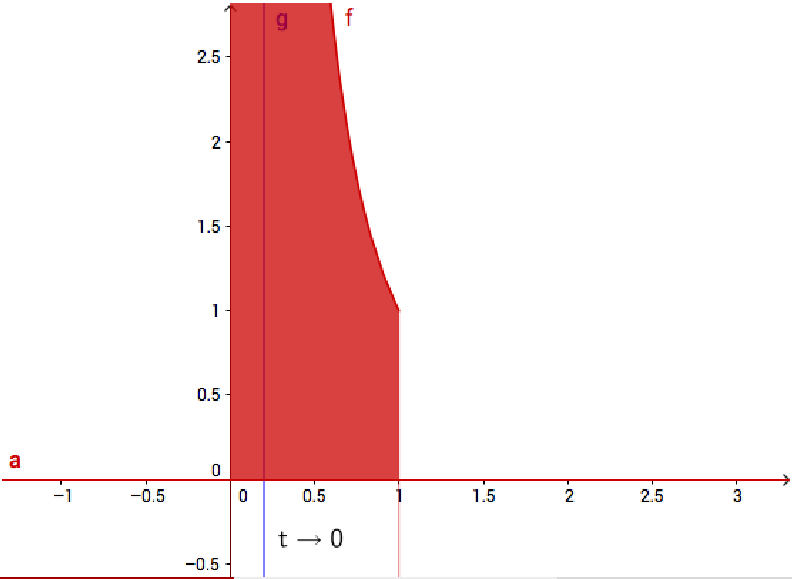
The improper integral:

- converges (C) if the limit exists and it is finite;
- diverges (D) if the limit does not exist/approaches infinity.

Stretching horizontally to infinity. Type I integrals



Stretching vertically to infinity. Type II integrals



Clicker question #1

What can you say about

$$\int_{-2}^{4} (1-x)^{-2/3}\,dx = \underbrace{\int_{-2}^{1} (1-x)^{-2/3}\,dx}_{A} + \underbrace{\int_{1}^{4} (1-x)^{-2/3}\,dx}_{B} =: C$$

- Integral A converges, B diverges, so integral C diverges
- Integral A diverges, B converges, so integral C diverges
- Both integrals A and B are infinite, but they cancel each other, so integral C is convergent
- Both A and B integrals are convergent, so C is convergent
- We can't compute the limits for this improper integral.

(Difficult) Improper integrals

Decide if $\int_2^\infty \frac{1}{x^4}dx$ converges [Yes, why?]

How about $\int_2^\infty \frac{1}{x^4+1}\,dx$?

Intuition: the functions $\frac{1}{x^4}$ and $\frac{1}{x^4+1}$ are very close to each other, so the second integral *should* converge also.

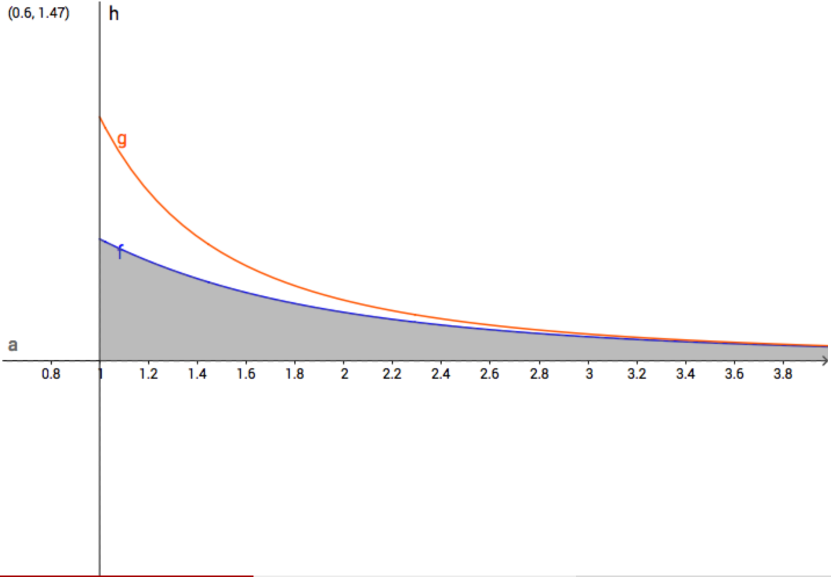
Does $\int_2^\infty \frac{1}{x^4+1}\,dx$ converge? How? Why?

The direct comparison test (DCT)

Assume that $0 \leq f(x) \leq g(x)$ for all $x \in (a, b)$, where a, b could be finite or infinite. We have the following:

- If $\displaystyle\int_a^b g(x)\,dx < \infty$ (C) then $\displaystyle\int_a^b f(x)\,dx < \infty$ (C) [because it is smaller]
- If $\displaystyle\int_a^b f(x)\,dx$ (D) then $\displaystyle\int_a^b g(x)\,dx$ (D) [because it is larger]

Graphic visualization of DCT



Examples for DCT

Example 1. Show that

$$\int_{\pi}^{\infty} \frac{2+\sin x}{x^{3/2}} dx$$

is convergent.

Example 2. Show that

$$\int_{2}^{\infty} \frac{1}{\ln x}\, dx$$

is divergent.

The limit comparison test (LCT)

Assume that $f(x), g(x) \geq 0$ for all $x \in (a, \infty)$ and

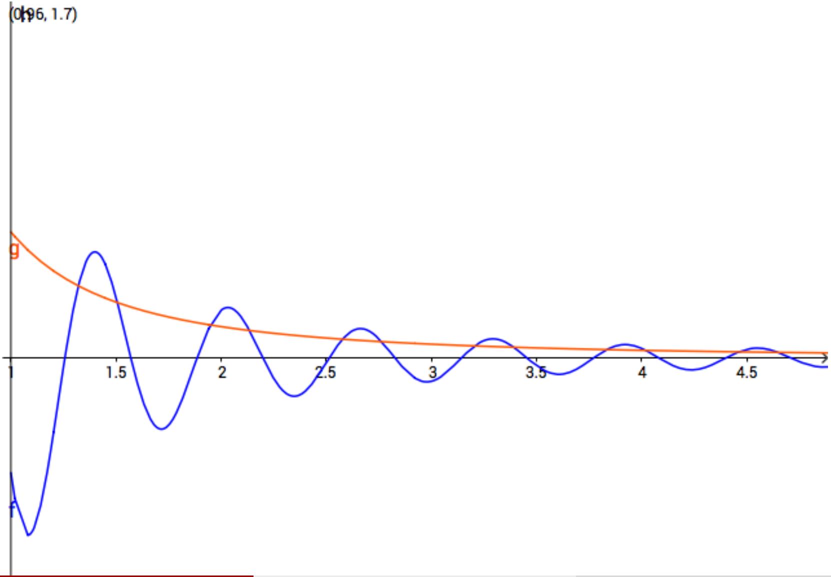
$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L, \qquad \text{for some } 0 < L < \infty$$

then

$$\int_a^\infty f(x)\,dx \quad \text{and} \quad \int_a^\infty g(x)\,dx$$

either both converge or both diverge.

Graphic visualization of LCT



Examples for LCT

Example 1. Show that

$$\int_2^\infty \frac{x-2}{x^{5/2}}\,dx$$

is convergent.

Example 2. Show that

$$\int_0^\infty e^{1/x}\,dx$$

is divergent.

Clicker question #2

Use the comparison test to decide which of the following integrals converge:

$$A := \int_{-\infty}^{-1} \frac{1}{5+x^4}\,dx, \quad B := \int_0^1 \frac{x^4+2}{(x-1)^3}\,dx, \quad C := \int_0^\infty \frac{1-e^{-x}}{x^2+1}\,dx.$$

- A and B are convergent, C is divergent
- A is convergent, B and C are divergent
- A and C are convergent, B is divergent
- they are all divergent
- they are all convergent

Wrapping up:

- Today we covered comparison tests for Improper Integrals (7.7).
- For next time finish working all suggested problems from section 7.7.
- Quiz covering improper integrals will be given tomorrow in recitation.