Calculus 1
Exam 1 Review

Find the derivative of

$$f(x) = \frac{e^x \sin(x)}{1 + x^2}$$

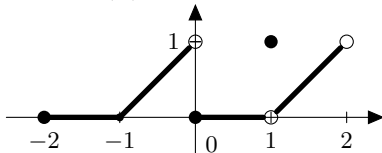If $r'(x) = \dfrac{e^x \sin x - e^x \cos x}{(\sin x)^2}$, what is $r(x)$?

If t is the number of years since 2003, the population $P(t)$ of China, in billions, can be approximated by the function

$$P(t) = 1.29(1.006)^t$$

1. Find $f(6)$ and $f'(6)$ to three decimal places.
2. What do these numbers tell us about the population of China?

Use the graph of $f(x)$, provided below, to answer the

questions.

(a) Is f(x) continuous at $x = 0$?

(b) Is f(x) continuous as $x = 1$?

(c) Is $f(x)$ differentiable at $x = -1$?

(d) Is $f(x)$ continuous at $x = \frac{1}{2}$?

(e) What is $f'\left(\frac{3}{2}\right)$?

(f) What is $\lim_{x \to 1/2} f(x)$?

(g) What type of discontinuity exists at $x = 0$?

(h) What type of differentiation problem exists at $x = -1$?

A laboratory study investigated the relationship between diet and weight in adults found that the weight of a subject, W, in pounds, was a function, $W = f(c)$ of the average number of calories per day, c, consumed by the subject.

1. Interpret the follows statements in term of weight and diet.
   1. $f(1800) = 155$
   2. $f'(2000) = 0$
   3. $f^{-1}(162) = 2200$
2. What are the units of $f'(c)$?

Let $f(x) = \dfrac{x^2 - x - 6}{x^2 - 9}$.

1. $\lim_{x \to 3} f(x) = $?
2. Is $f(x)$ continuous at $x = 3$?
3. Give $f(3)$ a definition, so that $f(x)$ is a continuous function at $x = 3$.
4. Is this new function continuous everywhere?

Use the table of values for $f(x)$ and $g(x)$ below to compute
the derivatives below. Clearly show your work.

x =	-2	-1	0	1	2
f(x) =	3	2	4	3	2
f'(x) =	1	2	5	2	5
g(x) =	2	3	-2	-1	5
g'(x) =	0	2	3	2	5

1. If $h(x) = f(x) \cdot g(x)$, compute $h'(-1)$.
2. If $h(x) = \dfrac{f(x)}{g(x)}$, compute $h'(1)$.

Find the equation of the line perpendicular to the tangent line to the curve $f(x) = x^3 - 3x^2 + 5$ at $x = 1$. (This line is called the normal line.)

(a) $y = -3x + 6$

(b) $y = -\dfrac{1}{3}x + \dfrac{10}{3}$

(c) $y = -3x$

(d) $y = \dfrac{1}{3}x + \dfrac{8}{3}$

(e) $y = 3x$

Let $f(x) = x - x^2$. Use the **<u>limit definition</u>** of the derivative to find $f'(x)$.

Find the derivative of the following functions. **Do not simplify your answers.**

1. $f(x) = \dfrac{e^x - \cos x}{x^2 + 2}$.

2. $g(x) = [3\sin(x^2) + e^x]^5$

By July 1st of 1915, the United States population was 100 million. By July 1st of 1968, the United States population was 201 million.

(a) Assuming the population of the United States grows exponentially, find a formula for this growth of the form $P(t) = P_0 e^{kt}$, where t is the number of years since July 1st, 1915.

(b) By July 1st of what year will the US population reach 400 million, according to your formula?

Let $R = f(a)$ be the revenue, in dollars, that a company makes from spending a dollars on advertising. Using complete sentences, answer the following questions.

(a) What does the sign of $f'(a)$ tell you about the company's advertising?

(b) What does it mean to write $f'(100,000) = 1/2$?

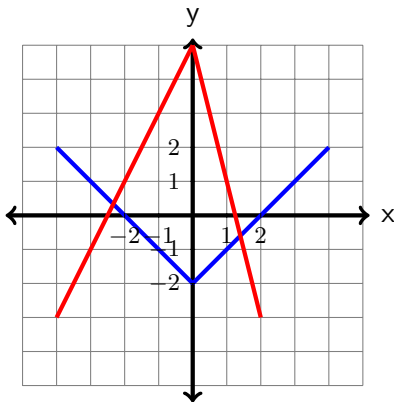(c) If $f'(100,000) = 1/2$, would you recommend changing the advertising budget? Why?

Find the value of k that make the following function
continuous.

$$f(x) = \begin{cases} kx - 1 & , \text{ for } x < 2 \\ (3 - x)^2 + 1, & \text{ for } x \geq 2 \end{cases} \qquad (1)$$
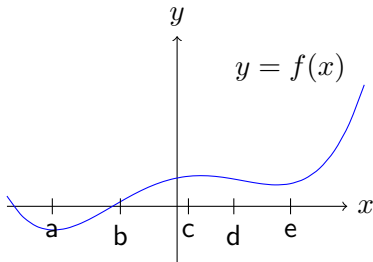
(a) $k = 1$

(b) $k = 3$

(c) $k = 2$

(d) $k = 1.5$

(e) None of the above

In the graph below, $f(x)$ is blue and $g(x)$ is red.

(a) If h(x) = f(x)g(x), compute $h'(1)$.

(b) If $k(x) = g(x)/f(x)$, compute $k'(2.5)$.

(c) If $r(x) = f(g(x))$, compute $r'(2)$.

Find an interval for which:

(a) $f'(x) > 0$

(b) $f(x)$ is increasing

(c) $f(x)$ concave down

(d) $f''(x) > 0$

Find all the points for which $f'(x) = 0$

Find the equation of the tangent line to the graph of the following function at $x = -1$.

$$f(x) = x^5 + 3x^2 - 4$$

(a) $y = -(x+1) - 2$

(b) $y = -x + 2$

(c) $y = x + 2$

(d) $y = -11x - 2$

(e) $y = -x$

Using the definition of the derivative in terms of a limit, find
the derivative of $f(x) = x^2 - 1$.