Calculus 1
The Derivative at a point

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Average Versus Instantaneous Rate of Change

The average rate of change of f over the interval from a to $a+h$ is given by

$$\frac{f(a+h)-f(a)}{h}$$

The Derivative

The derivative of f at a, written $f'(a)$, is defined as

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$

If the limit exists, then f is said to be differentiable at a.

To emphasize that $f'(a)$ is the rate of change of $f(x)$ as the variable x changes, we call $f'(a)$ the derivative of f with respect to x at $x=a$. When the function $y=s(t)$ represents the position of an object, the derivative $s'(t)$ is the velocity.

Clicker Question

Suppose we want to find how the volume, V, of a balloon changes as it is filled with air. We know $V(r) = 4/3\pi r^3$, where r is the radius in inches and $V(r)$ is in cubic inches. Which of the following represents the instantaneous rate at which the volume is changing when the radius is 1 inch?

a) $\frac{V(1.01)-V(1)}{0.01} = 12.69\text{in}^3$

b) $\frac{V(0.99)-V(1)}{-0.01} = 12.44\text{in}^3$

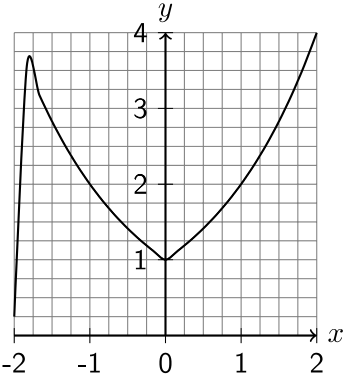
c) $\lim\limits_{h\to 0} \frac{V(1+h)-V(1)}{h}\text{in}^3$

d) All of the above

Visualizing the Derivative:

The derivative at point A can be interpreted as:

- The slope of the curve at A.
- The slope of the tangent line to the curve at A.



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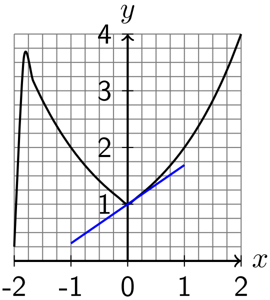


h	2^h	2^0	$\frac{2^h-1}{h}$
-0.0003		1	
-0.0002		1	
-0.0001		1	
0	1	1	Undefined
0.0001		1	
0.0002		1	
0.0003		1	

Complete the table on your own a

Clicker

For the function $g(x)$ shown below, arrange the following numbers in increasing order.

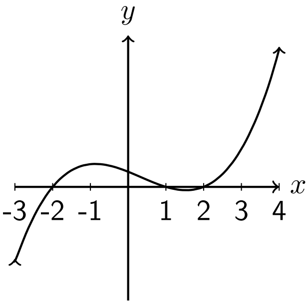
a) 0

b) $g'(-2)$

c) $g'(0)$

d) $g'(1)$

e) $g'(3)$



Computing the Derivative Algebraically

Find the derivative of $f(x) = 1/x$ at the point $x = 2$

Solution: Solution The derivative is the limit of the difference quotient, so we look at

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} \\ &= \lim_{h\to 0} \frac{2-(2+h)}{2h(2+h)} = \lim_{h\to 0} \frac{-h}{2h(2+h)} \end{aligned}$$

Since the limit only examines values of h close to, but not equal to, zero, we can cancel h. We get

$$f'(2) = \lim_{h\to 0} \frac{-h}{2h(2+h)} = \frac{-1}{4}$$

Thus, $f'(2) = ?1/4$.

Clicker

Let $f(x) = x|x|$. Then $f(x)$ is differentiable at $x = 0$.

a) True

b) False

Hint: Work out the limit yourself and see!