# Calculus 1
# The Mean Value Theorem (page 96)

October 2, 2017

# A Relationship Between Local and Global: The Mean Value Theorem

## The Mean Value Theorem

Let $f$ be a continuous function on the interval $[a, b]$ and differentiable on the interval $(a, b)$, then there exists a number $c$, with $a < c < b$, such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

In other words, $f(b) - f(a) = f'(c)(b - a)$.

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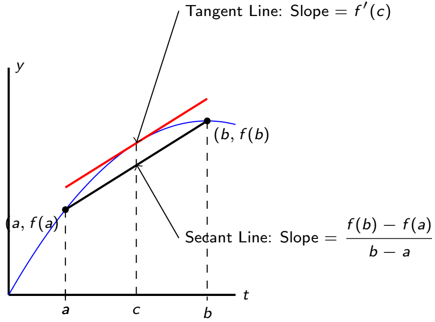
To understand this theorem geometrically, look at the figure. Join the points on the curve where $x = a$ and $x = b$ with a secant line and observe that the slope of secant line $= (f(b) - f(a))/(b - a)$.
Notice that there appears to be at least one point between a and b where the slope of the tangent line to the curve is precisely the same as the slope of the secant line.



# Clicker

Given the function $f(x) = x^3 + 2x^2 - 4x$, calculate $\dfrac{f(b) - f(a)}{b - a}$ when $a = -1$ and $b = 2$.

a) 1

b) 2

c) 3

d) 7

e) $\frac{7}{3}$

## Clicker

Given the function $f(x) = x^3 + 2x^2 - 4x$, calculate $f'(c)$.

a) $3x^2 + 2x - 4$

b) $3c^2 + 2c - 4$

c) $3x^2 + 4x - 4$

d) $3c^2 + 4c - 4$

Given the function $f(x) = x^3 + 2x^2 - 4x$ find the value of $c$ that satisfies the Mean Value Theorem on the interval $[-1, 2]$

a) $c = -2$

b) $c = 2$

c) $c = 2/3$

d) $c = 2/3$ and $2$

e) $c = -2$ and $2/3$

This theorem proves our geometric interpretation of relationship between the derivative?s sign and the increasing/decreasing property of the function.

### The Increasing Function Theorem

Suppose that $f$ is continuous on $a \leq x \leq b$ and differentiable on $a < x < b$.

- If $f'(x) > 0$ on $a < x < b$, then $f$ is increasing on $a \leq x \leq b$.
- If $f'(x) \geq 0$ on $a < x < b$, then $f$ is non-decreasing on $a \leq x \leq b$.

This theorem verifies what we know about a constant function ($f(x) = k$).

### The Constant Function Theorem

Suppose that $f$ is continuous on $a \leq x \leq b$ and differentiable on $a < x < b$. If $f'(x) = 0$ on $a < x < b$, then $f$ is constant on $a \leq x \leq b$.

# The Racetrack Principal

## The Increasing Function Theorem

Suppose that $g$ and $h$ are continuous on $a \le x \le b$ and differentiable on $a < x < b$, and that $g'(x) \le h'(x)$ for $a < x < b$.

- If $g(a) = h(a)$, then $g(x) \le h(x)$ for $a \le x \le b$.
- If $g(b) = h(b)$, then $g(x) \ge h(x)$ for $a \le x \le b$.

To understand this theorem think of a 100 meter race between Gary (g) and Harry (h). Here a = 0 and b = 100. Note that Harry is always faster than Gary. There are two scenarios to consider: They both are together at the start or they are both together at the finish.

If they both start together then Harry will always be ahead of Gary throughout the race because Harry is faster.

If they both finish together then Harry was behind Gary throughout the race until Harry caught up with Gary at the end of the race.

# Clicker

Which theorem would help you answer the following question?

If $f$ is differentiable on $[0, 1]$ and $f(0) < f(1)$, then there exists a $c$ in the interval $[0, 1]$ such that $f'(c) = \frac{f(1)-f(0)}{1-0}$.

a) The Racetrack Principle
b) The Constant Function Theorem
c) The Increasing Function Theorem
d) The Mean Value Theorem

# Clicker

A function intersects the x-axis at points a and b, where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function? There exists some point on the interval $(a, b)$ where

a) The slope is zero and the function has a local maximum.

b) The slope is zero but there is no local maximum.

c) There is a local maximum but there does not have to be a point at which the slope is zero.

d) None of the above have to be true.

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(d). The function $f(x) = 1/x^2?1$, where $a =?1$ and $b = 1$ is an example where none of the choices, (a), (b), or (c) are valid.

A function intersects the x-axis at points a and b, where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function for which the limit of the function exists at every point? There exists some point on the interval $(a, b)$ where

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d. An inverted absolute value function with a removable discontinuity at the tip is an example of a function for which none of the choices, (a), (b), or (c) are valid.

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A continuous function intersects the x-axis at points a and b, where $a < b$. The slope of the function at a is positive and the slope at b is negative. Which of the following is true for any such function for which the limit of the function exists at every point? There exists some point on the interval $(a, b)$ where

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c), The Extreme Value Theorem guarantees there will be a maximum.