Math 107-Lecture 3

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Announcements

- Practice Problems for the Gateway Exam are now posted. Their grades do NOT count towards your grade (only the homework grades in webwork will be imported into Canvas). Soon, practice exams will be posted (all problems included in one exam).

Plan for today

- Review of the substitution method
- Integration by parts
- Partial fractions (if time)

Review of the Substitution Method

In the differentiation framework we have the Chain Rule:

$$\frac{d}{dx} f\big(u(x)\big) = f'(u(x))u'(x)$$

Its equivalent in the integration framework is the substitution method:

$$\int f'(u(x)) \cdot u'(x)\ dx = f\big(u(x)\big) + C.$$

Example:

$$\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}} dx = \int e^u du = e^u + C = e^{\arcsin x} + C$$

where $u = \arcsin x$ and $du = \dfrac{1}{\sqrt{1-x^2}}$.

Integration by Parts

Recall the product rule

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).$$

In other words,

$$\begin{aligned}\int (f'(x)g(x) + f(x)g'(x))dx &= \int f'(x)g(x)dx + \int f(x)g'(x)dx \\ &= f(x)g(x) + C\end{aligned}$$

Or, rearranging the terms,

$$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$

(A constant will come out of the other intergrals anyway, so we don't need to include it here.) For definite integrals we have:

$$\int_a^b f'(x)g(x)dx = f(x)g(x)\big|_a^b - \int_a^b f(x)g'(x)dx$$

vdu, *udv* Formulas

Most often the integration by parts formula is presented as

$$\int \underbrace{u \;\; dv}_{u(x)v'(x)dx} = uv - \int \underbrace{v \;\; du}_{v(x)u'(x)dx}$$

Remark: The derivative jumps from one function to the other!
Integration Rule #3: Try integration by parts as the **last resort**.
Remember that the integrand must look like a **product** of the form $u(x) \cdot v'(x)dx$.

Examples. This is how the method works.

Example 1. $\displaystyle\int xe^x dx = ?$

1. Split the **integrand** (the function inside the integral) into a **product** $u(x) \cdot \overbrace{[\text{what's left}]}^{\text{“}v'(x)\text{”}}$

2. Decide what should be u and what should be dv (or v'). In this case, we want to move the derivative **onto** x so take $u = x$, $dv = e^x dx$

3. Find du and $v(x)$. We will have $du = dx$, $v = e^x$

4. Perform integration by parts

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C.$$

5 Check the result (optional): $\frac{d}{dx}(xe^x - e^x + C) = xe^x - e^x + e^x = xe^x$.

Examples (cont.)

Example 2. $\int \ln(x)dx$

1. We do not see a product, so we artificially write: $\ln x = 1 \cdot \ln x$
2. Take $u = \ln x$ and $dv = 1 \cdot dx$
3. We have $du = \frac{1}{x}dx$ and $v = x$.
4. Apply the substitution by parts formula.

Clicker Question #1

Compute

$$\int \ln(x)dx$$

- $\displaystyle\int \ln(x)dx = \ln(x) - x + C$
- $\displaystyle\int \ln(x)dx = \ln(x) + C$
- $\displaystyle\int \ln(x)dx = x\ln(x) + C$
- $\displaystyle\int \ln(x)dx = x\ln(x) - x + C$
- $\displaystyle\int \ln(x)dx = \frac{\ln(x)^2}{2} + C$

Examples (cont.)

Example 3. $\displaystyle\int \frac{(\ln x)}{x^2}\, dx$

1. What are the two terms in the product?
2. How should we choose the factors u, dv in the product? Why?
3. Compute du and v
4. Apply integration by parts formula.

Clicker Question #2

How can we find these antiderivatives?

$$I : \int x \sin(x^2)\, dx \qquad II : \int x \sin(x)\, dx$$

- both by substitution
- both by parts
- I by substitution and II by parts
- I by parts and II by substitution
- We can't do II by either parts or substitution

Partial Fractions

Motivation: Polynomials are **straight-forward** to integrate, how about rational functions? **Rational functions** are quotients of two polynomials, e.g.

$$\frac{x+2}{x^2-3}, \quad \frac{x+2}{x^2-3}, \quad \frac{x+2}{x^2-3}.$$

But NOT

$$\frac{\sqrt{x}}{x^2-3}, \quad \frac{x+2}{x^{3/2}-3}.$$

Idea: Write the rational functions in terms of very simple fractions (called partial fractions) which we know how to integrate.

Simple Example:

$$\frac{1}{x^2-1} = \frac{1/2}{x-1} - \frac{1/2}{x+1}$$

And we know how to integrate both terms on the right hand side. We will learn how to do this splitting next time in the **"Partial Fractions"** section.

Wrapping up:

- Today we covered Integration by Parts, the equivalent of the product rule for integration.
- For next time finish working on the suggested problems from section 7.2.
- Webwork 2 is due on Monday, 01/22 at 10:00 pm CST.
- For next Wednesday read section 7.3 (Integration by partial fractions); try solving some problems with it.