Math 107-Lecture 20

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Announcements

- Today - Review for Exam 2.
- Exam 2 is on Wednesday, March 13 (tonight), 6:30–8:00 pm.

Material on the exam

- Polar coordinates: conversion to Cartesian and vice-versa; intersections of curves; area formula in polar coordinates;
- Volumes - in Cartesian coordinates; Riemann sums (slices); Washer method (revolution bodies);
- Work through definite integrals;
- Sequences (Not strongly emphasized.) - recursive formulas, convergence, divergence.
- Geometric series: sums and convergence criteria; integral test.
- Section 9.4 is not on Exam 2, in particular, the Limit Comparison Test and other tests from Section 9.4 will not appear.

Areas, volumes, curve lengths

Problem 1. Consider the region in the first quadrant of the plane between $x = 1$ and $x = 2$, under the graph of $y = 3 - (x - 1)^2$. Write down the integral for the volume of the solid obtained by revolving this region around:

- the line $y = 4$.
- the line $x = 0$

Problem 2. Set up the integral, but do not compute the length of the curve given by

$$x(t) = e^t \sin(2t),\ y(t) = e^t \cos(2t)$$

for $t \in [0, \pi]$.

Polar coordinates

Problem 3. Consider the region in the first quadrant of the plane bounded by the spiral $r = 2\theta$, the circle $r = 2$, and the Ox axis.

- Find the intersection point(s) between the circle and the spiral.
- Write down and evaluate an integral for the area described above.

Problem 4. Set up, but do not evaluate, a definite integral which gives the area outside the circle $r = 3$ and inside the cardioid $r = 2 + 2\sin\theta$. Include in your work a sketch of the region whose area the integral computes.

Work

Problem 5. A mountain climber is about to haul up a 50 m length of hanging rope. How much work will it take if the rope weighs 0.624 N/m?

Problem 6. A rectangular swimming pool 50 ft long, 20 ft wide, and 10 ft deep is filled with water to a depth of 9 ft. Set up, but do not evaluate, an integral which gives the work required to pump all the water out over the top. (Take as the density of water 62.4 lb/ft^3.)

Remark: Be careful about units! In work problems lb (pound) usually denotes weight, i.e. force, rather than mass. In a work problem with units given in pounds (lbs) you do not need to add g = gravitational acceleration.

Clicker question #1

Does the sequence below converge or diverge?

$$s_0 = 2, \quad s_{n+1} = n + \cos(\pi s_n),\ n \geq 0.$$

(A) diverges (with alternating behavior)

(B) diverges ($\lim_{n\to\infty} s_n = \infty$)

(C) converges to 0

(D) converges to $\frac{3}{4}$

(E) don't know.

Clicker question #2

What can you tell about the series

$$2x^2 - 4x^4 + 8x^6 - 16x^8 + \ldots$$

(A) it is divergent for all $x \in \mathbb{R}$ except for $x = 0$

(B) it is convergent for $|x| < 2$ with the sum is $\dfrac{2x^2}{1+2x^2}$

(C) it is convergent for $|x| < \dfrac{1}{2}$ with the sum is $\dfrac{1}{1+2x^2}$

(D) it is convergent for $|x| < \dfrac{1}{\sqrt{2}}$ with the sum is $\dfrac{-2x^2}{1+2x^2}$

(E) it is convergent for $|x| < \dfrac{1}{\sqrt{2}}$ with the sum is $\dfrac{2x^2}{1+2x^2}$