Calculus I - Lecture 3
Friday, August 25, 2017

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Announcements

- Please submit all WebWork questions through WebWork (not through email).
- Slides are now posted on Canvas.
- The syllabus was updated (the schedule only).
- If you did not pass the CRA yet, please email your TA or me to help you design a plan to be successful with this test (and with the course!)
- Today: finish up some examples on exponentials, review inverses, logarithmic functions, and trigonometric functions.

Clicker Question

During 1988, Nicaragua's inflation rate averaged 1.3% a day. Which formula below represents the above statement? (Assume t is measured in days.)

(A) $I(t) = I_0 e^{0.013t}$

(B) $I(t) = I_0 (1.013)^t$

(C) $I(t) = I_0 (1.013)t$

(D) $I(t) = I_0 (1.3)^t$

(E) $I(t) = I_0 e^{1.3t}$

Inverses

Invertible

A function has an inverse if (and only if) its graph intersects any horizontal line at most once. If a function has an inverse, we say it is invertible.

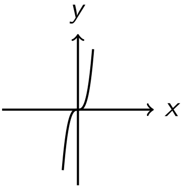
Inverse Function

If the function f is invertible, its inverse is defined as follows $f^{-1}(y) = x$ means $y = f(x)$.

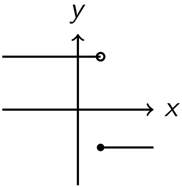
Clicker Question

Which of the following could be graphs of functions that have inverses?

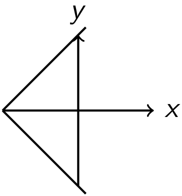
a)



c)



b)



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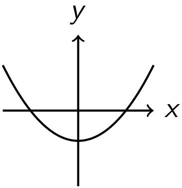

The inverse function. The logarithmic function

Inverse functions. If for each y in the range of f there exists exactly one value of x such that $f(x) = y$, then f has an inverse at y denoted by f^{-1} such that

$$f(x) = y \quad \iff \quad f^{-1}(y) = x$$

Hence

$$f(f^{-1}(y)) = y \quad \text{and} \quad f^{-1}(f(x)) = x.$$

The **inverse** of the exponential function $f(x) = e^x$ is the natural **logarithmic** function $f^{-1}(x) = \ln x$ so we have

$$\ln x = c \iff e^c = x$$

We also have the useful identities

$$\ln e^c = c \quad e^{\ln x} = x.$$

Page 30 in the textbook has some of the properties of the logarithmic function. Be sure to review them carefully!

Example of population growth problem

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Solution. We would like to use the natural base model so

$$P(t) = P(0)e^{kt} \implies P(0) = P_0 = 20.$$

The growth rate can be computed from

$$P(1) = P_0 e^k \implies \frac{P(1)}{P(0)} = e^k = 1 + 0.3 = 1.3$$

Hence, $k = \ln(1.3) \approx 0.2624$, so

$$P(t) = 2e^{.2624t}.$$

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From the model equation we have

$$\frac{P(t+1)}{P(t)} = \frac{P(0)e^{k(t+1)}}{P(0)e^{kt}} = e^k = 0.65 \implies P(t) = P_0(0.65)^t.$$

Example of half-life problem (solution continued)

We need to find t_h (half-life) such that $P(t_h) = \dfrac{P_0}{2}$ hence

$$P_0(0.65)^{t_h} = \frac{P_0}{2} \iff (0.65)^{t_h} = \frac{1}{2} \iff \ln(0.65^{t_h}) = \ln\frac{1}{2}$$

By using the properties of the logarithms we have

$$t\ln(0.65) = \ln\frac{1}{2} \implies t = \frac{\ln(1/2)}{\ln(0.65)} \approx 1.609.$$

The half life would be about 160.9 days, in which we would have half of the initial amount.

Clicker Question

What is the half life of a substance that decays according to the law $Q(t) = 10 \cdot 3^{-t}$?

(A) $\frac{1}{2}$

(B) 5

(C) $\log_3 \left(\frac{1}{2}\right)$

(D) $\log_2 3$

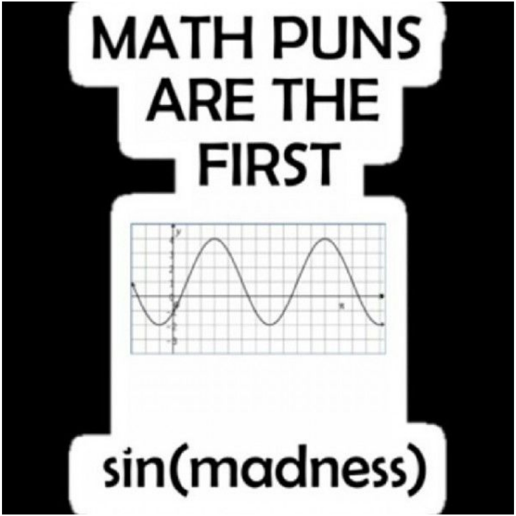
(E) $\log_3 2$

Review on trigonometry

- A radian is the measure of the angle that cuts off a sector of the unit circle of radius 1. (i.e. if we walk 1 length unit along a circle of radius 1). The whole circle has $2\pi \approx 6.28$ radians (corresponding to a 360° angle.
- Put your calculator in radian mode!

Sine and Cosine functions

- The coordinates of a point on the unit circle are $x = \cos t$, $y = \sin t$, $t \in \mathbb{R}$. Otherwise, in a right triangle $\sin t = \dfrac{\text{opp. side}}{\text{hyp}}$ and $\cos t = \dfrac{\text{adj. side}}{\text{hyp}}$.
- Recall the Pythagorean formula (fundamental identity in trigonometry)
$$x^2 + y^2 = \sin^2 t + \cos^2 t = 1.$$
- The sine and cosine functions are 2π periodic i.e.
$$\sin t = \sin(t + 2\pi \cdot k); \quad \cos t = \cos(t + 2\pi \cdot k).$$
- Amplitude= half the distance between max. and min. values.
- Period= minimum change in argument to cycle through the range. If f has period P then $f(x) = f(x + P)$.


MATH PUNS
ARE THE
FIRST
sin(madness)

The tangent function

- Defined as

$$\tan t = \frac{\sin t}{\cos t} = \frac{\text{opp. side}}{\text{adj. side}}.$$

- Undefined at $t = \frac{\pi}{2} + k\pi$. (vertical asymptote)
- Increasing function with range $= (-\infty, \infty)$.

Inverse trigonometric functions

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(No, we don't. Why?) Ok, maybe $\sin\theta = \frac{1}{2}$ or even $\sin\theta = 0.329748$. We will invert the trigonometric functions to obtain a value for the unknown angle.

- $\sin x = y \iff x = \arcsin y; \quad -\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$ and $y \in [-1, 1]$. (why?)
- $\cos x = y \iff x = \arccos y; \quad 0 \leq x \leq \pi$ and (still!) $y \in [-1, 1]$.
- $\tan x = y \iff x = \arctan y; \quad -\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$ and $y \in (-\infty, \infty)$.

Clicker question

What is $\sin\alpha$ when $\cos\alpha = \dfrac{1}{3}$? $(\alpha \in (0, \frac{\pi}{2}))$

(A) $\dfrac{2\sqrt{2}}{3}$

(B) $\dfrac{2}{3}$

(C) $-\dfrac{2\sqrt{2}}{3}$

(D) $\dfrac{2}{3\sqrt{2}}$

(E) $\dfrac{8}{9}$

Wrapping up

- We reviewed trigonometric functions.
- Work on the suggested problems from section 1.5 by Monday (from syllabus and webwork).
- Read section 1.7 (continuity) before lecture on Monday.
- Take the CRA today! (unless you passed :))
- Bring your your clickers to lecture on Monday!