Calculus 1
Section 4.6 Related Rates

Clicker Question 1

Let $P(t)$ be the population of California in the year t. Then $P'(2010)$ represents:

(a) The growth rate (in people per year) of the population.

(b) The growth rate (in percent per year) of the population.

(c) The approximate number of people by which the population changed in 2010.

(d) The approximate percent increase in population in 2010.

(e) The average yearly rate of change in the population since $t = 0$.

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(a) The derivative represents the **rate of change** of $P(t)$ in people per year.

A spherical snowball is melting in such a way that the instant at which its radius is 30cm, the radius is decreasing at a rate of 3cm/min. At what rate is the volume of snowball changing at that instant?

Solution

Since the snowball is spherical, we use the formula for volume: $V = \frac{4}{3}\pi r^3$. Both the volume and the radius are unknown functions of time t/ To find the rate of change we take the derivative the volume euqation with respect to time and get

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

We do not know $r(t)$ but we do know that when $r = 30$ is rate of change $(\frac{dr}{dt})$ is 3 cm/min. Using this information:

$$\frac{dV}{dt} = 4\pi(30)^2(-3) = -10800\pi \approx -33929cm^3/min.$$

Thus the instant the radius is 30cm the volume of the snowball is decreasing at a rate of 33929 cm^3/min.

Clicker Question 2

A spherical snowball of radius r has a surface area given by $S = 4\pi r^2$. As the snowball gathers snow its radius increases at a rate of 2.5 cm/min when the radius is 20 cm. How fast is the surface area of the snowball increasing?

1. $4\pi(20)^2$
2. $4\pi(2.5)^2$
3. $8\pi(20)(2.5)$
4. $8\pi(20)^2(2.5)$
5. No Idea how to even start

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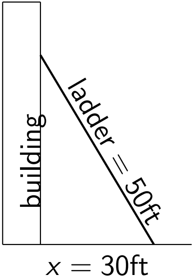
(c)

$$\frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi(20)(2.5)$$

Sliding Ladder

A 50ft ladder is placed against a large building. The base of the ladder is in an oil spill and slipping away from the base of the wall at a rate of 3ft/min. Find the rate of change of the height of that ladder the instant the ladder is 30 ft from the base.



Solution

The ladder forms a right triangle with the building and the ground, if the height of ladder is given by variable y then $x^2 + y^2 = 50^2$. Both x and y change in time as the ladder is sliding, we know $\frac{dx}{dt} = 3\text{ft/min}$, we want to find $\frac{dy}{dt}$. Differentiating with respect to t we get:

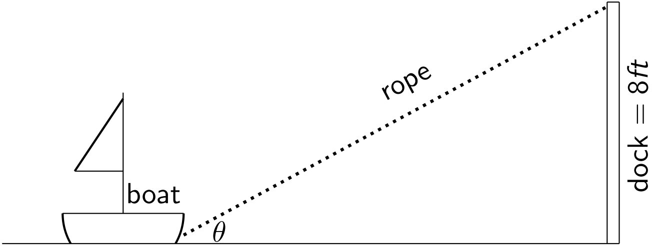
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dy}$$

Now just need to find the y value when $x = 30$. Using the Pythagorean theorem again, we get $y = 40$. Thus height of the ladder is decreasing at a rate of $\frac{30}{40}(3) = 2.25\text{ft/min}$ when the base is 30ft from the wall.

Another example

A boat is floating **away** from a dock at a steady rate of 0.5 ft/min. The dock is 8 ft. above the water, as shown in the sketch below. There is a rope attached to the boat (as shown) which is getting longer as the boat drifts away from the dock. Let θ be the angle between the rope and the water. What is the rate of change of θ when $\theta = \pi/3$



Solution

We know that $\frac{dx}{dt} = 0.5\text{ft/min}$. We need a relationship between x (the distance from the boat to the dock) and θ. Here $\tan(\theta) = \dfrac{8}{x}$. Our goal is to find $\dfrac{d\theta}{dt}$ so we differentiate $\tan(\theta) = \dfrac{8}{x}$ with respect to time using the chain rule:

$$\frac{1}{\cos^2(\theta)}\frac{d\theta}{dt} = -8x^{-2}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \cos^2(\theta)(-8x^{-2}\frac{dx}{dt})$$

We do not know the distance x, to find that use $\tan(\pi/3) = \sqrt(3) = 8/x$ thus $x = 8/\sqrt(3)$ so

$$\frac{d\theta}{dt} = \cos^2(\pi/3)(-8(8/\sqrt(3))^{-2})(0.5) = -\frac{3}{64}$$

The angle between the boat and water is decreasing by $\frac{3}{64}$ rad/min.

Clicker Question 3

A ruptured oil tanker causes a circular oil slick on the surface of the ocean. When its radius is 150 metes the radius of the slick is expanding by 0.1 meter/minute and its thickness is 0.02 meter. If the Volume stays constant how fast is the thickness of the oil spill deceasing at this instant? Recall the volume of a cylinder is given by $V = \pi r^2 h$

1. $\pi(150)^2(0.02)$
2. $2(0.1)(0.02)/150$
3. $2\pi(150)(0.1)(0.02)$
4. 0
5. No idea where to start

Solution

(b)

$$0 = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

so

$$\frac{dh}{dt} = \frac{-2\frac{dr}{dt}h}{r} = -2(0.1)(0.02)/(150)$$

Thus the height is decreasing at a rate of $2(0.1)(0.02)/150$ m/min.