Math 107-Lecture 26

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Taylor Series

The Taylor series for the function $f(x)$ near $x = a$ is given by:

$$f(a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)^2}{2!} + \cdots + f^{(n)}(a)\frac{(x-a)^n}{n!} + \cdots$$

Question: How can we find a Taylor series from a known one? (Taking so many derivatives can be a lot of work. . . .)

Some simple examples (hint: substitution will mostly do it)

Example 1. Find the Taylor series near 0 for the function f below and determine for what values of x it converges.

$$f(x) = \frac{x}{1 - (x/2)}$$

Example 2. Same for g

$$g(x) = \frac{x^2}{2 + x^4}$$

Example 3. How about for h?

$$h(x) = x^3 e^{x^2}$$

Clicker question

What is the Taylor series for the function $f(x) = \dfrac{1}{2-x}$ near $x = 1$?

- (A) $\displaystyle\sum_{n=0}^{\infty}(x-1)^n$
- (B) $\displaystyle\sum_{n=0}^{\infty}(2-x)^n$
- (C) $\displaystyle\sum_{n=0}^{\infty}(1-x)^n$
- (D) $\displaystyle\sum_{n=0}^{\infty}(1+x)^n$
- (E) $\displaystyle\sum_{n=0}^{\infty}(2+x)^n$

We can do more. A lot more!

Inside the interval of convergence we can also multiply, integrate, and differentiate a power series term by term. Using this approach we can find more easily the Taylor series around the origin for many functions.

Examples 4 & 5:

- $f(x) = \ln(1 - x)$
- $g(x) = \arctan x$

Example 6: Find the first four terms of the Taylor series near the origin for

$$\ln(1 - x^2)\arctan(x^3).$$

Using series for approximation of integrals

Recall that

$$\int e^{x^2}\,dx$$

cannot be expressed in terms of elementary functions, hence we cannot apply the FTC to compute

$$\int_0^1 e^{x^2}\,dx.$$

However we can get a good approximation (how good?) for this integral by using Taylor series.