Math 107-Lecture 5

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Announcements

- Paper Gateway Exam in recitation!
- The worksheet packet contains practice Gateway exams at the end.
- Practice Gateway Exam are now open on webwork. The grades do NOT count towards your final grade.
- Study Stops are now open; see schedule and updates at `http://success.unl.edu/current/study-stop-schedule`.

Plan for today

1. Review integration by partial fractions.
2. Trigonometric substitutions.

Motivation for trigonometric substitution

We learned that

$$\int \frac{1}{1+u^2}\,du = \arctan u + C, \quad \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

But how were they obtained? Recall the trigonometric identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

Or, after a few algebraic manipulations (ignore the absolute values for now):

$$a\sin\theta = \sqrt{a^2 - (a\cos\theta)^2}$$

$$a\sec\theta = \sqrt{a^2 + (a\tan\theta)^2}$$

$$a\tan\theta = \sqrt{(a\sec\theta)^2 - a^2}$$

Trigonometric substitutions

For expressions involving (usually some root of the following quantities):

- $a^2 - x^2$ use $x = a\sin\theta$ (ignore absolute values on $\cos\theta$.) Example: For $\int (16 - x^2)^{5/2} dx$ use $x = 4\sin\theta$.
- $a^2 + x^2$ use $x = a\tan\theta$ (ignore absolute values on $\sec\theta$.) Example: For $\int (16 + x^2)^{5/2} dx$ use $x = 4\tan\theta$.
- $x^2 - a^2$ use $x = a\sec\theta$ (The sign of $\tan\theta$ matches the sign of x.) Example: For $\int (x^2 - 16)^{5/2} dx$ use $x = 4\sec\theta$.

Note that $\sqrt{x^2 - a^2} \neq -\sqrt{a^2 - x^2}$!!!

Clicker question #1

Which trig substitution would you use for

$$\int \sqrt{9 - x^2}\, dx \qquad ?$$

- $x = \sin\theta$
- $x = 3\sin\theta$
- $x = 9\sin\theta$
- $x = 3\tan\theta$
- $x = 9\sec\theta$

Clicker question #2

Which trig substitution would you use for

$$\int \frac{1}{\sqrt{1+x^2}}\, dx \qquad ?$$

- $x = \sin\theta$
- $x = \tan\theta$
- $x = \sec\theta$
- Don't need it. The answer is $\arctan(x) + C$
- Don't need it. The answer is $\arcsin(x) + C$

Our initial examples ...

Let us compute together

- $\displaystyle\int \frac{1}{1+4x^2}\,dx$

- $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$

Wrapping up:

- Today we reviewed and finished section 7.4: integration with partial fractions and trigonometric substitutions.
- For next time finish working all suggested problems from section 7.4.
- For next lecture read Section 7.5 Numerical ‘integration.
- Paper Gateway Exam in recitation on Wednesday 02/01.