Calculus 1
Final Exam Review 1

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Evaluate $\lim_{x \to 0} \frac{1}{x}$.

a) e

b) 1

c) 0

d) -1

e) 5

f) Does Not Exist

Min/Max

Given $f(x) = x^3 - 3x^2 + 3x + 1$

a) Find the exact x-value(s) of any local maximum(s) and local minimum(s) of $f(x)$.

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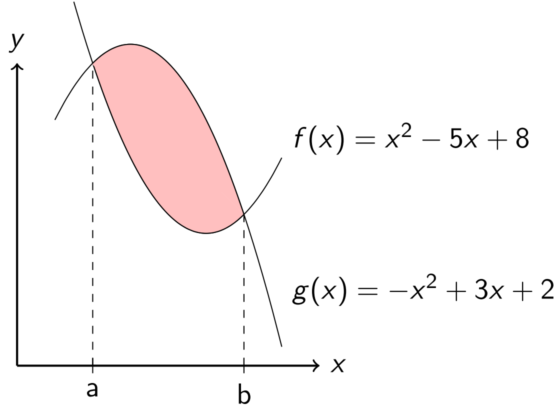
b) Does $f(x)$ have any inflection points? If so, find the exact ordered pair (coordinates) for any such points.

Area

Use the graph below and the knowledge that $f(x) = -x^2 + 3x + 2$ and $g(x) = x^2 - 5x + 8$ to find the area indicated.



Local Linearization

Find the linear approximation, $L(x)$ of $f(x) = \frac{1}{10+5x}$ near $x = 0$.

Local Linearization

Use your linear approximation to estimate $f(x)$ at $x = 1$. That is, find $L(1)$.

Local Linearization

Is $L(1)$ above or below the actual value of $f(x)$ at $x = 1$? *Be sure to explain your answer using complete sentences.*

Implicit Differentiation

A function is defined implicitly by the equation $e^{2x} + \ln(y) = x^2 - xy^3$.

a) Find the derivative $\dfrac{dy}{dx}$.

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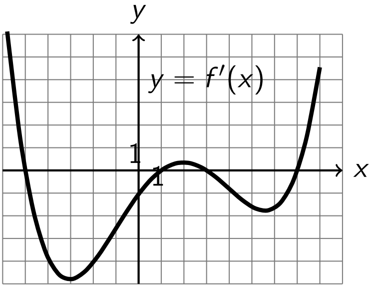
b) Find the equation of the line tangent to the graph at the point $(0, 1)$.

Interpreting Graphs

The following graph is of the derivative of a function $f(x)$. Use the graph to answer the following questions and justify your answers.



a) For what value of x in the interval $[-3, 2]$ will $f(x)$ have the largest slope?

b) For what value of x in the interval $[-3, 2]$ will $f(x)$ have its smallest value?

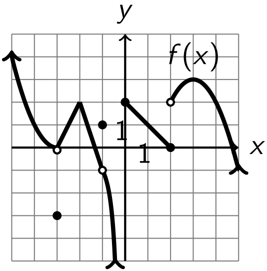
c) Give an interval where $f(x)$ is increasing.

d) Give an interval where $f''(x)$ is positive.

e) Give an interval where $f(x)$ is concave down.

Limits

Use the graph of $f(x)$ below to answer the following. Clearly write **DNE** if the value does not exist.



a) $\lim_{x \to 3} f(x)$

b) $\lim_{x \to -1} f(x)$

c) $\lim_{x \to 0^+} f(x)$

d) $\lim_{x \to 2^-} f(x)$

e) $f'(3)$

f) $f'(-2)$

g) $f''(1)$

Differential Equations

A man drops a coin off the top of the US Bank Center in Lincoln onto the concrete 220 feet below. Let $s(t)$ be the function that gives the coins height above the ground t seconds after the coin is dropped and $v(t)$ the velocity. For this problem you may neglect any air resistance and work in units of feet and seconds. For parts (a) and (b) you must use a differential equation to solve this problem to get any points.

- Write a function which gives the velocity as a function of time. You may assume that $a = \frac{d^2s}{dt^2} = \frac{dv}{dt} = -32$ and $v(0) = 0$.
- Write a function which gives the position as a function of time. You may still assume that $a = \frac{d^2s}{dt^2} = \frac{dv}{dt} = -32$ and $v(0) = 0$. You may also want to assume that $s(0) = 220$.
- How fast is the coin traveling when it hits the street?

Implicit Differentiation

Electricity is governed by Ohm's Law which states that voltage equals the product of resistance and current. A standard automotive system runs on 12 volts. Let x give the resistance of an electrical system and y the total current in an electrical system at a specific time t. In this situation Ohm's law can be stated as $12 = xy$. (You also may want to note that the standard unit of measure of resistance is an ohm, denoted by ω and the standard unit of measure of current is an amp denoted by A.)

- Find $\frac{dy}{dt}$ given $12 = xy$.
- Suppose that the resistance in a particular 12 volt electrical system currently 28ω. What is the current?
- Further Suppose that the resistance of this particular 12 volt electrical system is decreasing at a rate of $1\omega/s$. That is $\frac{dx}{dt} = -1$. At what rate is the current changing when the resistance is 28ω?

Related Rates

A spherical snow ball is melting. Its radius is decreasing at a rate of 0.2 cm per hour when the radius is 15 cm.

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Which of the following is the exact solution to $\frac{d}{dx}\int_1^x \ln(t^2+1)dt$?

a) $\ln(x^2+1)2x$

b) $\frac{2x}{x^2+1}$

c) $\ln(x^2+1)-\ln(2)$

d) $\ln(x^2+1)$

e) Does Not Exist

Derivative by Definition

Use the **limit definition** of the derivative to find the derivative of $f(x) = x^2 + 2$

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Which of the following is the exact solution to $\frac{d}{dx}\int_2^{\sin(x)} t^2 dt$?

a) $(\sin(x))^2$

b) $(\sin(x))^2\cos(x)$

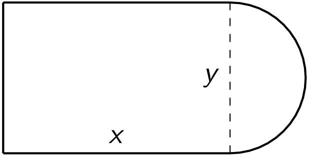
c) $\sin(t)$

d) $\sin(t^2)$

e) $\sin(x)2x$

Optimization

You fencing a new corral. The land owner has given you a very specific shape and asked that you use exactly 100 feet of fence. The shape is diagramed below. You may assume the shape is made of rectangles and semicircles.



- Find a formula for the area of the above shape.
- Find a formula for the perimeter of the above shape.
- Find the values of x and y that maximize the area of the shape, given the constraint that you use exactly 100 feet of fence.

Integrals

Evaluate the following integrals.

a) $\int_1^2 \left(\frac{1}{x^2} + x - \frac{1}{2}\right) dx$

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Integrals

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a) $\int_1^2 \left(\frac{1}{x^2} + x - \frac{1}{2}\right) dx$

b) $\int x^2 \left(1 + x^3\right)^5 dx$

c) $\dfrac{d}{dx} \int_1^{x^2} \sqrt{1+t}\,dt$

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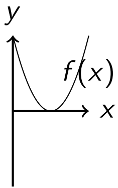
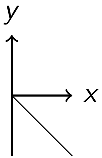
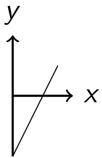
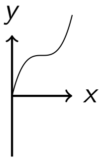
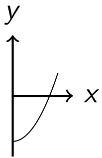
Given the graph of $f(x)$ below. Clearly circle the graph that could represent $f'(x)$.











Optimization

You are working for a soda-can manufacturing company. You have been asked to find the dimensions of an open cylindrical can that must hold $40cm^3$ of liquid and has the minimum surface area. What are the dimensions of such a can? (Recall: An open cylinder of radius r and height h has volume $\pi r^2 h$ and surface area $2\pi rh + \pi r^2$). Be sure to show how you know this is the minimum surface area and not the maximum.