Math 107-Lecture 14

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Announcements

- The last day to pass the Gateway exam is Friday, Feb. 22.
  If you haven't passed: Take the Gateway everyday!
- Today we will cover section 8.4 - Mass and Density.

Clicker question #1

Give a range of values for r, θ that describe the third quadrant of the circle of radius 3 centered at the origin:

(A) $0 \leq r \leq 3, \quad \pi/2 \leq \theta \leq \pi$

(B) $-3 \leq r \leq 3, \quad \pi/2 \leq \theta \leq \pi$

(C) $0 \leq r \leq 3, \quad -\pi/2 \leq \theta \leq 3\pi/2$

(D) $0 \leq r \leq 3, \quad \pi \leq \theta \leq 3\pi/2$

(E) $0 \leq r \leq 3, \quad -\pi/2 \leq \theta \leq \pi$

One more example for computing the area in polar coordinates

Example 1. Find the area of **one petal** of the rose curve given by

$$r(\theta) = 3\cos 3\theta,$$

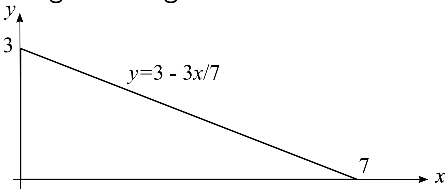
contained between $\theta = -\frac{\pi}{6}$ and $\theta = \frac{\pi}{6}$.

Solution. Applying the formula for the area of a region given in polar coordinates yields

$$\begin{aligned} A &= \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} r^2(\theta)\,d\theta \\ &= \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (3\cos 3\theta)^2\,d\theta \\ &= \frac{9}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1+\cos 6\theta}{2}\,d\theta = \frac{9}{4}\left[\theta + \frac{\sin 6\theta}{6}\right]\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \frac{3\pi}{4}. \end{aligned}$$

Example 1.

Suppose the triangle below is cut from a sheet of a material 0.1 in thick and having density $\rho = 5$ grams per cubic inch. Set up an integral that gives the **mass** M of this object.



Notation: M=mass, V = volume, A =area, h =thickness, ρ = density.

$$M = \rho \cdot V = \rho \cdot h \cdot A = \rho h \int_0^7 (3 - \frac{3x}{7})\, dx$$

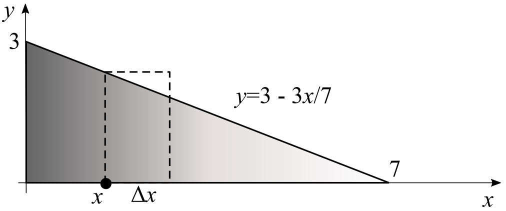
$$= \frac{5g}{\text{in}^3} \cdot (0.1)\text{in} \cdot (21 - 3 \cdot 7/2)\text{in}^2 = 5.25g.$$

Varying density in flat objects

Example 2. In the same setting suppose the density decreases as we move towards the right, satisfying $\rho(x, y, z) = 10 - \dfrac{x}{7}$ grams per cubic inch. Set up an integral that calculates the mass of this object.

$$M = \int_0^7 \underbrace{\rho(x)(3 - \frac{3x}{7})}_{\text{mass of bar at } x} h\,dx = h \cdot \int_0^7 (10 - \frac{x}{7})(3 - \frac{3x}{7})\,dx$$

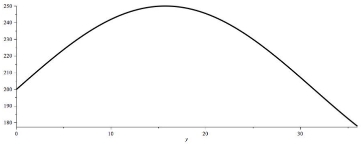
$$= (0.1) \int_0^7 (30 - \frac{33x}{7} + \frac{3x^2}{49})\,dx = 10.15g.$$



Varying density in a bar

Example 3. A cylindrical pole of height 3 ft has density that is specified in **grams per inch of length** (for example, constant density of 10 grams per inch of length means any one-inch slice of the pole has mass 10 grams) as measured from the ground:

$$\rho(y) = 200 + 50\sin(0.1y)$$



Varying density in a plate

Example 4. Find the mass of the triangular lamina with vertices $(0,0)$, $(0,3)$, $(2,3)$ given that the density at (x,y) is

$$\rho(x,y) = 2e^x.$$

Clicker question #2

What are the bounds for (x, y) that describe the triangular region with vertices $(0, 0)$, $(0, 3)$, $(2, 3)$?

(A) $0 \leq x \leq 2, \quad -3 \leq y \leq 3$

(B) $0 \leq y \leq 3, \quad 0 \leq x \leq 2$

(C) $0 \leq x \leq 3, \quad 2x \leq y \leq 3x$

(D) $0 \leq x \leq 2, \quad 0 \leq y \leq \frac{3x}{2}$

(E) $0 \leq x \leq 2, \quad \frac{3x}{2} \leq y \leq 3$

Wrapping up:

- For next time read section 8.5; solve the problems from section 8.4.
- There is an extension to take and pass the Gateway exam at the testing center for partial credit (5% instead of 7%);
- WeBWork on section 8.3 is due on Friday, 03/03.