Clicker Survey

How do you feel about the first test?

(a) Great

(b) Good

(c) Average

(d) No so good

(e) Terrible

Clicker Survey

Do you think you need to:

(a) Study about the same for the next test

(b) Study harder for the next test

(c) Study less for the next test

(d) Study Sooner for the next test

We have already used inverse functions to solve things such as $\tan(\arcsin(3/5)) = ?$
We now wish to take advantage of the fact that

$$f(f^{-1}(x)) = x$$

to find some derivatives of new functions.
Let's practice the idea on something we already know:

$$\text{If } f(x) = \sqrt{x}, \text{ then } (f(x))^2 = x.$$

Take the derivative of both sides:

$$2(f(x))\frac{df}{dx} = 1$$

$$\text{Therefore, } \quad \frac{df}{dx} = \frac{1}{2f(x)} = \frac{1}{2\sqrt{x}}$$

as expected.

The Derivative of $\ln(x)$

We use the chain rule to differentiate an identity involving $\ln x$.
Since $e^{\ln x} = x$, we can differentiate both sides. On the one hand we have:

$$\frac{d}{dx} e^{\ln x} = \frac{d}{dx} x = 1.$$

Also, by the chain rule,

$$\frac{d}{dx} e^{\ln x} = e^{\ln x} \frac{d}{dx} \ln(x) = x \frac{d}{dx} \ln(x)$$

Thus, dividing by x,

$$\frac{d}{dx} \ln x = \frac{1}{x}$$