# Math 107-Lecture 13

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# Announcements

- The deadline for passing the Gateway exam is Friday, February 22. The Gateway counts for 7% of the grade.

# Polar coordinates - Review

Relations between Cartesian and polar coordinates:

For the cartesian equivalent $(x, y)$ of the polar point $(r, \theta)$ we have

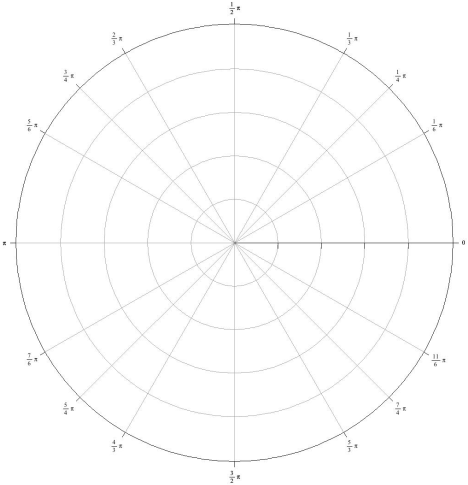
- $x = r\cos\theta$
- $y = r\sin\theta$
- $x^2 + y^2 = r^2$
- $\dfrac{y}{x} = \tan\theta$ [**Note:** You can not determine $\theta$ uniquely from this eq.; you also need to know in which quadrant you are].

Example: The circle of radius 10 centered at $(0, 0)$ is represented as

- $x^2 + y^2 = 100$ (in Cartesian coordinates)
- $r = 10$ (in polar coordinates).

# Points in polar coordinates

Plot polar points $(r, \theta)$ given by $A = (1, \pi/4)$, $B = (2, -\pi/4)$, $C = (2, \pi/4 + \pi)$, $D = (4, -\pi/4)$



# Clicker question #1

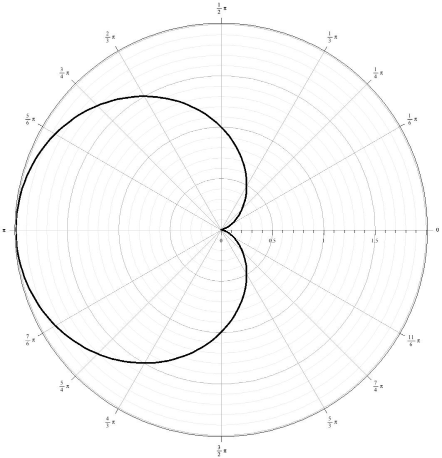
The curve $r = 1 - \cos\theta$ with $\theta \in [0, 2\pi]$ best resembles

A) a circle
B) a banana
C) a spiral
D) a heart
E) a star

*Hint:* Find first the intersection points of the curve with all axes; then take more points to better determine the shape.

# The answer is ...

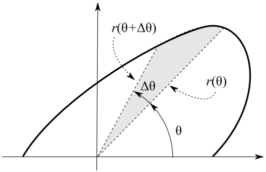
The cardioid $r = 1 - \cos\theta$ is represented graphically by

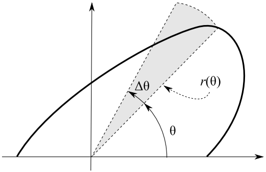


# Areas in polar coordinates - 5 min. introduction

Recall: Ares of a sector of opening $\theta$ (in radians) and radius $r$ is

$$A = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{r^2\theta}{2}$$




For a curve $r = f(\theta)$, with $f(\theta) \geq 0$ the area of the sector of opening between $\theta_0$ and $\theta_1$ (above left shaded region) is

$$A = \frac{1}{2} \int_{\theta_0}^{\theta_1} [f(\theta)]^2 \, d\theta.$$

# Examples for computing the area in polar coordinates

Example 1. The area of the quarter of the circle $r = 5$ with $\theta \in [0, \frac{\pi}{2}]$ is given by

$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} 5^2 d\theta = 25\frac{\pi}{4}.$$

Example 2. The area enclosed by the spiral $r = \theta$ with $\theta \in [0, \frac{\pi}{2}]$ is given by

$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} \theta^2 \, d\theta = \frac{\theta^3}{6}\Big|_0^{\frac{\pi}{2}} = \frac{\pi^3}{48.}$$

# Clicker question #2

Find the area enclosed by the cardioid $r = 1 - \cos\theta$. Recall that $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$.

A) $\pi^2 - \pi$

B) $\pi^2$

C) $3$

D) $3\pi$

E) $6\pi$

# Wrapping up:

- For next time read section 8.4.
- The deadline for passing the Gateway is Friday, February 22. The Gateway counts for 7% of the grade.