FFT

$$X(k) = \sum_{n=0}^{N-1} x(n)\cdot e^{-\frac{j2\pi}{N}kn} \quad , \quad k = 0, 1, 2, 3 \ldots N-1$$

Radix-2 Cooley Tukey

Stages $= \log_2(N) = 3$ → separate odd & even → calculate Xeven[] Xodd[] → butterfly

Twiddle Factors $e^{-\frac{j2\pi kn}{N}}$

$$x(n) = \{5, -2, 3, 8, 1, 4, -1, 7\}$$

$$x_{even} = \{\overset{0}{5}, \overset{1}{3}, \overset{2}{1}, \overset{3}{-1}\}$$

$$x_{odd} = \{-2, 8, 4, 7\}$$

$$\sum_{n=0}^{N-1}$$

$$K_{0\,\text{even}} = x(0)\, e^{-\frac{j2\pi\cdot 0\cdot 0}{4}} + x(1)\cdot e^{-\frac{j2\pi\cdot 1\cdot 0}{4}} + x(2)\cdot e^{-\frac{j2\pi\cdot 2\cdot 0}{4}} + x(3)\cdot e^{-\frac{j2\pi\cdot 3\cdot 0}{4}}$$

$$= 5 + 3 + 1 + (-1) = 8$$

$$K(1)_{\text{even}} = x(0)\, e^{-\frac{j2\pi\cdot 1\cdot 0}{4}} + x(1)\, e^{-\frac{j2\pi\cdot 1\cdot 1}{4_2}} + x(2)\, e^{-\frac{j2\pi\cdot 1\cdot 2}{4}} + x(3)\, e^{-\frac{j2\pi\cdot 1\cdot 3}{4_2}}$$

$$\boxed{K(0) = 8} = 5\cdot e^{0} + 3\cdot^{-j\pi/2} + 1\cdot e^{-j\pi} + (-1)\cdot e^{-j\frac{3}{4}\pi}$$

$K(1)_{\text{even}} = 4 - 2j$

$K(2)_{\text{even}} = 4$

$K(3)_{\text{even}} = 4 + 2j$

$$X_{even} = \{8, 4-2j, 4, 4+2j\}$$

k 0 2 3 4

$\boxed{N = 4}$

$n = 0 \to 3$

$$X_{odd} = \{-2, 8, 4, 7\}$$

$X(0)_{odd} = 17$, $X(1)_{odd} = -6-j$, $X(2) = -13$

$X(3) = -6 + j$

$$\underline{X_{odd}} = \{17, -6-j, -13, -6+j\}$$

Cooley tukey

[illegible] x = {3, 1, 4, 2, 6, 5, 0, 0}

[illegible] = [2, 3, 5, 1, 0, 4, 6, 7}

① bit reversly

{0, ①, 2, 3, 4, 5, 6, 7}
1 2 3 4 5 6 7 8

i = 0 → 8 - 1 = 7

index = x[i]

xout[i] = xin[index]

001

010
001
000

xout = {0, 1,

[0]

16
10000
00001

7}

011

110

{ 1/0 , 2/1 , 3/2 , 4/3 }

}

{ —/0 , —/1 , —/2 , —/3 }

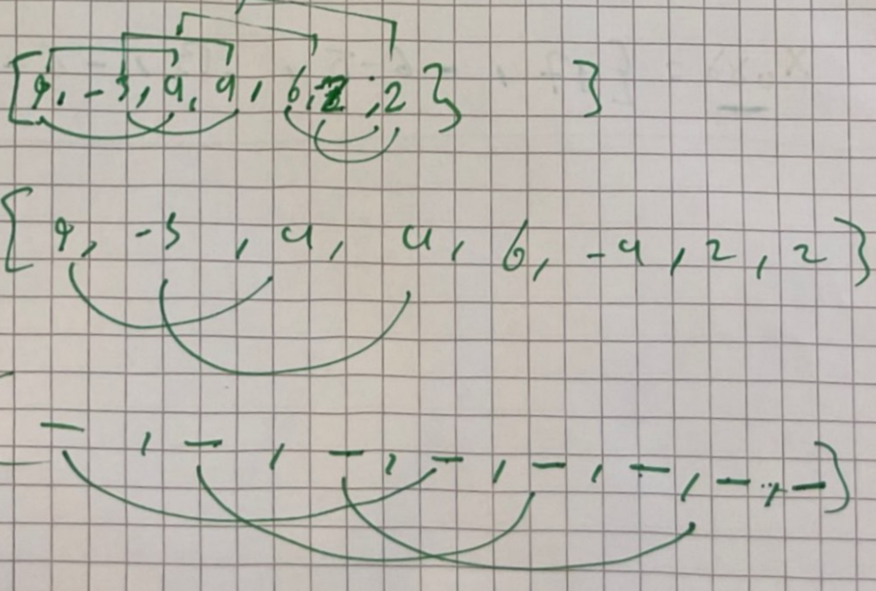
y = x[i]

= out(y)

{[3, 6], [4, 0], [1, 5], [2, 0]}

y[0] = 3 + 6

y[1] = 3 - 6



a)

using butterfly Algorithm

to do calculation between odd and even

$$X[k] = x_{even}[k] + W_8^k \, x_{odd}[k]$$

Twiddle Factor $W_8^k = e^{-\frac{j2\pi k}{8}}$

$$\boxed{\begin{aligned} X[k] &= x_{even}[k] + W_8^k \, x_{odd}\{k\} \\ X\{\underset{N/2}{k+4}\} &= x_{even}\{k\} - W_8^k \, x_{odd}\{k\} \end{aligned}}$$

$$X[0] = x_{even}[0] + W_8^0 \, x_{odd}[0]$$

$$X\{0+4\} = x_{even}[0] - W_8^0 \, x_{odd}[0]$$

$$X[1] = x_{even}[1] + W_8^1 \, x_{odd}[1]$$

$$X\{1+4\} = x_{even}[1] - W_8^1 \, x_{odd}[1]$$

$$X\{2\} = x_{even}\{2\} + W_8^2 \, x_{odd}\{2\}$$

$$X\{2+4\} = x_{even}[2] - W_8^2 \, x_{odd}[2]$$

$$X[3] = x_{even}\{3\} + W_8^3 \, x_{odd}[3]$$

$$X\{3+4\} = x_{even}\{3\} - W_8^3 \, x_{odd}\{3\}$$

$$X(K) = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

x out

k = 0 - N-1

Points 8 stages $\underline{\underline{3}}$

For loop stages **1 → 3**

Stag 1

sub FFt size = 2

BF width = 1

For Butterply $0 \leq \overset{1}{\text{BF width}}$

J Butterfly = 0

BF W_R = w[0]

For sub DFT size **J → FFT size** **+ sub FFT size**

0 2 4 6

i = 0

i_lower = i + BF width = 1

$\text{temp\_R} = \text{FFTout\_R}\{\overset{1}{\text{i\_lower}}\} \times \text{BFw\_R} - \text{FFTout\_I}[\overset{1}{\text{i\_lower}}] \times \text{BFw\_I}$

FFTout{1}×w

$\text{FFTout\_R}[\underline{\overset{1}{\text{i\_lower}}}] = \text{FFTout\_R}\{\overset{0}{i}\} - \text{temp\_R}$

$\text{FFtout R}\{\overset{0}{i}\} = \text{FFTout R}\{\overset{0}{i}\} + \text{temp\_R}$

---

i = 2

i_lower = 2 + 1 = 3

---

i = 4

i_lower = 5

i = 6

i_lower = 7

Stage 2

subFFTsize = 4
BFwidth = 2

For Butterfly $0 < 2$

0   1

$J = 0$

BFwR = w[0]

DFF ; $J \rightarrow$ FFTSize +

0   4

$\underline{i = 0}$

$i\_lower = 0 + 2 = \underline{2}$

$FFTout\_R\{\overset{2}{i\_lower}\} = FFToutR[\overset{0}{i}] - FFToutR\{\overset{2}{i\_lower}$
$FFTout\_R[\overset{0}{i}] = FFToutR\{\overset{0}{i}\} + FFToutR\{\overset{2}{i\_lower}\}$

$i = 4$

$i\_lower = 4 + 2 = 6$

$FFTout\_R[\overset{6}{i\_lower}] = FFToutR[\overset{4}{i}\} - FFToutR[\overset{6}{i\_lower}$
$FFTouR\{\overset{4}{i}\} = FFT\{\overset{4}{i}\} + FFT\{\overset{6}{i-lower})$

Stag. 2

1000
$\boxed{0010}$

Sub FFtsize = 4

BF width = 2

Buterrfl/s $0 < 2$

0 1

J = 1

BFwR = w[2}

DFT J -> FFTsize

1 5

$i = J = 1$

$i\text{-lower} = 1 + 2 = 3$

$\text{FFToutR}\{\overset{3}{i}\text{-lower}\} = \text{FFT}[\overset{1}{i}\} - \text{FFT}\{\overset{3}{i\text{-lower}}\}$

$\text{FFToutR}[\overset{1}{i}] = \text{FFT}[\overset{1}{i}\} + \text{FFT}\{\overset{3}{i\text{-lower}}\}$

$i = 5$

$i\text{-lower} = 7$

$\text{FFToutR}[\overset{7}{i\text{-lower}}\} = \text{FFT}\{\overset{5}{i}\} - \text{FFT}\{\overset{7}{i\text{-lower}}\}$

$\text{FFToutR}\{i\}^{5} = \text{FFT}\{i\}^{5} - \text{FFT}\{\overset{7}{i\text{-lower}}\}$

Stage 3

subFFTsize = 8

BFwidth = 4

Butterfly

0 < 4

0 1 2 3

J = 0

BFwR = W[0]

DFT

$\underline{J} \rightarrow$ FFTsize

i = 0

i_lower = i + BFwidth = 0 + 4 = 4

FFToutR{i_lower} = FFToutR{ i } - FFTout{i_lo

FFToutR{i} = FFT{i} + FFT{i_lower}

Butterfly

J = 1

BFw[R] = W[1]

DFT

i = 1

i_lower = 1 + 4 = 5

⋮

Butterfly

$j = 2$

$BFwR = w[2]$

DFT

$i = 2$

$i\text{-lower} = 6$ 2 + 4

BUTTERFLY

$j = 3$

$BFwR = w[3]$

DFT

$i = 3$

$i\ \text{lower} = 7$