Photometric Classification with Thompson Sampling

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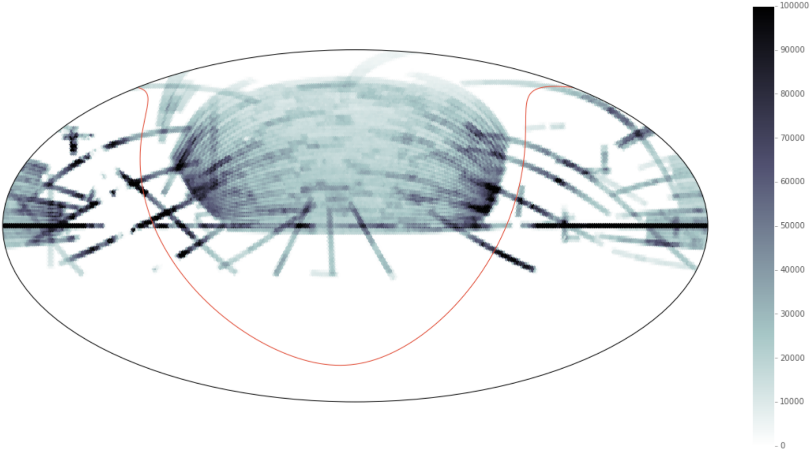
Sloan Digital Sky Survey

Photometric measurements of 800 million objects, out of which 3 million objects are spectroscopically labelled.



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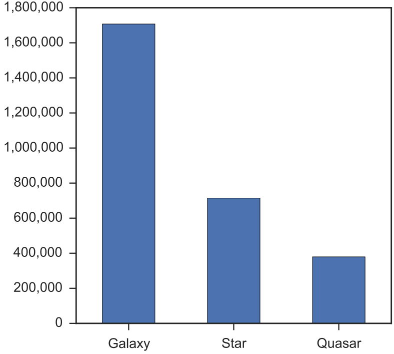
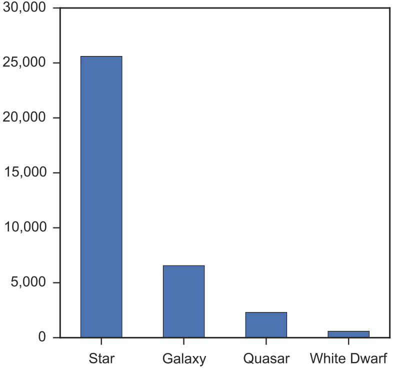


(a) SDSS Dataset



(b) VST ATLAS Dataset

Photometry vs Spectroscopy

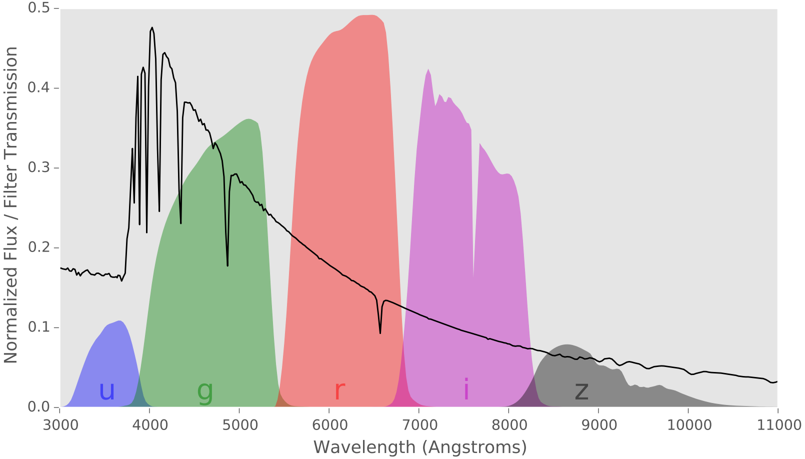


Figure: SDSS Filters and Vega Spectrum

Active Learning Motivation

- To construct the training set, one solution is to take a random sample objects for labelling.
- But labelling is expensive.
- Can we be smarter in choosing which objects for labelling?

Active Learning Algorithm

procedure $\textsc{ActiveLearner}(\mathcal{U},\ \mathcal{L},\ h,\ r\ n,\ t)$

    while $|\mathcal{L}| < n$ **do**

        $E \leftarrow$ random sample of size t from $\mathcal{U}$

        $\boldsymbol{x}_* \leftarrow \operatorname*{argmax}_{\boldsymbol{x} \in E} r(\boldsymbol{x})$

        $y_* \leftarrow$ ask the expert to label $\boldsymbol{x}_*$

        $\mathcal{L} \leftarrow \mathcal{L} \cup (\boldsymbol{x}_*, y_*)$

        $\mathcal{U} \leftarrow \mathcal{U} \setminus \boldsymbol{x}_*$

        $h_{\mathcal{L}}(\boldsymbol{x}) \leftarrow$ retrain the classifier

    end while

end procedure

Active Learning: Uncertainty Sampling Heuristics

- Pick the example whose prediction vector p displays the greatest Shannon entropy (information content):

$$r_S(\boldsymbol{x}) = -\sum_{c \in \mathcal{Y}} \mathbb{P}(y(\boldsymbol{x}) = c) \log \left[\mathbb{P}(y(\boldsymbol{x}) = c)\right]$$

- Pick the example with the smallest margin (difference between the two largest values in the prediction vector p):

$$r_M(\boldsymbol{x}) = \left|\mathbb{P}(y(\boldsymbol{x}) = c^{(1)}) - \mathbb{P}(y(\boldsymbol{x}) = c^{(2)})\right|$$

Active Learning: Query by Bagging Heuristics

1. Use bagging to train B classifiers $f_1, f_2, ..., f_B$.
2. Rank candidates by disagreement among f_i:
   - Margin-based disagreement: average the prediction of f_i and choose the example with the smallest margin.
   - Choose the example with the highest average Kullback-Leibler divergence from the average:

$$r_{QBB,KL}(\boldsymbol{x}) = \frac{1}{B}\sum_{b=1}^{B} D_{\mathrm{KL}}(P_b \| Q)$$

Active Learning: Loss Function Heuristic

- Expected squared loss can be decomposed into three terms:

$$\mathbb{E}[\text{Squared Loss}] = \text{Bias}^2 + \text{Variance} + \text{Noise}$$

- Pick the example that will cause the greatest drop in variance of the unlabelled pool:

$$r_V(\boldsymbol{x}) = \sum_i^k \mathbb{P}(y(\boldsymbol{x}) = i) V_{\mathcal{L} \cup \boldsymbol{x}}$$

Active Learning: Classifier Certainty Heuristic

- The entropy of the classifier's predictions on $\mathcal{U}$ is

$$CC_{\mathcal{L}} = -\sum_{\boldsymbol{u}\in\mathcal{U}} \sum_{c\in\mathcal{Y}} \mathbb{P}(y(\boldsymbol{u}) = c) \log \left[\mathbb{P}(y(\boldsymbol{u}) = c)\right]$$

- Pick the example that is expected to increase the classifier's prediction certainty by the the greatest amount:

$$r_{CC}(\boldsymbol{x}) = -\sum_{c\in\mathcal{Y}} \mathbb{P}(y(\boldsymbol{x}) = c) CC_{\mathcal{L}\cup\boldsymbol{x}}$$

Multi-arm bandit Problem

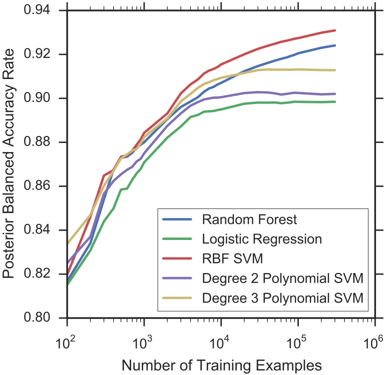
- A gambler stands in front a slot machine with n levers.
- Each lever emits a reward of unknown distribution.
- Goal: maximise lifetime rewards

Thompson Sampling

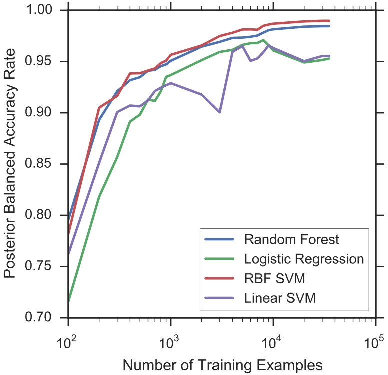
Algorithm 1 Thompson sapmling

procedure ThompsonSampling($\mathcal{R}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$)
  for each $t \in \{1, 2, ..., n\}$ **do**
    for each $r \in \mathcal{R}$ **do**
      $\boldsymbol{\nu}'_r \leftarrow$ draw a sample from $N(\boldsymbol{\mu}_{\boldsymbol{r}}, \boldsymbol{\sigma}_{\boldsymbol{r}})$
    end for
    $r_* \leftarrow \operatorname*{argmax}_{r \in \mathcal{R}} \boldsymbol{\nu}'_r$
    Observe reward w_*
    Update $\boldsymbol{\mu}_*$
    Update $\boldsymbol{\sigma}_*$
  end for
end procedure

Learning Curves with Random Sampling



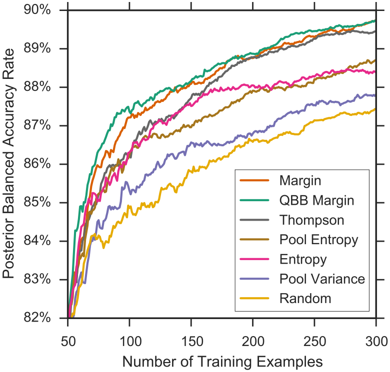
(a) With SDSS data



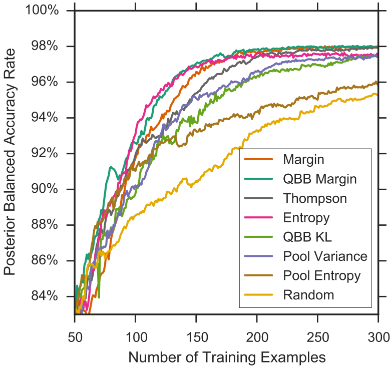
(b) With VST ATLAS data

Learning Curves with Acive Learning

Balanced Pool with Logistic Regresion



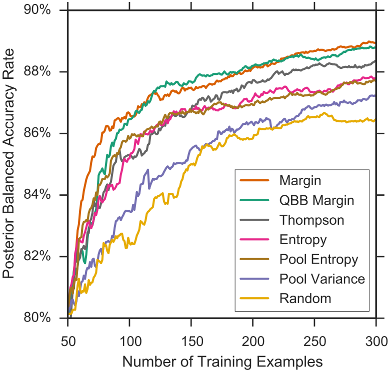
(a) SDSS Dataset



(b) VST ATLAS Dataset

Learning Curves with Active Learning

Unbalanced Pool with Logistic Regresion



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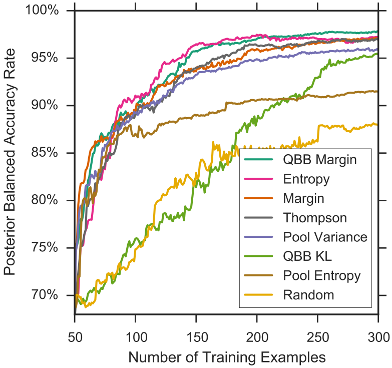
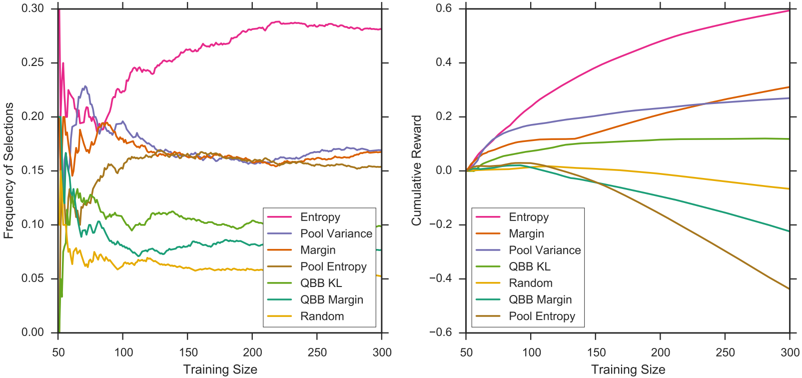


(b) VST ATLAS Dataset

Heuristic Selection

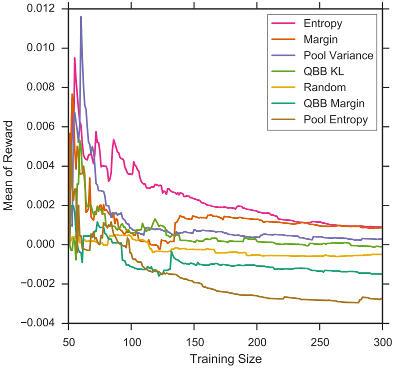
VST ATLAS, Unbalanced, Logistic Regression.



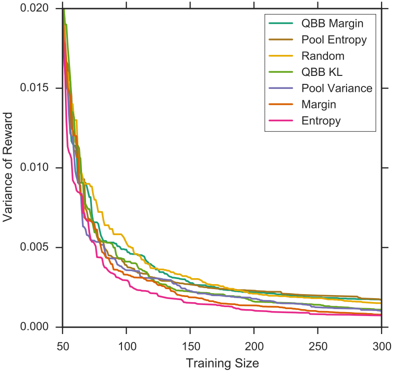
(a) Frequency (b) Cumulative Rewards

Drifting Rewards

VST ATLAS, Unbalanced, Logistic Regression.



(a) Frequency



(b) Cumulative Rewards

Concluding Remarks

- Dynamic Thompson Sampling approach to address the reward drifting problem
- Theoretical analysis of convergence.
- Batch active learning.