

# **Phenomenology of the Higgs and Flavour Physics in the Standard Model and Beyond**

DISSERTATION

zur Erlangung des akademischen Grades

doctor rerum naturalium  
(Dr. rer. nat.)  
im Fach Physik

eingereicht an der  
Mathematisch-Wissenschaftlichen Fakultät  
Humboldt-Universität zu Berlin

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## Abstract

This thesis investigates some aspects for the future of Higgs measurements after a decade of its discovery, focusing on the potential for the future runs of the Large Hadron Collider.

The first part provides an overview of the Higgs theory and measurements, with some meta-analysis on the most recent results and focus on the Standard Model Effective Field theory (SMEFT). The second part is concerned with single-Higgs production, and two-loop calculation of  $Zh$  production via gluon fusion. Then a SMEFT analysis of the interplay between Higgs self-coupling and four heavy quark operators stemming from Higher order effects.

The third part focuses on the Higgs pair production, an essential process for measuring Higgs-self coupling. Employing multivariate analysis to study its potential for probing light Yukawa couplings.

Lastly, some models aims to explain the recent flavour anomalies are proposed, in the light of a global SMEFT Bayesian analysis.

**Keywords:** Higgs Physics, Standard Model Effective Field Theory, Flavour observables, Statistical data analysis.

## Zusammenfassung

Diese Dissertation untersucht einige Aspekte für die Zukunft der Higgs-Messungen nach einem Jahrzehnt seiner Entdeckung, im Rahmen der Zukunft des LHC's. Der erste Teil bietet einen Überblick über die Higgsphysik und -Messungen, mit einigen Metaanalysen zu den aktuellen Ergebnissen und einem Schwerpunkt auf der Standardmodell-Effektivfeld-Theorie (SMEFT). Der zweite Teil befasst sich mit der Einzel-Higgs-Produktion. Zusätzlich werden Ergebnisse für  $Zh$ -Produktion via Gluon-Fusion in nächstführender Ordnung in der starken Kopplungskonstante im Niederenergielimes präsentiert. Dann eine SMEFT-Analyse der Interaktion zwischen Higgsselbstkopplung und vier Heavy-Quark-Operatoren, die von Korrekturen höherer Ordnung stammen.

Der dritte Teil konzentriert sich auf die Higgspaarproduktion, ein wesentlicher Prozess zur Messung der Higgsselbstkopplung. Unter Verwendung multivariater Analyse zur Untersuchung seines Potenzials zur Untersuchung leichter Yukawa-Kopplungen. Schließlich werden einige Modelle vorgeschlagen, um die letzten Flavour-Anomalies im inspiriert von einer globalen SMEFT-Bayesschen Analyse zu erklären.

**Schlagwörter:** Higgs Physik, Standardmodell-Effektivfeld-Theorie, Flavour Anomalies, Statistische Datenanalyse



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## List of abbreviations

**Colliders and working groups .**

<b>CERN</b>	Conseil européen pour la recherche nucléaire.
<b>LHC</b>	Large Hadron Collider
<b>HL-LHC</b>	High-Luminosity LHC
<b>CMS</b>	Compact Muon Solenoid
<b>ATLAS</b>	A Toroidal LHC ApparatuS
<b>LEP</b>	Large Electron-Positron Collider
<b>ALEPH</b>	Apparatus for LEp PHysics
<b>SLC</b>	Stanford Linear Collider
<b>FCC</b>	Future circular collider
<b>HXSWG</b>	Higgs cross-section working group
<b>PDG</b>	Particle data group

**Higgs and Standard Model physics.**

<b>SM</b>	Standard Model
<b>QCD</b>	Quantum chromodynamics
<b>QED</b>	Quantum electrodynamics
<b>EFT</b>	Effective field theory
<b>SMEFT</b>	Standard Model effective field theory
<b>HEFT</b>	Higgs effective field theory
<b>EW</b>	Electroweak
<b>VEV/ vev</b>	Vacuum expectation value
<b>EWSB</b>	Electroweak symmetry breaking
<b>EWPO</b>	Electroweak precision observables
<b>EWChL</b>	Electroweak chiral Lagrangian
<b>SSB</b>	Spontanious symmetry breaking

**$SU(N)$**  Special unitary (group) of dimension  $N$

**ggF** Gluon fusion (processes)

**$q\bar{q}A$**  Quark anti-quark annihilation (processes)

**PDF** Parton distribution functions

**STXS** Simplified template cross-sections

**Higher order computations.**

**RGE** Renormalisation group equation or evolution

**LO, NLO ...** Leading order, Next to leading order etc.

**HTL** Heavy top limit

**HPL** Harmonic polylogarithms

**GPL** Generalised polylogarithms

**HE** High energy expansion

**Flavour.**

**CKM** Cabibbo-Kobayashi-Maskawa-Matrix

**$\mathcal{CP}$**  Charge conjugation and parity

**MFV** Minimal flavour violation

**AFV** Aligned flavour violation

**SFV** Spontaneous flavour violation

**PDD** Phenomenological data-driven

**PMD** Phenomenological model-driven

**FCNC** Flavour-changing neutral currents

**LUV** Lepton universality violation

**Data analysis/statistics.**

**MC** Monte Carlo (simulation)

**ML** Machine learning

**BDT** Boosted decision tree

**XGBoost** EXtreme gradient boosted decision tree

<b>DNN</b>	Deep Neural Networks
<b>MCMC</b>	Markov chain Monte Carlo (Bayesian analysis)
<b>PCo</b>	Principle component
<b>FDR</b>	False discovery rate
<b>ANOVA</b>	Analysis of variation
<b>HDPI</b>	Highest density posterior interval
<b>CI</b>	Credible interval (Bayesian statistics)
<b>CL</b>	Confidence interval (Frequentiststatistics)

**New Physics.**

<b>4F</b>	Four-fermion
<b>NP</b>	New physics
<b>BSM</b>	Beyond the Standard Model
<b>VLQ</b>	Vector-like quarks
<b>LQ</b>	Leptoquarks
<b>2HDM</b>	Two-Higgs-doublet model
<b>CHM</b>	Composite Higgs model
<b>MSSM</b>	Minimal supersymmetric Standard Model
<b>SILH</b>	Strongly interacting light Higgs

# 1 Introduction

Since the discovery of the Higgs particle in 2012 by a collaborative efforts between ATLAS [1] and CMS [2] experiments at the Large Hadron Collider (LHC), and the Standard Model of particle physics (SM) [3–5], has been completed [6–10], albeit leaving us with more questions than what is has answered. The most prominent question was: *What are the properties of this newly-discovered particle ?*

Answering this very question has become the preeminent goal of the LHC. Higgs measurements are getting progressively accurate, and our understanding of this particle is approaching few percent-level. The future runs of the LHC, will open the doors of Higgs-precision era. However, increased luminosity, i.e. data acquisition from the LHC, without improving the theoretical prediction of Higgs processes is futile. Therefore, to insure the success of the experimental efforts in probing Higgs couplings and properties at the precision-level, it is imperative to include higher-order calculations for Higgs production cross-sections.

An example of such processes, is the associated production of Higgs with  $Z$  bosons, which suffers from higher theoretical uncertainties compared to its sister process the  $Wh$  production, due to the presence of the gluon fusion sub-process  $gg \rightarrow Zh$ . These uncertainties can be improved by including the two-loop corrections to the gluon fusion sub-process efficiently using state-of-the-art analytic techniques [11].

After a decade of *Higgs physics*, and over ten-thousand Higgs-related publications, we still have a lot to learn about the Higgs boson. In particular, the structure of the Higgs potential is still unknown, as well as its couplings to the light quarks and leptons. We want to know more about the Higgs potential, as it will reveal to us whether there are new scalars beyond the Higgs boson which we have observed. Measuring Higgs coupling to light fermions is essential in understanding the source of masses, and explaining the significant hierarchy between the masses across the three generations of matter.

The High Energy Physics community, anticipated that the Higgs discovery, particularly given its mass  $m_h < 130$  GeV, will be followed by the detection of a *Zoo* of particles stemming from Supersymmetric extension of the SM, cf. [12]. Unfortunately, this was not the case, and we have not seen any new particles discovered after the Higgs boson. This motivated parametrising new physics (NP) effects in a model-independent manner, specially within the Standard Model Effective Field Theory (SMEFT) framework [13, 14]. In this formalism, all NP interactions are summarised in 56 terms, which make very little assumptions about what physics beyond the SM might be, guaranteeing a model-independent approach to collider searches.

The use of SMEFT in higher-order calculations of Higgs rates has revealed insights on the Higgs potential by the appearance of Higgs trilinear self-coupling within electroweak loop corrections of single-Higgs processes [15]. With the help of precision measure-

ments of the Higgs, the trilinear coupling could be constrained[16]. Nevertheless, more SMEFT observables can also enter in single-Higgs loops that alters the constraining power of these measurements. In fact, the interconnectivity between the Higgs and top-quark sectors are further emphasized within the SMEFT framework, as recent global fits have established strong correlations between observables from both sectors as well as the electroweak precision observables (EWPO) [17]. Strong correlations between the top and EWPO are also seen at loop-level [18], thus one expects to see similar correlations emerging from loop effects of top operators on Higgs processes.

The observation of Higgs pairs is slated for the High-Luminosity (HL) LHC operating phase. This rare –if observed– process will be the pièce de résistance of the LHC Higgs physics programme [19], directly measuring the Higgs self-interaction. Also, untangling Higgs potential measurements from the top-sector interactions. Furthermore, this process could be of great utility in probing Higgs coupling to light quarks, as well. The full potential of Higgs pair production can be exploited when it is treated as a multivariate problem by implementing interpretable machine learning analysis technique [20]. In this manner, it is possible to have a simultaneous measurement of the two most elusive Higgs interactions, the trilinear self-coupling and light Yukawas.

Recent measurements, by Belle and Babar, in addition to the LHCb experiment at CERN, of  $B$ -mesons semi-leptonic decays showed some tension with the SM predictions of lepton flavour universality of electroweak couplings [21–25], with up to  $\sim 3\sigma$  deviation from the SM [26–29]. These anomalies require typically models with some flavour violation, that makes model-building for explaining these anomalies as tree-level an Augean task [30–39]. Additionally, to complicate things further, these anomalies are in dissonance with EWPO. Hence, promoting for a more careful treatment of these anomalies; introducing them at loop-level in SMEFT and performing a global fit combining both flavour and EWPO data. Thus allowing for a SMEFT guided UV-model building for these anomalies, with extended Higgs and top sectors.

**This work is structured as follows:** We start by an introduction to the theory of the Higgs field and its role in the SM in chapter 2, followed by a theoretical constraints on the Higgs. In chapter 3, a review of the state-of-the-art Higgs measurements and the constraints on Higgs couplings derived from the latest LHC data. After that, I present the basics of Effective Fields Theories relevant to Higgs physics at the LHC in chapter 4. The second part of the thesis focuses on the production of -single- Higgs at the LHC, starting with an overview in chapter 5, followed by a discussion of the use of  $p_T$ -expansion technique for obtaining an analytic expression for the virtual correction of the gluon fusion  $Zh$  production in chapter 6. chapter 7 showcases the potential for single-Higgs processes to probing four-fermion operators from the top sector of SMEFT along with the trilinear self-coupling by performing Higher-order computations of these processes in SMEFT, followed by a Bayesian fit.

The third part of the thesis focuses on the production of Higgs in pairs, namely at the HL-LHC in chapter 8. Then, I show the potential for employing Higgs pair production to

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probe light quark coupling to the Higgs. In addition the methods of multivariate analysis to maximise the efficiency of extracting the di-Higgs signal using Interpretable machine learning. The last part of the thesis, has only [chapter 10](#) and it is about potential UV models for the  $B$  anomalies, inspired by a global SMEFT fit and minimal flavour violation (MFV).



# List of publications

1. **L. Alasfar**, J. de Blas and R. Gröber  
*“Higgs probes of top quark contact interactions and their interplay with the Higgs self-coupling,”*  
arXiv:2202.02333 [hep-ph].
2. **L. Alasfar**, G. Degrassi, P. P. Giardino, R. Gröber and M. Vitti  
*Virtual corrections to  $gg \rightarrow ZH$  via a transverse momentum expansion*  
JHEP **05** (2021), 168  
arXiv:2103.06225 [hep-ph].
3. **L. Alasfar**, A. Azatov, J. de Blas, A. Paul and M. Valli  
*B anomalies under the lens of electroweak precision*  
JHEP **12** (2020), 016  
arXiv:2007.04400 [hep-ph].
4. **L. Alasfar**, R. Corral Lopez and R. Gröber  
*Probing Higgs couplings to light quarks via Higgs pair production*  
JHEP **11** (2019), 088  
arXiv:1909.05279 [hep-ph].



# Part I

## Higgs Physics



## 2 The Standard Model Higgs boson

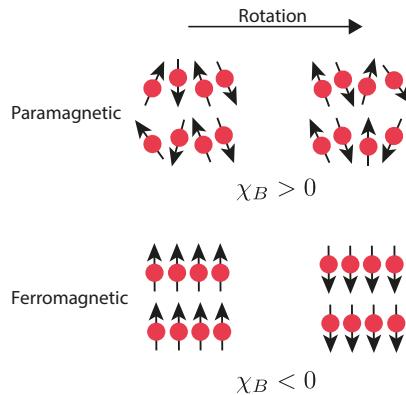
It's very nice to be right sometimes...  
it has certainly been a long wait.

---

Peter Higgs

### 2.1 Spontaneous symmetry breaking

Before talking about symmetry breaking, we need to discuss the concept of symmetry in physics. Symmetry is a crucial part in studying physical systems. It manifests not only as a geometric feature of physical objects but also in the dynamics of physical systems. For example, one can find symmetries in the equation of motion, Lagrangians/Hamiltonians and actions. The magnetisation of materials is a good example of the role that symmetry plays in describing physical behaviour. For instance, paramagnetic materials have a positive magnetic susceptibility  $\chi_B$  due to the random arrangement of their electrons' spins. The paramagnetic material spins arrangement will therefore possess rotational symmetry. The material has no *preferred direction* in space [40]. Au contraire ferromagnetic materials with the electrons' spins aligned in a certain direction, will not have such symmetry as there will be a preferred direction, see Figure 2.1.



**Figure 2.1.** In paramagnetic materials, the spins are randomly distributed such that a rotation performed on the system will keep the spin distribution invariant. However, for ferromagnetic materials, where the spins are aligned in a single direction, the symmetry is broken, and the system has a preferred direction.

In particle physics and quantum field theory, symmetry plays an essential role in the

taxonomy and dynamics of elementary particles and their bound states, i.e. hadrons, cf. [41, 42]. There are two types of symmetries considered when studying elementary particles and their quantum fields: external and internal symmetries. The first is the symmetry of the spacetime background. Typically, this is a four-dimensional Poincaré symmetry. However, in some models, higher spacetime dimensions or non-flat geometries are considered. Though there is no current evidence of higher dimensions or indications of non-flat spacetime from colliders and cosmological observations [43]. The second class of symmetries is internal symmetries stemming from the quantum nature of these particles/fields. Because their state is described by a ray in complex Hilbert/Fock spaces, internal symmetries are simply symmetries of rotations in these spaces that keep the action variation unchanged. Internal symmetries are usually described in terms of simple or product of simple Lie groups, e.g.  $SU(N)$ <sup>1</sup>, and particles/fields will be arranged as multiplets in some representation of the groups. If the rotations of the states could be parametrised by constants, the symmetry is called global. Alternatively, if these transformations are themselves functions of the spacetime, the symmetry is then called local or **gauged**.

Gauge symmetries describe rotations in the state space that depend on spacetime, the generator of the gauge transformations could propagate between two spacetime points. This is the way particle interactions are described in quantum field theory. The generators of these gauge transformations are called gauge bosons, and they mediate the interactions between the particles and hence transform under the adjoint representation of the gauge group. Gauge symmetries are the basis of describing the fundamental interactions of nature, called gauge theories.

An example of a gauge theory that is realised in nature is the **Standard Model** (SM), which is a gauge theory based on the group  $G_{\text{SM}} := SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ . The first simple group is for the *strong* interaction described by quantum chromodynamics (QCD). The product of the two remaining groups  $SU(2)_L \otimes U(1)_Y$  forms the Weinberg-Salam *electroweak* (EW) model [3–5], where  $SU(2)_L$  describes the weak interaction which only couples to *left handed* fermions and  $U(1)_Y$  is the weak hypercharge  $Y$  gauge group, defined by the formula

$$Y = 2(Q - T_3). \quad (2.1)$$

Where  $Q$  is the electric charge and  $T_3$  is the third component of the weak isospin. A description of the matter content of the SM and their multiplicities with respect to  $G_{\text{SM}}$  is shown in Table 2.1

The SM has been very successful at describing particle interactions even when challenged by numerous precision tests at LEP and SLD [47–50], and later at DØ [51] and the LHC [52, 53]. Nevertheless, it fails to describe the ground state if only the fermion and gauge sectors are considered. The reason for this shortcoming is that the  $W^\pm$  and  $Z$  bosons are massive, this violates the EW gauge symmetry. This can be easily seen by

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<sup>1</sup>Gauge theories based on finite groups have been proposed in the literature, but their phenomenological significance is yet to be further investigated [44, 45]

Particle/Field	$G_{\text{SM}}$ multiplicity	mass [GeV]
<b>Quarks</b>		
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	$m_u = 2.16 \cdot 10^{-3}, m_d = 2.67 \cdot 10^{-3}$
$U = u_R, c_R, t_R$	$(\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	$m_c = 0.93 \cdot 10^{-2}, m_s = 1.27$
$D = d_R, s_R, b_R$	$(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	$m_t = 172.4, m_b = 4.18$
<b>Leptons</b>		
$L = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$m_e = 0.511 \cdot 10^{-3}, m_\mu = 1.05 \cdot 10^{-2}$
$E = e_R, \mu_R, \tau_R$	$(\mathbf{1}, \mathbf{1})_{-1}$	$m_\tau = 1.77, m_\nu = ??$
<b>Gauge bosons</b>		
$g/G_\mu^A, A = 1 \dots 8$	$(\mathbf{8}, \mathbf{1})_0$	0.0
$\gamma/A_\mu$	$(\mathbf{1}, \mathbf{1})_0$	0.0
$W_\mu^\pm$	$(\mathbf{1}, \mathbf{3})_0$	80.379
$Z_\mu$	$(\mathbf{1}, \mathbf{3})_0$	91.1876
<b>The Higgs boson</b>		
$h$	$(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	125.10

**Table 2.1.** The SM constituents, their multiplicities with respect to the SM gauge group  $G_{\text{SM}} := SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  and masses. The mass of the neutrinos  $\nu$  is zero according to the SM prediction, but observations suggest that they are massive, and only the difference between the three masses is known [46]. The values of the masses are taken from the Particle Data Group (PDG) [43], and used throughout this thesis.

looking at the mass term of a spin 1 field  $B_\mu^A$

$$\mathcal{L} = m_B B^{A,\mu} B_\mu^A, \quad (2.2)$$

and performing an  $SU(N)$  gauge transformation

$$B_\mu^A \rightarrow B_\mu^A + \partial_\mu \Lambda^A + g \varepsilon_{BC}^A B_\mu^B \Lambda^C. \quad (2.3)$$

We see that the mass term does not preserve gauge symmetry. Secondly, because the SM is a chiral theory, as only left-handed fermions are doublets under  $SU(2)_L$ . Thus, the Dirac mass term

$$\mathcal{L}_D = m_D \bar{\psi}_L \psi_R + \text{h.c.}, \quad (2.4)$$

cannot be a singlet under  $SU(2)_L$ , therefore violates the EW symmetry. Despite quark and lepton masses being forbidden by the EW symmetry, we have already measured their masses, and since they also carry charges this mass has to be a Dirac mass.

In order for the EW model to be consistent in the ground state like it is in the interaction states. A mechanism for spontaneous symmetry breaking (SSB) needed to be introduced.

### 2.1.1 Nambu-Goldstone theorem

Coming back to the example of the paramagnetic-ferromagnetic materials, when a ferromagnetic metal is heated above a certain temperature, known as the Curie Temperature  $T_C$ , it will undergo a phase transition and become paramagnetic. In the mean-field theory approximation the magnetic susceptibility is related to the temperature of the metal via the relation

$$\chi_B \sim (T - T_C)^{-\gamma}, \quad (2.5)$$

where  $\gamma$  is called a critical exponent. We see that if the metal temperature  $T > T_C$  the metal is in an *disordered phase* and when  $T < T_C$  it is in the *ordered phase*, i.e.  $\chi_B$  is the order parameter of this system. At the Curie temperature, the system will be at the *critical point* and the susceptibility is divergent. The exponent  $\gamma$  cannot be used to describe the system at the critical point.

There is a “pictorial” description of the metal at the critical point which is helpful in understanding the Nambu-Goldstone theorem. Starting at  $T > T_C$ , the metal would be in a paramagnetic phase, where the spins are randomly arranged. As the temperature becomes lower and lower, thermal fluctuations start to lessen. In some regions of the metal, the spins will start to get aligned. With continued cooling, nearing  $T_C$ , these turned spins will affect their neighbours by flipping their direction. At the critical point  $T = T_C$ , the system behaves in a peculiar manner, when one would see regions of spins in “up” and others in “down” directions. The system will resemble a fractal of these regions, becoming scale-invariant. Additionally, waves of oscillating local magnetisation will propagate. These waves, or spinless quasiparticles (called Magnons) are Goldstone bosons emerging from SSB Which will manifest at  $T < T_C$  as the spins will be arranged

in a certain single direction and the metal becomes ferromagnetic.

### The Nambu-Goldstone theorem

When a continuous symmetry has a conserved currents but broken in the ground state (vacuum) is called to be spontaneously broken. There is a scalar boson associated with each broken generator of this spontaneously broken symmetry. The modes of these bosons are fluctuations of the order parameter.

This theorem first emerged from condensed matter physics, particularly superconductors [54, 55]. However, it soon got applied to relativistic quantum field theories [56].

## 2.2 The Braut-Englert-Higgs mechanism

In order to solve the aforementioned shortcomings of the Weinberg-Salam model, Nambu-Goldstone theorem has been first proposed by P. W. Anderson [57]. However, the way that Anderson formulated his theory was unfamiliar to particle physicists and used a non-relativistic picture to illustrate how photons could gain mass in an electron plasma with a plasma frequency  $\omega_p$

$$m_\gamma^{\text{plasma}} = \frac{\hbar\omega_p}{c^2} \quad (2.6)$$

Later on, a theory that explains the mass generation of the EW gauge bosons has been published in an almost simultaneous manner by R. Braut and F. Englert [6], P. Higgs [7, 8] and G. Guralnik, C. R. Hagen, and T. Kibble [9, 10]<sup>2</sup>. The Higgs mechanism starts by considering the SSB of the electroweak sector of the SM via the pattern

$$SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_Q \quad (2.7)$$

This is achieved by the vacuum expectation value (vev) of a complex scalar field  $\phi \sim (\mathbf{1}, \mathbf{2}, +1/2)$ , with the Lagrangian

$$\mathcal{L} = D_\mu \phi^* D^\mu \phi - V, \quad V := \mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2, \quad (2.8)$$

with  $V$  denoting the Higgs potential, illustrated in Figure 2.2, which gives non-vanishing vacuum for  $\mu^2 < 0$ . The field  $\phi$  is given explicitly by

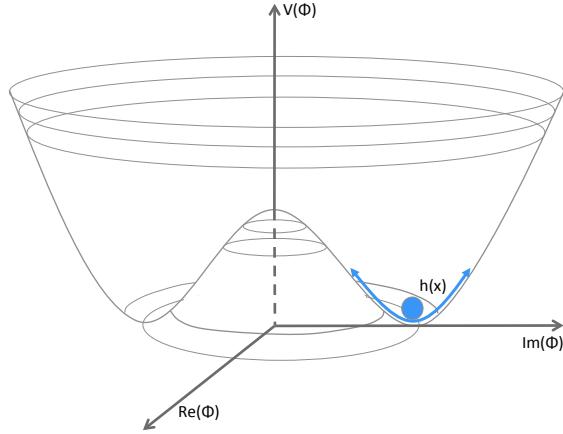
$$\phi = \begin{pmatrix} \phi^1 + i\phi^2 \\ \frac{1}{\sqrt{2}}(h + v) - i\phi^3 \end{pmatrix} \quad (2.9)$$

The covariant derivative

$$D_\mu = \partial_\mu - ig_2 \frac{\sigma_a}{2} W_\mu^a - ig_1 \frac{1}{2} B_\mu, \quad (2.10)$$

---

<sup>2</sup>All of these authors have contributed to the theory of SM (SSB). By calling it the “Braut-Englert-Higgs” mechanism or the “Higgs” boson. I, by no means, have intended to ignore the role played by the rest, rather, I wanted to stick the most widely-used terminology in the field.



**Figure 2.2.** The characteristic shape of the Higgs potential showing a non-zero vacuum. While the physical Higgs boson is an oscillation within the energy well illustrated in the diagram with blue arrows., this illustration is taken from [58].

dictates the coupling between the Higgs field and the EW gauge bosons and  $g_3$ ,  $g_2$  and  $g_1$  are, respectively, the coupling constants of  $SU(3)_C$ ,  $SU(2)_L$  and  $U(1)_Y$ . The minimum of the scalar potential is obtained by

$$\frac{\partial V}{\partial \phi} |_{\phi \rightarrow v} = 0, \quad (2.11)$$

which for a tachyonic mass  $\mu^2 < 0$  will have a real non-vanishing values  $v$  corresponding to the vev of this field  $\langle \phi \rangle = (\frac{0}{\sqrt{2}})$ .

According to Nambu-Goldstone theorem, the three broken generators of  $SU(2)_L \otimes U(1)_Y$  will become massive, and they are the  $W^\pm$  and  $Z$  bosons, while the photon will remain massless. We will have three massless Goldstone bosons  $G^\pm = \frac{1}{2}(\phi^1 \pm i\phi^2)$  and  $G^0 = \phi^3$  that are “eaten” by the aforementioned massive  $W^\pm$  and  $Z$  bosons, where they become their longitudinal polarisations. In order to see this more concretely, we start by looking at the terms of the EW Lagrangian where the field  $\phi$  couples to the gauge bosons, in the unbroken phase

$$D_\mu \phi^* D^\mu \phi = \frac{1}{2} |\partial_\mu \phi|^2 + \frac{1}{8} g_2^2 |\phi|^2 |W_\mu^1 + iW_\mu^2|^2 + \frac{1}{8} |\phi|^2 |g_2 W_\mu^3 - g_1 B_\mu|^2 \quad (2.12)$$

After SSB, we write the gauge bosons in the mass basis

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2), \\ Z_\mu &= \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 W_\mu^3 - g_1 B_\mu), \\ A_\mu &= \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 W_\mu^3 + g_1 B_\mu). \end{aligned} \quad (2.13)$$

From this, the electric charge is identified as the coupling constant to the photon  $A_\mu$

$$e = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}. \quad (2.14)$$

It is useful to define the Weinberg angle  $\theta_W$ , an important EW parameter relating the electric charge to the weak coupling  $g_2$

$$\sin \theta_W = \frac{e}{g_2} \approx 0.231214, \quad (2.15)$$

typically the sin and cos of the Weinberg angle are denoted by  $s_W$  and  $c_W$ , respectively. We use the unitary gauge, to absorb the Goldstone bosons into the  $W^\pm$  and  $Z$  longitudinal polarisations. In this gauge the Higgs doublet can be written as

$$\phi \rightarrow \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(h + v). \end{pmatrix}, \quad v = 246 \text{ GeV}. \quad (2.16)$$

With these substitutions, one can read off the masses of the gauge bosons from their bilinear terms in (2.12)

$$m_W = \frac{v g_2}{2} \quad m_Z = \frac{v}{2} \sqrt{g_1^2 + g_2^2} \quad m_A = 0. \quad (2.17)$$

Since  $\phi$  is a complex doublet. We have seen that it has four components, and three of them correspond to the Goldstone bosons, thus one remains physical field  $h(x)$  which is what we now identify with the ‘‘Higgs boson’’ discovered in the Summer of 2012 [1, 2]. The couplings between the Higgs and the electroweak bosons is related to their mass via the vev

$$g_{hVV} = \frac{2m_V^2}{v}, \quad g_{hhVV} = \frac{2m_V^2}{v^2}. \quad (2.18)$$

By substituting (2.16), into the Higgs potential (2.8) one can also write the mass of the physical Higgs boson in terms of the vev

$$m_h = \sqrt{2\lambda}v. \quad (2.19)$$

The Higgs boson mass is related to the  $\mu$  parameter via the relation

$$m_h^2 = -2\mu^2, \quad (2.20)$$

One can see that the mass term after SSB changes its sign, characterising the order-parameter for this system, analogous to the magnetic susceptibility for the magnetisation of materials example. One could also identify the self-couplings of  $h$ , the trilinear and quartic couplings

$$g_{hhh} = 3\lambda v = 3\frac{m_h^2}{v}, \quad g_{hhhh} = 3\lambda = 3\frac{m_h^2}{v^2}. \quad (2.21)$$

### 2.3 Yukawa interaction

It is possible to also use the Higgs vev to give fermions their masses by introducing Yukawa-interaction terms, first introduced by S. Weinberg [5]

$$\mathcal{L}_{\text{Yuk}} = -y_e \bar{L} \phi E - y_d \bar{Q} \phi D - y_u \bar{Q} \tilde{\phi} U + \text{h.c.}, \quad (2.22)$$

with  $\tilde{\phi} = i\sigma_2\phi$  and  $y_e, y_d, y_u$  are  $3 \times 3$  matrices. These matrices are free parameters in the SM. As the Higgs boson acquires a the vev, the fermions will acquire a mass  $m_f = vy'_f$  and the Higgs boson coupling to the fermions is given by

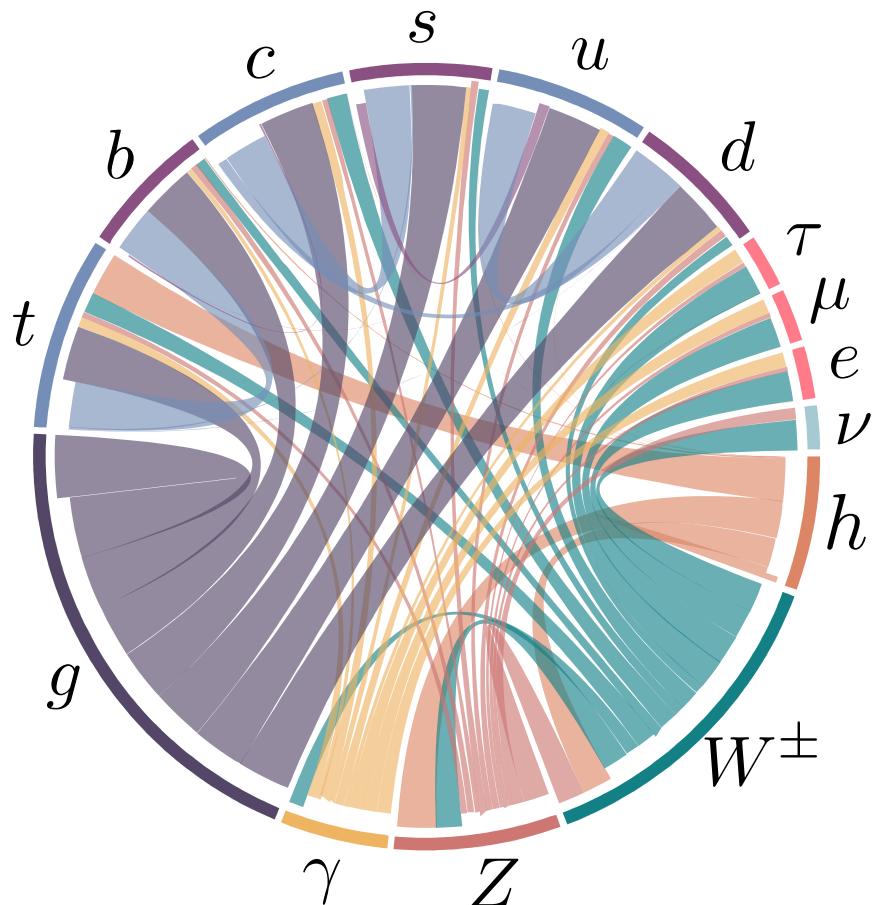
$$g_{h\bar{f}f} = \frac{m_f}{v}, \quad (2.23)$$

and the Yukawa matrices will be fixed in the mass basis  $y'_f$  by measurements of the fermion masses.

Leptonic Yukawa matrix is diagonal, with a degeneracy between the flavour and masses basis, this manifests as lepton family number conservation (the lepton family operator commutes with the Hamiltonian.). However, for the quarks, the situation is more complicated. One can rotate these matrices to the mass basis via a bi-unitary transformation via the unitary matrices  $\mathcal{V}_q, \mathcal{U}_q$  for  $q = u, d$

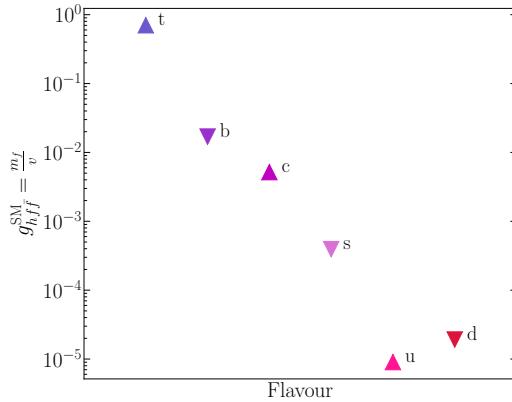
$$y_q \longrightarrow y'_q = \mathcal{V}_q^\dagger y_q \mathcal{U}_q = \text{diag}(m_{q_1}, m_{q_2}, m_{q_3}). \quad (2.24)$$

However, there is no degeneracy here as the Hamiltonian does not commute with the quark flavour operator. This is because the transformation matrices for the up and down-type quarks are not the same. The charged EW quark currents contain flavour mixing described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [59, 60]. Figure 2.3 shows all the SM couplings' strengths, with the thickness of the chord is proportional to the strength of the coupling, one can see the Higgs couplings in orange. In this figure, we cannot easily see Higgs coupling to the fermions, except for its couplings to the third generation. Strictly speaking, if we further examined the Yukawa coupling using a logarithmic scale and focused on the quark sector as Figure 2.4 illustrates. We observe that these Yukawa couplings span about 6 orders of magnitudes with marked hierarchy



**Figure 2.3.** A chord diagram showing the SM couplings, with the coupling strength illustrated by the chord thickness. Higgs couplings are coloured in orange.

amongst generations. As these couplings are in fact free parameters in the SM, and only determined by the experimental measurements of the quark (or equally applies lepton) masses. This hierarchy of quark masses therefore cannot be explained by the SM Braut-Englert-Higgs mechanism, and sometimes known as the “old” flavour puzzle. In later chapters, we will examine the experimental effort to better measure these couplings and how Higgs pair production can be used to probe them in chapter 9. Even the potential of using techniques from *interpretable machine learning* to further improve Higgs pair sensitivity to probing light quarks Yukawa couplings ??.



**Figure 2.4.** The SM Yukawa couplings are proportional to the quark masses, because of the the Higgs Yukawa couplings span about 6 orders of magnitude, as seen in the case of quarks here. This large hierarchy cannot be explained by the SM.

## 2.4 The Higgs and EW precision observables

One of the most valuable sources for the study of new physics (NP) above the EW scale is provided by indirect tests of the SM via the so-called the EW precision observables (EWPO). These include, in particular, the very precise measurements at the  $Z$  pole performed at the Large Electron-Positron (LEP) collider and the Stanford Linear Collider (SLC). In corroboration with the Higgs-boson discovery and the experimental information collected at LHC and Tevatron, they provide strong constraints on theories beyond the SM (BSM) that lead to important deformations of the standard EW sector [18, 58, 61–68].

Higgs physics is deeply intertwined with the EW sector as many of the Higgs parameters are linked to EWPO. For instance, the Higgs vev is determined from Fermi’s constant  $v = (\sqrt{2}G_F)^{-1/2}$ , which is in turn fixed by the muon lifetime  $\tau_\mu$  measurements [69–72]. This can be seen when we examine the theoretical prediction for  $\tau_\mu$

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} \left(1 - \frac{8m_e^2}{m_\mu}\right) \left[1 - 1.810 \frac{\alpha}{\pi} + (6.701 \pm 0.002) \left(\frac{\alpha}{\pi}\right)^2\right], \quad (2.25)$$

then comparing this formula with  $\tau_\mu$  experimental measurements. This confrontation leads to a very precise measurement on  $G_F$  [43]

$$G_F = 1.1663787(6) \cdot 10^{-5} \text{GeV}^{-2}, \quad (2.26)$$

given the value of the fine structure constant  $\alpha^{-1} = 137.03599976(50)$ .

Another important EWPO is the ratio between the  $W$  and  $Z$  masses

$$\rho = \frac{m_W^2}{c_W^2 m_Z^2}. \quad (2.27)$$

At leading order (LO), this parameter is equal to unity in the SM. The  $\rho$  parameter depends on the representation of the scalar sector of the EW model having  $\phi_i$  scalars with  $T_i$  weak isospin and  $T_{3,i}$  being its third component, and a vev  $v_i$ , via the relation [73, 74]

$$\rho = \frac{\sum_i [T_i(T_i + 1) - T_{3,i}^2] v_i^2}{2 \sum_i T_{3,i}^2 v_i^2}. \quad (2.28)$$

From (2.28) one can see that a real triplet scalar, for instance, would not fit the experimental EW measurement of  $\rho$ . Hence, a complex doublet is the simplest scalar possible for the EW symmetry breaking. However, radiative corrections to the EW gauge bosons mass from vacuum polarisation diagrams could potentially cause  $\rho$  to deviate significantly from unity. This is not the case, as the experimentally measured value of  $\rho$  [43]

$$\rho_{\text{exp}} = 1.00038 \pm 0.00020 \quad (2.29)$$

Additionally, it is possible to think of an extended Higgs sector, where there are multiple scalars with different  $SU(2)_L$  multiplicities. Or, a composite Higgs sector, where the Higgs boson is a pseudo Nambu-Goldstone boson, cf. [75, 76]. How can such models be built assuring the  $\rho$  parameter is protected from change? The answer to this question lies in a symmetry of the Higgs Lagrangian known as custodial symmetry.

#### 2.4.1 Custodial symmetry

After SSB, a residual global symmetry known as the custodial symmetry protects the  $\rho$  parameter from obtaining large radiative corrections at higher orders in perturbation theory. This symmetry must be kept in extended or composite Higgs models. This symmetry can be seen by rewriting the Higgs potential as

$$V = \frac{\lambda}{4} \left( \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 - 2\mu^2 \right)^2. \quad (2.30)$$

This potential is invariant under  $SO(4) \simeq SU(2)_L \otimes SU(2)_R$  rotations. However, when the Higgs field acquires a non-vanishing vev,  $\phi_4 \rightarrow h + v$ , the potential becomes

$$V = \frac{\lambda}{4} \left( \phi_1^2 + \phi_2^2 + \phi_3^2 + h^2 + 2vh + v^2 - 2\mu^2 \right)^2, \quad (2.31)$$

which is only invariant under  $SO(3) \simeq SU(2)_V$  transformations, the diagonal part of the original group. This global SSB pattern comes alongside the EW-SSB of the gauge group  $SU(2)_L \otimes U(1)_Y$  as global  $SU(2)_L$  is itself the gauged  $SU(2)_L$  group. Additionally the  $T_3$  component of the  $SU(2)_R$  global group is the gauged  $U(1)_Y$  and the  $T^3$  component of the custodial group  $SU(2)_V$  is gauged as well and identified to be the electric charge operator, i.e. the generator of  $U(1)_Q$ .

$$\underbrace{SU(2)_R}_{\supset U(1)_Y} \otimes \overbrace{SU(2)_L}^{\text{gauged}} \longrightarrow \underbrace{SU(2)_V}_{\supset U(1)_Q}. \quad (2.32)$$

This pattern indicates that the symmetry is already broken by the gauging of the diagonal part of  $SU(2)_R$  (the hypercharge). The custodial symmetry is only *approximate* in the limit of  $g_1 \rightarrow 0$ , and  $\rho = 1$  is a consequence of  $g_1 \neq 0$ . The symmetry breaking pattern  $\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$  also allows us to identify the Goldstone bosons as the custodial triplet and the physical Higgs  $h$  as the custodial singlet, explaining the electric charge pattern they have.

We could use the isomorphism between the special orthogonal and special unitary groups to parametrise the Higgs doublet as an  $SU(2)_L \otimes SU(2)_R$  bidoublet

$$\mathcal{H} = (\tilde{\phi} \ \phi) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_4 - i\phi_3 & \phi_1 + i\phi_2 \\ \phi_1 - i\phi_2 & \phi_4 + i\phi_3 \end{pmatrix}, \quad (2.33)$$

with the bi-unitary transformations

$$\mathcal{H} \longrightarrow \mathcal{U}_L \mathcal{H} \mathcal{U}_R^\dagger \quad (2.34)$$

which leaves any traces of the form  $\text{Tr}(\mathcal{H}^\dagger \mathcal{H})$ , invariant. The Higgs potential could be rewritten in terms of the bidoublet

$$V = -\frac{\mu^2}{2} \text{Tr}(\mathcal{H}^\dagger \mathcal{H} + \frac{\lambda}{4} (\text{Tr}(\mathcal{H}^\dagger \mathcal{H}))^2) \quad (2.35)$$

The vev is hence written in this representation as

$$\langle \mathcal{H} \rangle = \frac{v}{\sqrt{2}} \mathbb{1}_{2 \times 2}. \quad (2.36)$$

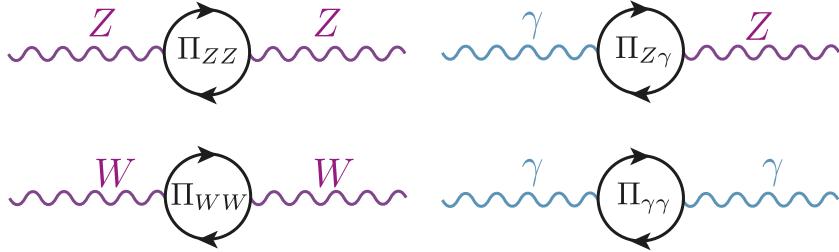
We can also look at the Yukawa sector, and observe that in the case where  $y_u = y_d = y$ , we can also write the left-handed and right-handed quarks as  $SU(2)_L \otimes SU(2)_R$  bidoublets and  $SU(2)_R$  doublets, respectively. Hence, the quark part of the Yukawa Lagrangian in (2.22) becomes

$$\mathcal{L}_{yuk} \supset \frac{y}{\sqrt{2}} (\bar{u}_L \ \bar{d}_L) \begin{pmatrix} \phi_4 - i\phi_3 & \phi_1 + i\phi_2 \\ \phi_1 - i\phi_2 & \phi_4 + i\phi_3 \end{pmatrix} \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \quad (2.37)$$

which is invariant under custodial transformations, but when  $y_u \neq y_d$ , this Lagrangian term breaks custodial symmetry. Thus, the differences between the up-type and down-type quark masses  $m_u - m_d$  are considered **spurions** of the custodial symmetry and one expects to see radiative corrections to  $\rho$  being proportional to these spurions.

In order to see this more concretely, we start by examining the radiative corrections that could lead to a deviation of  $\rho$  from unity( $\Delta\rho$ ). These corrections are known as the **oblique correction**, that come from electroweak vacuum polarisations  $\Pi_{VV}(p^2)$ , as shown in Figure 2.5, for more details on these corrections and their calculation see refs.. [77, 78]

The one-loop correction to the  $\rho$  parameter is given in terms of the  $\Pi_{VV}$  by



**Figure 2.5.** The oblique corrections, are radiative correction with electroweak gauge bosons propagators. Namely vacuum polarisations of the  $Z$ ,  $W^\pm$  and  $\gamma$  bosons.

$$\Delta\rho = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2} \quad (2.38)$$

Where the dominant contributions are given by [79]

$$\Delta\rho = \frac{3G_F}{8\sqrt{2}\pi^2} \left( (m_t^2 - m_b^2) - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2} \ln \frac{m_t^2}{m_b^2} \right) + \dots \quad (2.39)$$

Since  $m_b \ll m_t$ , the correction is non-vanishing, and (2.39) shows clearly how the radiative corrections are proportional to the spurions of the custodial symmetry. However, this radiative correction is absorbed into the SM definition of  $\rho$ , i.e. the  $\overline{\text{MS}}$  definition of the  $\rho$ -parameter  $\rho^{\overline{\text{MS}}}$ .

One can study new physics (NP) effects that violates custodial symmetry, by looking at deviations from  $\rho = 1$  coming from the NP degrees of freedom. Given the experimentally measured value of  $\rho$  (2.29) many NP models violating custodial symmetry can already be excluded. Nevertheless,  $\rho$  alone does not capture the full story of EWPO's. For instance, adding a new quark doublet would not necessarily violate the custodial symmetry though it still can be excluded by EWPO. It is hence useful to introduce new parameters known as **the oblique parameters** [78, 80–83]<sup>3</sup>

<sup>3</sup>The are also called the Peskin–Takeuchi parameters, however, W. Marciano and J. Rosner also D. Kennedy and P. Langacker published the same parametrisation proposals almost simultaneously. Therefore, I preferred not to use this eponym, instead calling them the oblique parameters, as they

## The oblique parameters

$$\begin{aligned}
 S &:= \frac{4c_W^2 s_W^2}{\alpha} \left[ \frac{\Pi_{ZZ}^{\text{NP}}(m_Z^2) - \Pi_{ZZ}^{\text{NP}}(0)}{m_Z^2} - \frac{c_W^2 - s_W^2}{c_W s_W} \frac{\Pi_{Z\gamma}^{\text{NP}}(m_Z^2)}{m_Z^2} - \frac{\Pi_{\gamma\gamma}^{\text{NP}}(m_Z^2)}{m_Z^2} \right], \\
 T &:= \frac{\rho^{\overline{\text{MS}}} - 1}{\alpha} = \frac{1}{\alpha} \left[ \frac{\Pi_{WW}^{\text{NP}}(0)}{m_W^2} - \frac{\Pi_{ZZ}^{\text{NP}}(0)}{m_Z^2} \right], \\
 U &:= \frac{4s_W^2}{\alpha} \left[ \frac{\Pi_{WW}^{\text{NP}}(m_W^2) - \Pi_{WW}^{\text{NP}}(0)}{m_W^2} - \frac{c_W}{s_W} \frac{\Pi_{Z\gamma}^{\text{NP}}(m_Z^2)}{m_Z^2} - \frac{\Pi_{\gamma\gamma}^{\text{NP}}(m_Z^2)}{m_Z^2} \right] - S.
 \end{aligned} \tag{2.40}$$

The NP contributions to the EW vacuum polarisations  $\Pi_{VV}^{\text{NP}}(p^2)$  could either come from loop or tree-level effects. Typically both  $T$  and  $U$  are related to custodial symmetry violation. However,  $U$  has an extra suppression factor of  $m_{\text{NP}}^2/m_Z^2$  compared to  $T$  and  $S$ . The most recent fit result for these parameters is [43]

$$\begin{aligned}
 S &= -0.01 \pm 0.10, \\
 T &= 0.03 \pm 0.13, \\
 U &:= 0.02 \pm 0.11.
 \end{aligned} \tag{2.41}$$

But since  $T$  and  $S$  tend to give stronger constraint on NP, due to the suppression factor of  $U$ . One can perform a two-parameter fit of  $S$  and  $T$  setting  $U = 0$ , thus shown in Figure 2.6, with the numerical values [43],

$$\begin{aligned}
 S &= 0.00 \pm 0.07, \\
 T &= 0.05 \pm 0.06.
 \end{aligned} \tag{2.42}$$

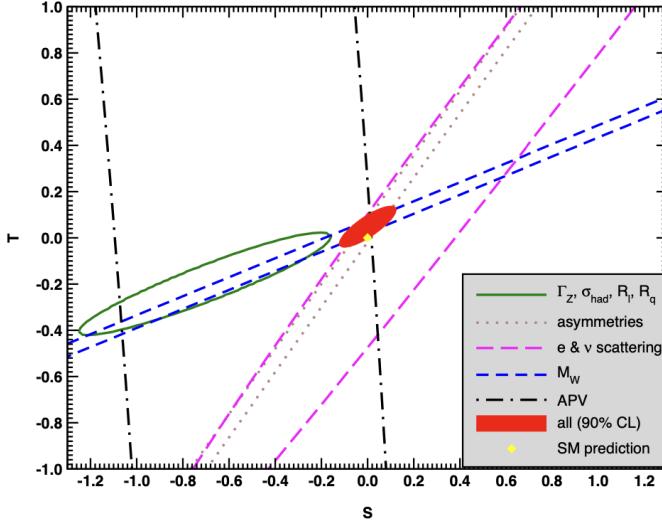
The oblique parameters are important in constraining effective operators in the Higgs sector , namely

$$\begin{aligned}
 \hat{O}_S &= \phi^\dagger \sigma_i \phi W_{\mu\nu}^i B^{\mu\nu}, \\
 \hat{O}_T &= |\phi^\dagger D_\mu \phi|^2.
 \end{aligned} \tag{2.43}$$

For example,  $\hat{O}_S$  appears in Technicolour models causing large deviations of  $S$  compared to its measured value [81, 84–86]. Moreover, The constraints on  $T$  parameter is important for top mass generation ans well as modifications to  $Zb\bar{b}$  coupling in such models [87, 88]. We will revisit the  $\hat{O}_T$  when we discuss the Higgs and effective field theories in chapter 4.

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stem from the oblique corrections .



**Figure 2.6.** Fit results from various EWPO's for  $T$  and  $S$  setting  $U = .$  The contours show  $1\sigma$  contours (39.35% for closed contours and 68% for the rest). This plot is obtained from the PDG [43].

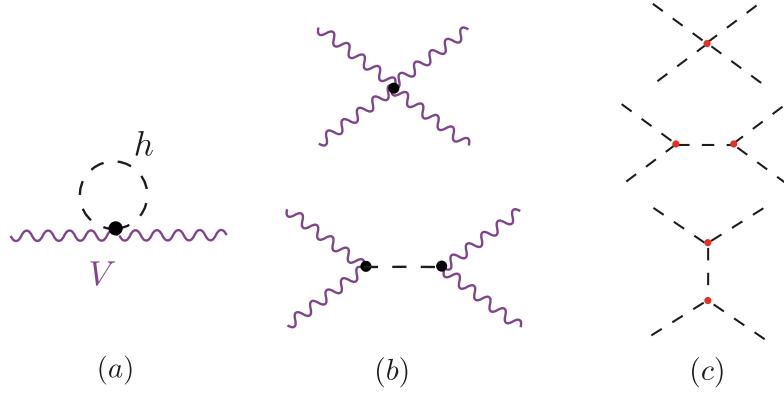
## 2.5 Theoretical constraints on the Higgs

### 2.5.1 Electroweak precision data fits

Even prior to the discovery of the Higgs boson at LHC in 2012, many theoretical aspects of the Higgs sector provided marked bounds on the Higgs properties, particularly its mass. For instance, using the EWPO measurements at LEP provided an input for a fit based of radiative effects coming from the Higgs boson to such observables [47] as in diagram (a) of Figure 2.7, the bounds improved with the improvements of EWPO measurements, these bounds were known as the “blue band” plots seen with their progression in Figure 2.8.

### 2.5.2 Partial-wave unitarity

Another bound on Higgs mass emerged from studying the longitudinally polarised elastic scattering amplitudes of the EW vector bosons  $V_L V_L \rightarrow V_L V_L$  at high energies  $E \gg m_W$ , where the Goldstone equivalence theorem holds [89], see diagrams (b) in Figure 2.7. This bound comes from applying the partial wave perturbative unitarity on the EW boson scattering amplitude. I will derive here this bound starting from the **Optical theorem**, which a direct result from the unitarity of the **S** matrix.



**Figure 2.7.** Diagrams contributing to theoretical bounds on the Higgs, (a) shows an example of radiative corrections to EWPO from the Higgs bosons. The diagrams in (b) show an elastic scattering of EW vector bosons leading to a bound on the Higgs mass from perturbative unitarity, similarly in (c) diagrams for  $hh \rightarrow hh$  scattering leading to constraints on Higgs self-coupling.

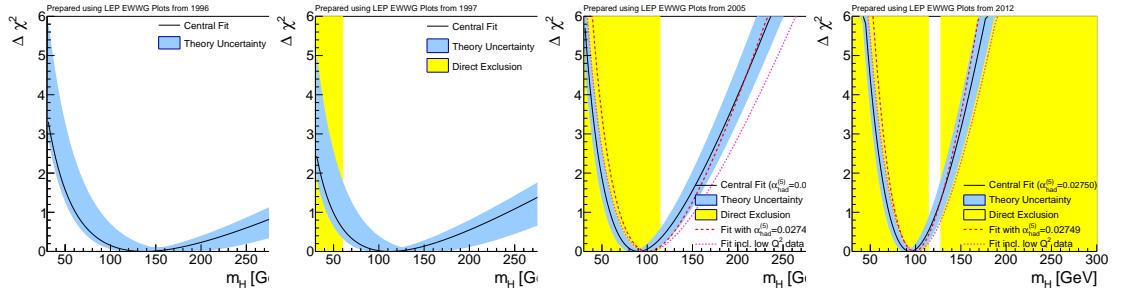
## The optical theorem

Let  $\mathcal{M}_{aa}$  be a covariant matrix element for an elastic scattering process with for a particle  $a$  then the following relation applies

$$\sum_f \int d\Phi_n(p_a, p_i^f) |\mathcal{M}_{af}|^2 = 2\Im(\mathcal{M}_{aa}), \quad (2.44)$$

where the sum is over all intermediate  $n$ -particle states  $f$  with momenta  $p_i^f$  and  $d\Phi_n(p_a, p_i^f)$  is the  $n$ -particle phase space.

If we only consider a  $2 \rightarrow 2$  process with momentum states,  $|p_1, p_2\rangle \rightarrow |k_1, k_2\rangle$ , then



**Figure 2.8.** Progression of the “blue band” plots with LEP data from 1996 up to 2012 prior to the announcement of the Higgs boson discovery. These plots were taken from [58], based on data from LEP [47].

the LHS of (2.44), after expanding the 2-particle phase space, simplifies to

$$\begin{aligned} & \int \frac{d^3 k_1}{(2\pi)^3 2E_1} \int \frac{d^3 k_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2) |\mathcal{M}(s, t)|^2, \\ & = \frac{1}{16\pi} \int_{-1}^1 d(\cos \theta) |\mathcal{M}(s, t)|^2, \end{aligned} \quad (2.45)$$

with the Mandelstam variables

$$\begin{aligned} s &= k_1 + k_2, \\ t &= k_1 - p_1, \\ u &= k_1 - p_2, \\ s + t + u &= 4m \end{aligned} \quad (2.46)$$

By using the relation between the Mandelstam variable  $t$ , and the scattering angle for the elastic scattering

$$t = \frac{1}{2}(s - 4m^2)(\cos \theta - 1) \quad (2.47)$$

We could expand the matrix element  $\mathcal{M}(s, t)$  in terms of *partial waves*, isolating  $s$  from scattering angle dependence

$$\mathcal{M}(s, t) = 16\pi \sum_j (2j+1) a_j P_j(\cos \theta). \quad (2.48)$$

Where  $a_j$  are called the  $j$ th partial wave amplitude, and  $P_j(\cos \theta)$  are the Legendre polynomials

$$P_j(z) = \frac{1}{j!} \frac{1}{2^j} \frac{d^j}{dz^j} (z^2 - 1)^j \quad (2.49)$$

Which satisfies the orthonormality condition

$$\int_{-1}^1 dz P_j(z) P_k(z) = \frac{1}{2j+1} \delta_{jk} \quad (2.50a)$$

$$P_j(1) = 1 \quad \forall j. \quad (2.50b)$$

We hence get for the LHS of (2.44) scattering

$$\begin{aligned}
 & \int \frac{d^3 k_1}{(2\pi)^3 2E_1} \int \frac{d^3 k_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2) |\mathcal{M}(s, t)|^2, \\
 &= \frac{1}{16\pi} \int_{-1}^1 d(\cos \theta) \left[ 16\pi \sum_j (2j+1) a_j(s) P_j(\cos \theta) \right] \times \\
 & \quad \left[ 16\pi \sum_k (2k+1) a_k^*(s) P_k(\cos \theta) \right], \\
 &\Rightarrow = 32\pi \sum_j (2j+1) |a_j(s)|^2. \tag{2.51}
 \end{aligned}$$

And the RHS of (2.44)

$$2\Im(\mathcal{M}_{aa}) = \underbrace{2\Im(\mathcal{M}(s, 0))}_{t \text{ is integrated out.}} = 32\pi \sum_j (2j+1) \Im(a_j(s)). \tag{2.52}$$

Otherwise large cancellations needed,  $a_j(s)$ 's are hierachal. Thus, we could compare the partial wave amplitudes term-by-term

$$|a_j(s)|^2 \leq \Im(a_j(s)) \Rightarrow \Re(a_j(s))^2 + \Im(a_j(s))^2 \leq \Im(a_j(s)) \tag{2.53}$$

Rearranging terms, we get

$$\Re(a_j(s)) + \left( \Im(a_j(s)) - \frac{1}{2} \right)^2 \leq \frac{1}{4} \tag{2.54}$$

The partial wave amplitude must remain within the unitarity circle for the perturbation theory to be valid.

$$\Re(a_j(s)) \leq \frac{1}{2} \tag{2.55}$$

This is known as the perturbative partial wave unitarity bound.

When (2.55) is applied for  $V_L V_L \rightarrow V_L V_L$ , in the Goldstone boson equivalence theorem regime in particular for  $V = W$  boson, we get for the  $S$ -wave ( $j = 0$ ) partial amplitude

$$a_0 \sim \frac{m_h^2}{16\pi v^2} \left( 2 + \mathcal{O}\left(m_h^2/s\right) \right). \tag{2.56}$$

Looking at the asymptotic behaviour as  $s \rightarrow \infty$ , we obtain the bound

$$\frac{m_h^2}{8\pi v^2} < \frac{1}{2} \Leftrightarrow m_h \leq 870 \text{ GeV}. \tag{2.57}$$

Indeed this bound is obsolete now after the Higgs mass measurement, however it is very important to demonstrate the power of this technique in constraining Higgs parameters. As this method can be applied to any elastic scattering with the Higgs acts as a mediator

like  $ZZ \rightarrow ZZ$ ,  $WW \rightarrow ff$  and constrain the corresponding couplings  $g_{ZZh}$ ,  $g_{f\bar{f}h}$  and so on. An important bound can be derived by examining the Higgs elastic scattering  $hh \rightarrow hh$  shown in (c) of Figure 2.7 in order to set bounds on Higgs self-interactions  $g_{hhh}$  and  $g_{hhhh}$ . This is what exactly has been done in ref. [90] where they have found that the  $S$ -wave partial amplitude for this process is given by

$$a_0 = -\frac{1}{2} \frac{\sqrt{s(s-4m_h^2)}}{16\pi s} \left[ g_{hhh}^2 \left( \frac{1}{s-m_h^2} - 2 \frac{\log \frac{s-3m_h^2}{m_h^2}}{s-4m_h^2} \right) + g_{hhhh} \right], \quad (2.58)$$

which leads to unitarity bounds on the trilinear  $g_{hhh}$  and the quartic  $g_{hhhh}$  couplings

$$\left| g_{hhh}/g_{hhh}^{\text{SM}} \right| \lesssim 6.5 \quad \text{and} \quad \left| g_{hhhh}/g_{hhhh}^{\text{SM}} \right| \lesssim 65. \quad (2.59)$$

A more stringent constraint can be obtained by looking at the one-loop correction to the  $hh \rightarrow hh$  scattering amplitude, within the full kinematic range. The unitarity bound here is obtained by looking at the one-loop amplitude at the threshold, and is given by

$$\left| g_{hhh}/g_{hhh}^{\text{SM}} \right| \lesssim 6. \quad (2.60)$$

It should be noted that the unitarity bounds on the trilinear coupling depends on the ansatz used estimating the size of the NP contributions to the scattering amplitudes. These bounds are, hitherto, the strongest on these two couplings even when compared to the ones coming from current experimental searches.

### 2.5.3 Other bounds

Further theoretical bounds can be obtained by studying quantum effects on the Higgs potential. For example, if we looked at the solution of the renormalisation group equation (RGE) for the Higgs self-coupling  $\lambda$  with the boundary condition  $\lambda(v) = \lambda_0$  and ignoring other SM particle-contributions

$$\lambda(Q^2) = \frac{\lambda_0}{1 - \frac{3}{4\pi^2} \log \frac{Q^2}{v^2}} \quad (2.61)$$

We see that the running of  $\lambda$  will hit a pole, known as Landau pole when the denominator vanishes. This will happen at the scale

$$Q_c = v e^{4\pi^2/3\lambda_0} = v e^{4\pi^2 v^2 / 3m_h^2} \quad (2.62)$$

This indicates that the theory will break down at scales larger or equal to  $Q_c$ . Since the “critical scale” is a function of the Higgs mass, this allows us to set an upper limit on the Higgs mass assuming the SM will be valid up to a certain scale  $Q_c$ . This bound is known as **quantum triviality** bound [91]. This is because the low energy behaviour of (2.61) leads to a vanishing interaction, and if we want the Higgs Lagrangian to be perturbative

for all scales, then  $\lambda$  has to be vanishing and the theory becomes non-interacting or *trivial*.

Another bound coming from the RGE of  $\lambda$  is the **stability bound**, which considers the stability of the Higgs potential given the running of  $\lambda$  by requiring that the Higgs potential is an operator bounded from below. This bound is obtained by approximating the solution of the RGE at small  $\lambda$

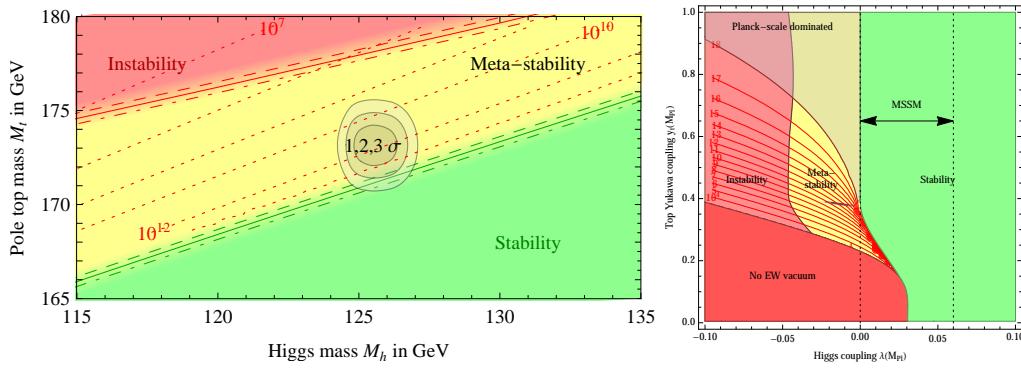
$$\lambda(Q^2) \sim \lambda_0 + \frac{1}{16\pi^2} \left[ -\frac{12m_t^4}{v^4} + \frac{3}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right] \log \frac{Q^2}{v^2} \quad (2.63)$$

For the Higgs potential to be bounded from below  $\lambda(Q^2)$  ought to be  $\lambda(Q^2) > 0$ . With this relation for  $\lambda_0$  in terms of the mass, we get a bound on  $m_h$

$$m_h^2 > \frac{v^2}{8\pi^2} \left[ -\frac{12m_t^4}{v^4} + \frac{3}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right] \log \frac{Q^2}{v^2} \quad (2.64)$$

Which leads to  $m_h \approx 130$  GeV if we assume that the SM is valid up to the Grand Unified Theory (GUT) scale of  $\sim 10^{16}$  GeV and  $m_h \approx 180$  GeV for  $Q$  being at the Planck scale  $\sim 10^{19}$  GeV.

More sophisticated calculations and discussion for the Higgs potential and vacuum stability has been a subject of great interest in pre and post-Higgs discovery eras cf. [91–94] and the most state-of-the-art calculation for the vacuum stability at two-loop level has been performed in ref. [95] where they also included finite temperature effects to construct a phase diagram in the  $m_t - m_h$  and  $m_t - \lambda(M_p)$  planes as shown in Figure 2.9. Indicating that the measured Higgs mass is likely compatible with a metastable vacuum rather than absolute stability. This indicates that there is a finite probability for the Higgs vacuum (false vacuum) to decay into a lower energy state (true vacuum) via quantum tunnelling.





# 3 Experimental measurements of the Higgs boson

The observation of the Higgs boson, along with the extensive measurement of its properties and couplings has been on the top of the LHC programme priorities [96]. In the time this thesis was written, the particle physics community will be celebrating a decade since the Higgs boson's discovery. Looking back 10 years ago, when I have witnessed the discovery of the Higgs boson via news press-conference in the summer of 2012, and decided to be a part of this enormous step that humanity has taken, I feel astonished by the progress made in understanding this newly discovered particle!

In this chapter, I will start by an overview of the extraordinary LHC and its experiments in section 3.1. Then, I will provide a state-of-the-art status review of the experimental measurements of the Higgs boson properties in section 3.2 and its cross-sections and couplings in section 3.3. At the end I will discuss the challenges and outlook for the future runs of the LHC section 3.4. The rest of the thesis will be aiming to address a small part of these challenges.

## 3.1 Overview of the Large Hadron Collider

The Large Hadron Collider (LHC) is the largest particle accelerator in the CERN accelerators complex, with a circumference of about 26 km and over 9590 superconducting magnets cooled to 1.9 K. It was built as an upgrade to the Large electron positron collider (LEP) which ended its operation in the year 2000. The LHC contains four main experiments situated at the four beam collision points and detectors, these experiments are: ATLAS, CMS, LHCb and ALICE, there also smaller experiments such as LHCf, MilliQan, TOTEM and others. For more details about the LHC cf. [97, 98] or the LHC technical design report [99] for an in-depth review.

The LHC started operation in September 2008, with low energy proton beams, then gradually increased to an energy of 3.5 TeV per proton to reach a centre of mass energy  $\sqrt{s}$  of 7 TeV, and data-taking period started from 2011 . By 2012, its energy has increased to  $\sqrt{s} = 8$  TeV and operated at this energy for about year and half, then stopping in mid 2013 concluding what is known as **Run-I**. In 2015, **Run-II** started with almost double the energy  $\sqrt{s} = 13$  TeV, and lasted for ca. 3 years. As this thesis being written, preparations are being made to get **Run-III** started and will last until 2024. During these runs, heavier nuclei such as  $^{207}\text{Pb}$  and  $^{131}\text{Xe}$  have been collided either with protons or with themselves [100].

From, 2025 and beyond, the **High-Luminosity LHC** (HL-LHC) era will commence,

see Figure 3.2. Where the LHC will be shutdown for extensive upgrades [101] to potentially increase its energy to  $\sqrt{s} = 14$  TeV and higher collision rates hence the term *high luminosity*. Which leads us to an important notion in particle physics phenomenology *integrated luminosity*.

The performance of colliders depends on many factors, but for phenomenological studies, like this thesis, the most important of which are the centre-of-mass energy  $\sqrt{s}$  and the integrated luminosity  $\mathcal{L}$ . This is mainly due to the fact that particle colliders experiments are basically “counting experiments”, and all of the bounds on physical observables or model parameters are obtained from the number of signal versus background events, and the number of expected events  $N_{\text{expec}}$  for a given resonance  $R$  and a subsequent decay final state  $X$  at any collider experiments is given by

$$N_{\text{expec}} = \sigma(pp \rightarrow R) \text{BR}(R \rightarrow X) \mathcal{L} \epsilon_{\text{SEL}}. \quad (3.1)$$

Here  $\epsilon_{\text{SEL}}$  is the selection efficiency, which depends on many factors like the detector geometry and particle identification performance etc., as well as the signal one searches for, it can be improved by better detected or selection cuts. The production cross-section increases typically with quadratically with  $\sqrt{s}$ , hence comes the need for higher energies but this can only achieved by building new colliders from scratch. The integrated luminosity, on the other hand, can be increased by running the experiment for a longer time, without the need for a new collider, This is because the integrated luminosity is the time integral of the collider’s luminosity  $L(t)$  over its operation time  $T$

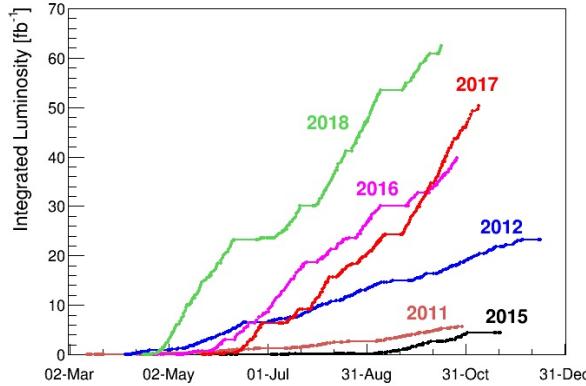
$$\mathcal{L} = \int^T L(t). \quad (3.2)$$

Therefore, we see that the integrated luminosity for the LHC experiments will increase over time, when more collisions taking place, as seen in figure Figure 3.1 showing the integrated luminosity for ATLAS and CMS experiments. As the protons travel in the LHC in **bunches**, when these bunches cross, the protons inside of them collide at a certain frequency  $f$ . When two bunches with  $N_1$  and  $N_2$  protons per bunch, respectively collide, each bunch will have an effective cross-section  $4\pi\sigma_i$  corresponding to their physical sizes  $\sigma \sim 16 \mu\text{m}$ , the luminosity is therefore given -approximately- by

$$L = \frac{f N_1 N_2}{4\pi\sigma_1\sigma_2}, \quad (3.3)$$

which is for the LHC averages to  $\sim 10^{34}$  collisions  $\text{cm}^{-2} \text{s}^{-1}$  [102, 103].

The total physics-viable  $pp$ -collisions integrated luminosity for Run-I was  $4.57 \text{ fb}^{-1}$  for 7 TeV and  $20.3 \text{ fb}^{-1}$  for 8 TeV (ATLAS [104]) and  $5.55 \text{ fb}^{-1}$  at 7 TeV and  $21.8 \text{ fb}^{-1}$  at 8 TeV (CMS [105]). As for Run-II the integrated luminosity is  $139 \text{ fb}^{-1}$  at 13 TeV (ATLAS [106]) and  $137 \text{ fb}^{-1}$  at 13 TeV (CMS [105]). The expected integrated luminosity by the end of Run-III is  $300 \text{ fb}^{-1}$  [107] and  $3000 \text{ fb}^{-1}$  by the end of the HL-LHC at energy of 14 TeV [101].



**Figure 3.1.** The integrated luminosity of the CMS and ATLAS experiments combined over the period from 2011-2018, source [102].

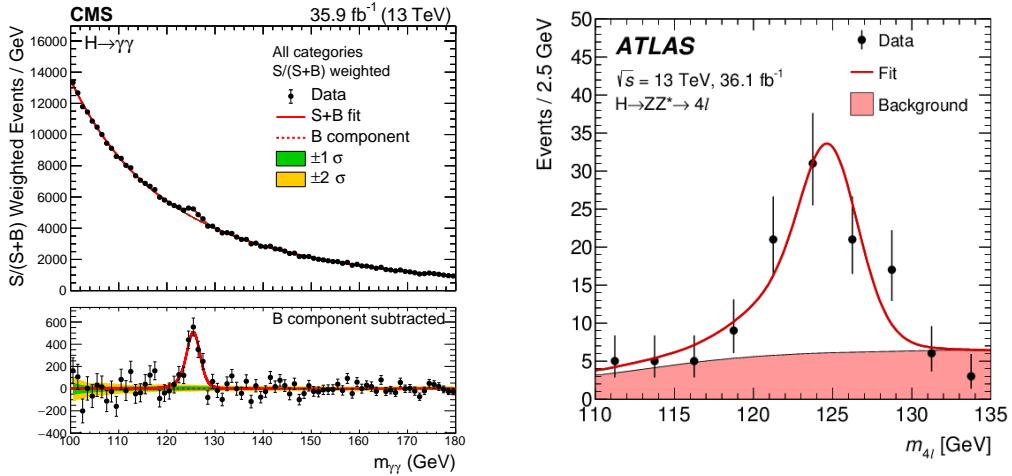


**Figure 3.2.** A timeline of the LHC operation showing Run-I, Run-II and future planned runs of the LHC, including the HL-LHC, source [100].

## 3.2 Higgs properties

### 3.2.1 Higgs boson mass measurements

In order to measure the mass of the Higgs boson with high precision, one need to consider final states that can be reconstructed with high momentum and mass resolutions, this is typically achieved when no hadronic constituents in the decays involved, such as



**Figure 3.3.** The invariant mass distributions of diphoton  $m_{\gamma\gamma}$  (CMS [108]) and four lepton  $m_{4\ell}$  (ATLAS [109]) final states showing a clear peak at the Higgs mass, with smooth background. These final states are ideal for Higgs mass measurements.

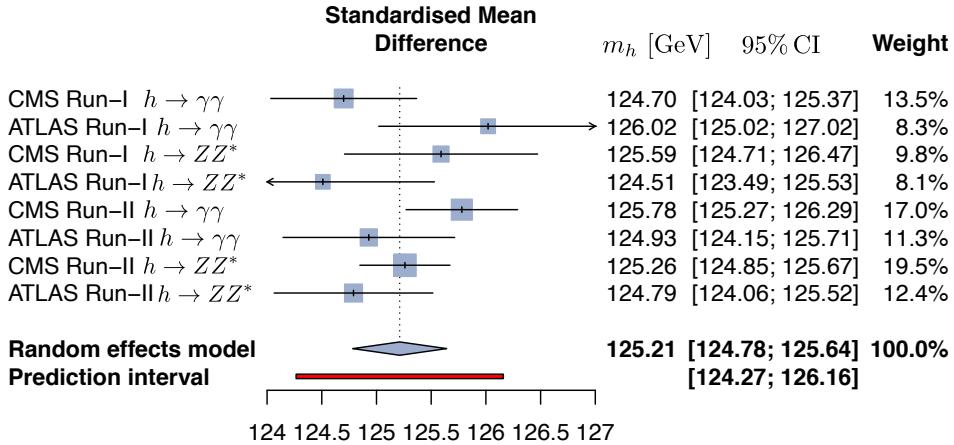
$h \rightarrow \gamma\gamma$  and  $h \rightarrow ZZ^* \rightarrow 4\ell$ . Reconstructing the invariant mass distributions  $m_{\gamma\gamma}$  and  $m_{4\ell}$  one observes that the Higgs peak is narrow over a relatively smooth background, see Figure 3.3, which is ideal for the measurement of the Higgs mass. It should be noted that the width of the resonance is due to the detector resolution and does not correspond to the actual Higgs width.

There have been consistent improvements of the Higgs mass measurements since its discovery. In Figure 3.4, I have performed a meta-analysis on ATLAS and CMS measurements of the Higgs mass in Run-I and Run-II of the LHC for both diphoton and  $ZZ^*$  final states based on the data from the studies [108–111] using a random effects model [112]. The pooling of the studies yielded a mass measurement of  $m_h = 125.21 \pm 0.10$ , which translates to a 0.11% accuracy, the heterogeneity of the studies was found to be  $I^2 = 49\%$  ( $p = 0.05$ ). Different measurements combination techniques were used in [108] and [43] yielded different central values but all of the results agree within the uncertainties.

### 3.2.2 Higgs full width

The SM values of the Higgs boson full width is  $\Gamma_h = 4.1$  GeV and it can be accessed in the LHC by looking at the ratio of on-shell vs off-shell Higgs production and decay to the  $ZZ^{(*)}$  state, and  $ZZ^{(*)} \rightarrow 4\ell, 2\ell 2\nu$ , namely

$$\frac{\sigma(gg \rightarrow h \rightarrow ZZ^*)}{\sigma(gg \rightarrow h^* \rightarrow ZZ)} = \kappa_g^2 \kappa_Z^2 \frac{4m_Z^2}{m_h \Gamma_h}, \quad (3.4)$$



**Figure 3.4.** A meta analysis preformed to combine all the measurements of the Higgs mass from Run-I and Run-II, the combined result was obtained from pooling all of the studies using the random effects model method.

where the  $\kappa$  here denotes the ratio between the measured or modified coupling with the Higgs and the SM prediction, i.e.

$$\kappa_X := \frac{g_{XXh}}{g_{Xh}^{\text{SM}}} \quad (3.5)$$

Which is commonly used in reporting experimental constrains/ of the Higgs couplings, the  $\kappa$ -formalism will be discussed more in chapter 4.

Unfortunately, it is not possible to directly measure the Higgs full width at the LHC, as this requires full reconstruction of the collision event and study the recoil mass which is only possible at lepton colliders [113, 114]. Alas, it is still possible to extract bounds on  $\Gamma_h$  using (3.4). ATLAS used this method to constrain the full width of the Higgs using Run-II data [115], while CMS has performed the same analysis using Run-I and Run-II data combined [116], the results are 95% CL bounds of  $\Gamma_h$

$$\Gamma_h < 14.4 \text{ GeV} \quad (\text{ATLAS}) \quad 0.08 \text{ GeV} < \Gamma_h < 9.16 \text{ GeV} \quad (\text{CMS}), \quad (3.6)$$

with the combined bound being  $\sim 3\Gamma_h^{\text{SM}}$ .

### 3.2.3 Higgs spin and parity

According to the SM predictions, the Higgs boson is a scalar and  $\mathcal{CP}$  even ( $J^p = 0^+$ ). However, the discovery of a peak in the  $m_{\gamma\gamma}$  distribution on its own would not automatically imply that the particle discovered is scalar, it could be a spin-2 boson, or a pseudoscalar ( $J^p = 0^-$ ). In order to study the  $J^p$  properties of the Higgs, one needs to examine the differential distributions of angular variables such as rapidity  $y$  or transverse momentum  $p_T$ . Both ATLAS and CMS collaborations studied using Run-I data the angular distributions of the Higgs decays  $h \rightarrow ZZ^*$ ,  $h \rightarrow WW^*$  and  $h \rightarrow \gamma$ , to study an anomalous  $VVh$  coupling. Then test the alternative hypothesis for  $J^p$  against the SM [117, 118]. The analysis results show that the SM  $0^+$  hypothesis is favoured at  $> 99.9\%$  CL.

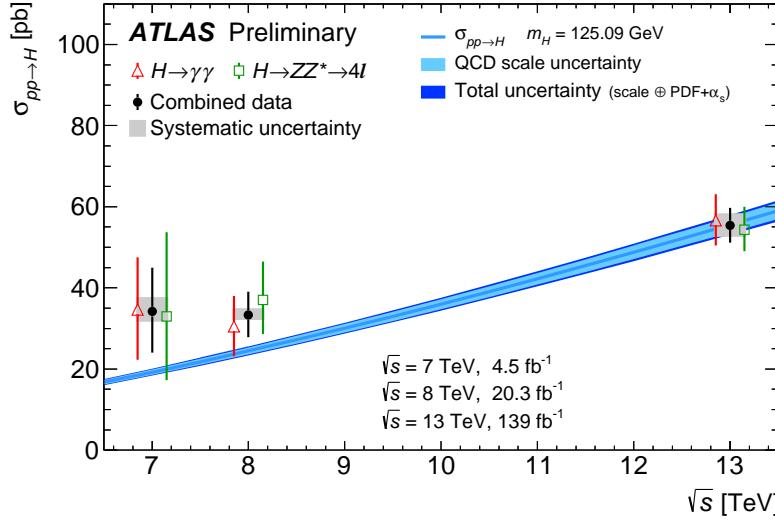
## 3.3 Measurements of Higgs rates and couplings

### 3.3.1 Higgs cross-sections

The total inclusive Higgs cross-section has been measured using the final states  $h \rightarrow \gamma\gamma$  and  $h \rightarrow ZZ^* \rightarrow 4\ell$  and their combinations. The measurements have been done at the three energies the LHC was operating at: 7 TeV, 8 TeV [119] and 13 TeV [120? , 121]. As shown in Figure 3.5, the measured inclusive cross-section is in agreement with the SM prediction across all of the LHC operation energies.

In addition to the inclusive cross-section measurements, differential cross-sections of the Higgs has been measured for  $p_T$  and  $y$  as we have seen in subsection 3.2.3 for Higgs's  $J^p$  determination. Additionally, the differential cross-sections for other variables have been measured, and they include  $N_{\text{jets}}, p_T^{\text{jet}}, m_{jj}, \delta\phi_{jj}$  and others using the channels  $h \rightarrow ZZ^*$ ,  $h \rightarrow WW^*$  and  $h \rightarrow \gamma$ . The most recent results using the full Run-II data can be found in refs. [121–124].

A collection of measurements of Higgs production and decay rates has been carried out by both ATLAS and CMS. These measurements also carried out in, what is known as the “Standard Template Cross-Sections” (STXS) framework. The STXS's are fiducial cross-sections in exclusive phase-space regions or bins stratified by the Higgs boson production channels. They have the advantage of standardisation of cuts and final results such that measurements could be easily combined across analyses. More details about the STXS framework can be found in the reports of LHC Higgs cross-sections working group (HXSWG) cf. [125]. Table 3.1 presents a summary of the state-of-art measurements of the Higgs rates separated into production and decay channels using the total LHC Run-II data from ATLAS and CMS experiments. The HL-LHC projections from CMS experiment are given as a comparison. The results in this table are written in terms of the signal strength, which is directly extracted from measuring the number



**Figure 3.5.** The total inclusive cross-section measurements by ATLAS collaboration [122] for 7, 8 and 13 TeV using  $h \rightarrow \gamma\gamma$  and  $h \rightarrow ZZ^* \rightarrow 4\ell$ . channels and their combination (black points) compared to the SM prediction with the uncertainties shown as blue line with light and dark blue bands for QCD scale uncertainties and total uncertainties, respectively.

of events dividing them by the standard model,

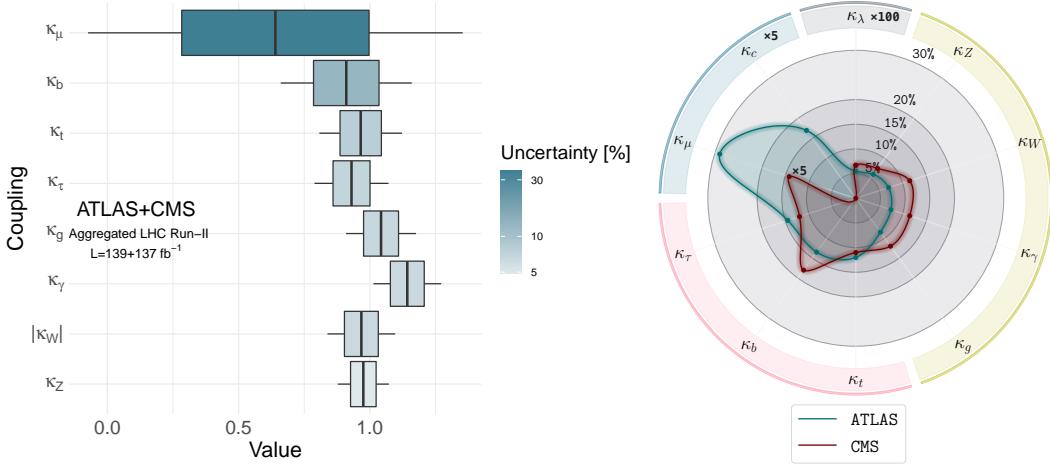
$$\mu_{\text{Exp}} := \frac{\sigma \cdot \text{BR}}{\sigma^{\text{SM}} \cdot \text{BR}^{\text{SM}}} \quad (3.7)$$

Production	Decay	$\mu_{\text{Exp}} \pm \delta\mu_{\text{Exp}}$ (symmetrised)		Ref.	
		LHC Run-II			
		CMS $137 \text{ fb}^{-1}$	ATLAS $139 \text{ fb}^{-1}$		
ggF	$h \rightarrow \gamma\gamma$	$0.99 \pm 0.12$ $1.030 \pm 0.110$		$1.000 \pm 0.042$ [126–128]	
	$h \rightarrow ZZ^*$	$0.985 \pm 0.115$ $0.945 \pm 0.105$		$1.000 \pm 0.040$	
	$h \rightarrow WW^*$	$1.285 \pm 0.195$ $1.085 \pm 0.185$		$1.000 \pm 0.037$ [126, 128, 129]	
	$h \rightarrow \tau^+\tau^-$	$0.385 \pm 0.385$ $1.045 \pm 0.575$		$1.000 \pm 0.055$	
	$h \rightarrow b\bar{b}$	$2.54 \pm 2.44$ —		$1.000 \pm 0.247$ [128, 129]	
	$h \rightarrow \mu^+\mu^-$	$0.315 \pm 1.815$ —		$1.000 \pm 0.138$ [128, 129]	
VBF	$h \rightarrow \gamma\gamma$	$1.175 \pm 0.335$ $1.325 \pm 0.245$		$1.000 \pm 0.128$ [126–128]	
	$h \rightarrow ZZ^*$	$0.62 \pm 0.41$ $1.295 \pm 0.455$		$1.000 \pm 0.134$	
	$h \rightarrow WW^*$	$0.65 \pm 0.63$ $0.61 \pm 0.35$		$1.000 \pm 0.073$ [126, 128, 129]	
	$h \rightarrow \tau^+\tau^-$	$1.055 \pm 0.295$ $1.17 \pm 0.55$		$1.000 \pm 0.044$	
	$h \rightarrow b\bar{b}$	— $3.055 \pm 1.645$		— [126]	
	$h \rightarrow \mu^+\mu^-$	$3.325 \pm 8.075$ —		$1.000 \pm 0.540$ [128]	
$t\bar{t}h$	$h \rightarrow \gamma\gamma$	$1.43 \pm 0.30$ $0.915 \pm 0.255$		$1.000 \pm 0.094$ [126–128]	
	$h \rightarrow VV^*$	$0.64 \pm 0.64 (ZZ^*)$ $0.945 \pm 0.465 (WW^*)$ $1.735 \pm 0.545$		$1.000 \pm 0.246 (ZZ^*)$ $1.000 \pm 0.097 (WW^*)$ —	
	$h \rightarrow \tau^+\tau^-$	$0.845 \pm 0.705$ $1.27 \pm 1.0$		$1.000 \pm 0.149$ [126, 128, 129]	
	$h \rightarrow b\bar{b}$	$1.145 \pm 0.315$ $0.795 \pm 0.595$		$1.000 \pm 0.116$	
$Vh$	$h \rightarrow \gamma\gamma$	$0.725 \pm 0.295$ $1.335 \pm 0.315$		$1.000 \pm 0.233 (Zh)$ $1.000 \pm 0.139 (W^\pm h)$ [126–128]	
	$h \rightarrow ZZ^*$	$1.21 \pm 0.85$ $1.635 \pm 1.025$		$1.000 \pm 0.786 (Zh)$ $1.000 \pm 0.478 (W^\pm h)$ [126, 128, 129]	
	$h \rightarrow WW^*$	$1.850 \pm 0.438$ —		$1.000 \pm 0.184 (Zh)$ $1.000 \pm 0.138 (W^\pm h)$ [128, 130]	
	$h \rightarrow b\bar{b}$	— $1.025 \pm 0.175$		$1.000 \pm 0.065 (Zh)$ $1.000 \pm 0.094 (W^\pm h)$ [126, 128]	
$Zh$ CMS	$h \rightarrow \tau^+\tau^-$	$1.645 \pm 1.485$		[129]	
	$h \rightarrow b\bar{b}$	$0.94 \pm 0.32$			
$W^\pm h$ CMS	$h \rightarrow \tau^+\tau^-$	$3.08 \pm 1.58$		[129]	
	$h \rightarrow b\bar{b}$	$1.28 \pm 0.41$			

**Table 3.1.** The experimental single Higgs production and decay rates measurements from the complete data of LHC Run II and projections for the HL-LHC. The uncertainties were symmetrised here.

### 3.3.2 Constraints on Higgs couplings

The measurements of the Higgs rates and their combination have been used to set bounds on the Higgs couplings, the most recent bounds have been reported by ATLAS using the Higgs inclusive rates and STXS for the full Run-II data [131], and by CMS using Higgs rates shown in Table 3.1 [129]. In Figure 3.6, I present the aggregation the ATLAS and CMS bounds on the Higgs coupling modifiers in the  $\kappa$  formalism defined in eq. (3.5). The aggregation of these bounds was preformed taking into account the between experiment effects, as described in [132] assuming there is no correlation between ATLAS and CMS measurements.



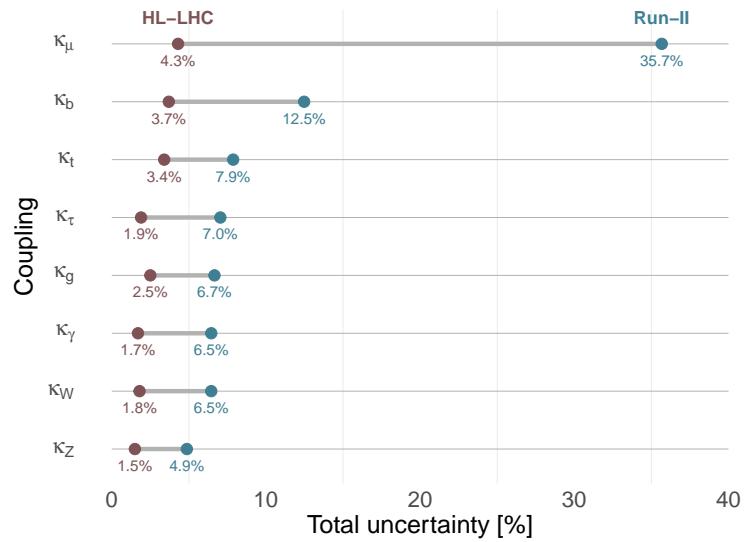
**Figure 3.6.** (left) Meta-analysis aggregation the most recent bounds from ATLAS [131] and CMS [129] on the Higgs couplings modifiers  $\kappa$ . (right) The individual 68% CI uncertainties on the coupling modifiers from ATLAS and CMS.

The measured bounds on the Higgs coupling to the gauge bosons, including the effective couplings to  $\gamma$  and  $g$ , as well as the couplings to the third-generation fermions are within few percent of the SM prediction. The bounds on the coupling to the  $W$  boson seems to favour a negative value in CMS fits, due to the channel used to constraint it  $h \rightarrow WW$  which depends on  $\kappa_W^2$ , thus making the best fit value of  $\sim -1$  within the SM prediction. An independent analysis on the relative signs of  $\kappa_W$  and  $\kappa_t$  was preformed using  $th/t\bar{t}h$  processes in ref. [133], hence only the absolute value of  $\kappa_W$  is reported in my combination of the analysis results. Additionally, the observation of the decays  $h \rightarrow b\bar{b}$  [134–136] and  $h \rightarrow \tau\tau$  [137, 138] leading to direct measurements of the beauty and  $\tau$  Yukawa couplings has made their bounds comparable to the gauge bosons and top couplings with the Higgs, having less than 10% uncertainty. Au contraire, bounds on the Yukawa couplings of second and first generation fermions remain very weak.

Recently, searches for the decay  $h \rightarrow \mu\mu$  [139, 140] using the whole Run-II data conducted by both collaborations, showed an evidence ( $3\sigma$ ) for observing this decay. Improving the constraints on  $\kappa_\mu$ , though as seen in Figure 3.6, the uncertainty remains

high  $\sim 36\%$ . Searches for the Higgs decaying to charm pairs is significantly more challenging, thus only quoted an upper 95% CL bounds on  $|\kappa_c|$  of 8.5 for ATLAS [141, 142] and 70 for CMS [143]. There is no planned direct searches for the first generation Yukawa couplings (*direct*) measurements planned for the LHC as it is not possible to directly access decays of the Higgs to up or down quarks. Other methods for probing these couplings will be extensively discussed in chapter 9.

By the end of the HL-LHC, it is projected that the couplings of the Higgs, including the couplings with gauge bosons, third generation fermions as well as the muon Yukawa will be measured at few percent level [19]. This is highlighted by Figure 3.7, showing the improvement in the  $\kappa$  measurement uncertainty expected by the HL-LHC compared to Run-II. Because



**Figure 3.7.** Dumbbell plot illustrating the improvement of the uncertainties on the Higgs coupling's measurement projected for the HL-LHC compared to the current combined CMS and ATLAS measurements from Run-II data.

## 3.4 Challenges and outlook

The future runs of the LHC hold a lot of potential for further understanding of the 10-year old Higgs boson. Although, for some processes and couplings there will still be a lot of challenges. For instance, the observation of  $h \rightarrow c\bar{c}$  will require highly efficient charm-tagging, which is expected to improve at the HL-LHC by a factor of 2.5 [144]. The signal strength with rare decay  $h \rightarrow Z\gamma$  currently is constrained to 3.6 times the SM values at 95% CL [145] and it is projected to be measured at the HL-LHC with  $\sim 10\%$  uncertainty.

One of the couplings of the Higgs which we did not discuss above is the Higgs self-interaction (trilinear and quartic), as I have shown in subsection 2.5.2 that the perturbative unitarity bound derived in ref. [90] is the strongest on these couplings so far. In view of the fact that to experimentally measure Higgs self-couplings, it is imperative to observe multiple Higgs production. Namely, to access the trilinear self-coupling Higgs pair production must be observed. Similarly, triple Higgs production observation is needed for the quartic Higgs coupling measurement. Woefully, these processes are experimentally arduous to detect, due to their low inclusive cross-section  $\sim 30$  fb for  $hh$  [146] and  $< 0.1$  fb for  $hhh$  at LHC maximum expected operational energy of 14 TeV. The triple Higgs production will remain challenging even for future colliders, e.g for the FCC-hh at 100 TeV, this process has a cross-section of only  $\sim 5$  fb [147]. To put these numbers in the context of single Higgs production, recall that the inclusive single-Higgs production cross-section of  $\sim 70$  pb at the current LHC operation energy. The triple Higgs production thus, will not be accessible at the LHC and consequently the quartic self-coupling will not be measured. However, there is a promising outlook for the HL-LHC to measure the trilinear self-coupling.

In chapter 7 I will discuss the potential for using single-Higgs processes to indirectly probe the trilinear coupling, as proposed by several studies in refs. [148–155] and the challenges accompanying it. Later in chapter 8 the Higgs pair production at the LHC will be overviewed along the current and future searches for this process and the bounds from them on the trilinear Higgs self-coupling.

Light quark Yukawa couplings are another example of formidable couplings to probe at the LHC. chapter 9 will be dedicated to overviewing the potential for Higgs pair production in the measurement of these elusive couplings. The focal point of that chapter will be the use of multi-variate analysis in signal vs background separation.



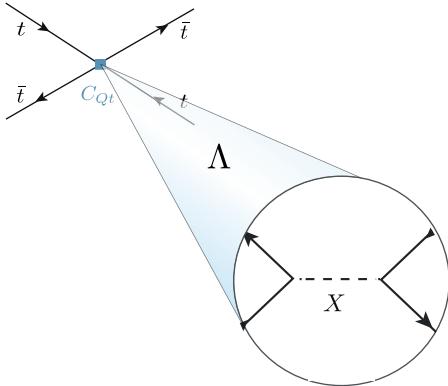
## 4 Higgs and effective field theories

The study of the Higgs properties, couplings and rates aims to shed light on the structure of its potential, how and why it is responsible for the EW symmetry breaking. Explaining the vacuum expectation value and the mass of the Higgs has been the aim of many theoreticians and phenomenologists. This is because the SM provides no insights on the nature of the Higgs potential and its parameters. In the SM, these are input parameters that needs to be provided from experimental observations. The Higgs potential shown in eq. (2.8) is the minimal one that could cause the EW symmetry breaking, but nature may not have taken this minimalist approach.

In order to test whether the Higgs potential is in the minimalist SM form or there are other more complex structures involved. One can start by measuring Higgs rates and confronting them against the SM prediction as overviewed in the previous chapter, using the  $\kappa$  formalism. Alas, this approach does not help in understanding what would the new physics (NP) structures be more likely to case a certain deviation. Conversely, we are interested in knowing what are the allowed NP structures given the current (or future) measurements of the Higgs rates. Of course, by looking at concrete models, one-by-one, confronting them with Higgs data one would get an insight on the aforementioned questions. However, this is a tedious task, as there are numerous ways NP might manifest

In order to make the search for NP more accessible and model-agnostic, we could revert to **effective field theories** (EFT), one of the most perspicacious concepts of quantum field theory. In the EFT framework, the interactions mediated by NP at small scale of an arbitrary complexity can be systematically simplified by approximating these interactions via integrating the UV degrees of freedom, leaving only numerable operators added to the SM. The premise of EFT's can be simply illustrated in Figure 4.1. The LHC-for example- might not be able to resolve the UV degrees of freedom at their scale  $\Lambda$ , rather one can only observe the effective interactions they mediates. These new interactions are depend on a set of free parameters known as **Wilson coefficients**, that would be constrained or set from experiments. These “phenomenological Lagrangians” as called by Weinberg [156], are not necessarily renormalisable but still allow for robust predictions that can be tested at colliders, including higher order effects .

This chapter is organised as follows: In ?? the Higgs sector of Standard Model effective field theory (SMEFT) will be presented along with the parametrisation of single and di-Higgs rates in terms of the SMEFT Wilson coefficients. In contrast to the SMEFT formalism, section 4.2 will present a non-linear EFT formalism known as the Chiral Lagrangian (EWChL)) or (Higgs)EFT . Finally I will conclude this chapter with section 4.3.



**Figure 4.1.** The premise of EFT stems from observing interactions at collider energy reach without being able to resolve the details of the NP mediating them, as the NP degrees of freedom have an energy scale  $\Lambda$  higher than the collider's reach.

## 4.1 The Standard Model effective field theory

There is no unique way of defining an EFT for the Higgs boson  $h$ . One could consider the field  $h$  as an EW singlet or as a part of the doublet  $\phi$  like the SM. The first ansatz is more compatible with a heavier Higgs and the effective coupling based on it could be derived from the EWChL as we shall see in section 4.2. However, after the discovery of the Higgs having a mass close to  $m_Z$ , the second option for an EFT seemed more fitting, though more restrictive. Assuming that the NP resonances would occur at masses  $\Lambda \gg m_Z$ , one can integrate them out yielding a set of effective operators of mass dimension  $> 4$ . Hence, one can think of the SM Lagrangian of mass dimension 2 and 4 as a part of a more general EFT that contains the same fields and symmetries, known as the Standard Model Effective field theory (SMEFT).

From simple dimensional analysis, we know that the Higher dimensional operators need to contain an inverse mass with some power  $p = 4 - d$  in the couplings, we have a clear power counting in the SMEFT Lagrangian, such that we could collect all operators of the same mass dimension  $d$  into a  $d$ -mass-dimensional Lagrangians taking the form

$$\mathcal{L}^{(d)} = \frac{1}{\Lambda^{d-4}} \sum_i C_i \mathcal{O}_i. \quad (4.1)$$

For any  $d > 4$  the Lagrangian in eq. (4.1) is not renormalisable in the strict sense, yet it is still predictive, via fitting the Wilson coefficients  $C_i$  order-by-order to experimental measurements. This power-counting property allows for predictability even when we, in principle, have infinite number of free Wilson coefficients, as all of these operators are suppressed by the NP scale (irrelevant operators w.r.t. the renormalisation group). In order to illustrate this, we let  $\Lambda = 1$ , then the effects of dimension-six operators will be in percent level, while dimension-eight operators will have effects of order  $\sim 10^{-4}$ , allowing

us to ignore the dimension-eight and higher operators. Regarding dimension-five, we have only one operators called the Weinberg operator [157]

$$\mathcal{O}_{\nu\nu} = (\tilde{\phi} L_p)^T \mathcal{C} (\tilde{\phi}^\dagger L_q), \quad (4.2)$$

were  $\mathcal{C}$  is the charge conjugation operator. The Weinberg operator violates leptonic number and generates neutrino masses after EW symmetry breaking, similar effects are generated from dimension-seven operators [158]. These effects do not yield considerable collider phenomenology. Hence, I shall be discussing SMEFT with dimension-six operators only, for studies on Higher dimensional SMEFT operators cf. [158–161].

The SMEFT Lagrangian up to dimension-six operators is given by

$$\mathcal{L}_{\text{SMEFT}}^{d=6} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i C_i \mathcal{O}_i. \quad (4.3)$$

The study of dimension-six effective operators in characterising NP effects at energies beyond colliders reach has been first proposed in [162, 163]. Nowadays, phenomenological studies of EFT's with dimension-six operators primarily focus on using a set of complete and non-redundant “basis”. This is due to the fact that different effective operators will correspond to same observables e.g. same scattering amplitudes of SM particles. This is the case if the operators can be related by using equations of motion, Fierz transformations, integration by parts or field redefinitions. This leads to non-trivial and counter-intuitive relations between operators. Thus making the construction of basis for the dimension-six SMEFT Lagrangian of eq. (4.3) a cumbersome task. Such task has been accomplished recently by [13, 14] forming what is known as the **Warsaw Basis**. Another set of basis is the strongly-interacting light Higgs basis (SILH), originally proposed by [164], before the Warsaw basis, and completed in ref. [165, 166]. A more recent set of basis has been published in [167] using a subset of couplings characterising the interactions of mass eigenstates in the effective Lagrangian.

The complete  $d = 6$  SMEFT is described by 2499 independent parameters [14]. However, if one suppresses the flavour indices, assuming SMEFT is flavour blind, then their inventory is significantly reduced. The Warsaw basis for example, assuming Baryon number conservation and dropping the flavour indices one has only 59 operators, listed in Table 4.1. It should be noted that all of the basis of SMEFT will produce the same phenomenology, though the choice of basis is sometimes helpful in simplifying the analysis. In this thesis, I will mainly focus on Warsaw basis.

The SMEFT operators can either modify SM parameters (couplings, masses) or introduce new vertices that do not exist in the SM, like four-fermion operators, or both like  $\mathcal{O}_{\phi e}$ . An example of operators modifying SM parameters is  $\mathcal{O}_{\phi D}$ , which leads to modification of the  $Z$  boson mass after EW symmetry breaking

$$\frac{C_{\phi D}}{\Lambda^2} |\phi^\dagger D_\mu \phi|^2 \rightarrow \frac{C_{\phi D} v^4}{16 \Lambda^2} (g_2^2 + g_1^2) Z^\mu Z_\mu. \quad (4.4)$$

Additionally, from field redefinitions, we get indirect contributions to the  $W$  mass from

$X^3$		Pure Higgs		$\psi^2 \phi^3 + \text{h.c.}$	
$\mathcal{O}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{\phi\square}$	$(\phi^\dagger \phi) \square (\phi^\dagger \phi)$	$\mathcal{O}_{e\phi}$	$(\phi^\dagger \phi) (\bar{l}_p e_r \phi)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{\phi D}$	$(\phi^\dagger D_\mu \phi)^* (\phi^\dagger D_\mu \phi)$	$\mathcal{O}_{u\phi}$	$(\phi^\dagger \phi) (\bar{q}_p u_r \widetilde{\phi})$
$\mathcal{O}_W$	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_\phi$	$(\phi^\dagger \phi)^3$	$\mathcal{O}_{d\phi}$	$(\phi^\dagger \phi) (\bar{q}_p d_r \phi)$
$\mathcal{O}_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \phi^2$		$\psi^2 X \phi + \text{h.c.}$		$\psi^2 \phi^2 D$	
$\mathcal{O}_{\phi G}$	$\phi^\dagger \phi G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \phi W_{\mu\nu}^I$	$\mathcal{O}_{\phi l}^{(1)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\bar{l}_p \gamma^\mu l_r)$
$\mathcal{O}_{\phi \widetilde{G}}$	$\phi^\dagger \phi \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \phi B_{\mu\nu}$	$\mathcal{O}_{\phi l}^{(3)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu^I \phi) (\bar{l}_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{\phi W}$	$\phi^\dagger \phi W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{\phi} G_{\mu\nu}^A$	$\mathcal{O}_{\phi e}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\bar{e}_p \gamma^\mu e_r)$
$\mathcal{O}_{\phi \widetilde{W}}$	$\phi^\dagger \phi \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{\phi} W_{\mu\nu}^I$	$\mathcal{O}_{\phi q}^{(1)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\bar{q}_p \gamma^\mu q_r)$
$\mathcal{O}_{\phi B}$	$\phi^\dagger \phi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{\phi} B_{\mu\nu}$	$\mathcal{O}_{\phi q}^{(3)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu^I \phi) (\bar{q}_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{\phi \widetilde{B}}$	$\phi^\dagger \phi \widetilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \phi G_{\mu\nu}^A$	$\mathcal{O}_{\phi u}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\bar{u}_p \gamma^\mu u_r)$
$\mathcal{O}_{\phi WB}$	$\phi^\dagger \tau^I \phi W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \phi W_{\mu\nu}^I$	$\mathcal{O}_{\phi d}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\bar{d}_p \gamma^\mu d_r)$
$\mathcal{O}_{\phi \widetilde{WB}}$	$\phi^\dagger \tau^I \phi \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \phi B_{\mu\nu}$	$\mathcal{O}_{\phi ud} + \text{h.c.}$	$i(\widetilde{\phi}^\dagger D_\mu \phi) (\bar{u}_p \gamma^\mu d_r)$
$(\bar{L}L)(\bar{L}L)$			$(\bar{R}R)(\bar{R}R)$		
$\mathcal{O}_{ll}$	$(\bar{l}_p \gamma_\mu l_r) (\bar{l}_s \gamma^\mu l_t)$		$\mathcal{O}_{ee}$	$(\bar{e}_p \gamma_\mu e_r) (\bar{e}_s \gamma^\mu e_t)$	
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{q}_s \gamma^\mu q_t)$		$\mathcal{O}_{uu}$	$(\bar{u}_p \gamma_\mu u_r) (\bar{u}_s \gamma^\mu u_t)$	
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$		$\mathcal{O}_{dd}$	$(\bar{d}_p \gamma_\mu d_r) (\bar{d}_s \gamma^\mu d_t)$	
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r) (\bar{q}_s \gamma^\mu q_t)$		$\mathcal{O}_{eu}$	$(\bar{e}_p \gamma_\mu e_r) (\bar{u}_s \gamma^\mu u_t)$	
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r) (\bar{q}_s \gamma^\mu \tau^I q_t)$		$\mathcal{O}_{ed}$	$(\bar{e}_p \gamma_\mu e_r) (\bar{d}_s \gamma^\mu d_t)$	
			$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r) (\bar{d}_s \gamma^\mu d_t)$	
			$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r) (\bar{d}_s \gamma^\mu T^A d_t)$	
$(\bar{L}L)(\bar{R}R)$			$(\bar{L}R)(\bar{L}R) + \text{h.c.}$		
$\mathcal{O}_{le}$	$(\bar{l}_p \gamma_\mu l_r) (\bar{e}_s \gamma^\mu e_t)$		$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \epsilon_{jk} (\bar{d}_s^k d_t)$	
$\mathcal{O}_{lu}$	$(\bar{l}_p \gamma_\mu l_r) (\bar{u}_s \gamma^\mu u_t)$		$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \epsilon_{jk} (\bar{q}_s^k T^A d_t)$	
$\mathcal{O}_{ld}$	$(\bar{l}_p \gamma_\mu l_r) (\bar{d}_s \gamma^\mu d_t)$		$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t)$	
$\mathcal{O}_{qe}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{e}_s \gamma^\mu e_t)$		$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \epsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	
$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{u}_s \gamma^\mu u_t)$		$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r) (\bar{d}_s q_{tj})$	
$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{u}_s \gamma^\mu T^A u_t)$				
$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r) (\bar{d}_s \gamma^\mu d_t)$				
$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r) (\bar{d}_s \gamma^\mu T^A d_t)$				

**Table 4.1.** Complete list of the dimension-six SMEFT operators in the Warsaw basis [13]. The  $\mathcal{CP}$  violating operators contains the dual fields  $\tilde{X}$ . The flavour labels of the form  $p, r, s, t$  on the  $\mathcal{O}$  operators are suppressed on the left hand side of the tables.

$C_{\phi D}$ , combining both effects as a deviation in the  $\rho$  parameter, we get

$$\delta\rho = \frac{v^2}{2\Lambda^2} C_{\phi D}. \quad (4.5)$$

Which allows us to constrain  $C_{\phi D}$  from the  $T$  parameter

$$T = \frac{-2\pi v^2}{\Lambda^2} \frac{(g_1^2 + g_2^2)}{g_1^2 g_2^2} C_{\phi D} \quad (4.6)$$

Another operator that affects the oblique parameters directly is  $\mathcal{O}_{\phi WB}$ , as it modifies the  $S$  parameter in the following way

$$S = \frac{16\pi v^2}{g_1 g_2 \Lambda^2} C_{\phi WB} \quad (4.7)$$

SM coupling modifications by SMEFT operators related to EWPO's are investigated in [168], and chapter 10. Additionally, the contributions of the SMEFT Wilson coefficients to SM parameters are not only from tree-level effects like in eq. (4.4) but could also come at loop-level, either from finite or RGE contributions.

SMEFT is suitable as a low energy limit for supersymmetric models [169] or some classes of composite Higgs models [170, 171]

### 4.1.1 Single Higgs processes in SMEFT

Single Higgs production and decay processes are modified at LO by a relatively long list of operators summarised in eqs. (4.8), (4.9) and (4.10). Explicit formulae for the Higgs rates dependence on the Wilson coefficients of these operators can be found in [172]

SMEFT operators modifying Higgs rates at LO

Higgs operators

$$\begin{aligned} &C_{\phi D}, \mathcal{O}_{\phi\square}, \mathcal{O}_{\phi G}, \mathcal{O}_{\phi W}, \mathcal{O}_{\phi B}, \mathcal{O}_{\phi WB}, \mathcal{O}_{\phi l}^{(1)}, \\ &\mathcal{O}_{\phi l}^{(3)}, \mathcal{O}_{\phi e}, \mathcal{O}_{\phi q}^{(1)}, \mathcal{O}_{\phi q}^{(3)}, \mathcal{O}_{\phi u}, \mathcal{O}_{\phi d}, \mathcal{O}_{\tau\phi}, \mathcal{O}_{t\phi}, \mathcal{O}_{b\phi}, \mathcal{O}_{tb\phi}. \end{aligned} \quad (4.8)$$

Top-quark operators

$$\mathcal{O}_{tG}, \mathcal{O}_{tW}, \mathcal{O}_{tB}, \quad (4.9)$$

other

$$\mathcal{O}_G, \mathcal{O}_{ll}^{(1)}, \mathcal{O}_{Qq}^{(1),(3)}, \mathcal{O}_{tu}, \mathcal{O}_{td}^{(1),(8)}, \mathcal{O}_{Qu}^{(1),(8)}, \mathcal{O}_{Qd}^{(1),(8)}. \quad (4.10)$$

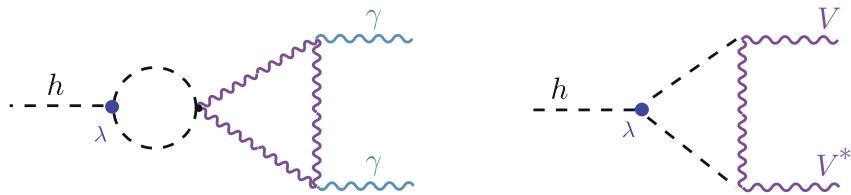
The third generation quarks are denoted by  $Q$  while the first and second generation quarks are assumed to have the same coupling and denoted by  $q, u, d$ .

Some of these operators are strongly constrained from EWPO data such as  $\mathcal{O}_{\phi D}$  and  $\mathcal{O}_{\phi WB}$ , while others still have weak bounds from current measurements and insensitive

to EWPO's. Global fits on SMEFT Wilson coefficients can be found in ref. [18], where Higgs and EW data were used to fit a subset of the SMEFT Wilson coefficients of the operators listed above. The fit also includes RGE and NLO (even NNLO for  $m_W$ ) effects. While in [173], a global fit for a larger set of operators, but only with LO effects, including EW, Higgs and top data. More recent study [174] has utilised EWPO data to constrain the four-fermion operators appearing in Higgs rates at LO in addition to operators with four heavy quarks, using their NLO effects to EW bosons pole masses. We shall see in chapter 7 that the latter operators with also contribute to Higgs rates at NLO. A wider scope analysis including a wide range of Higgs, top, di-boson and EWPO data has been preformed in [17].

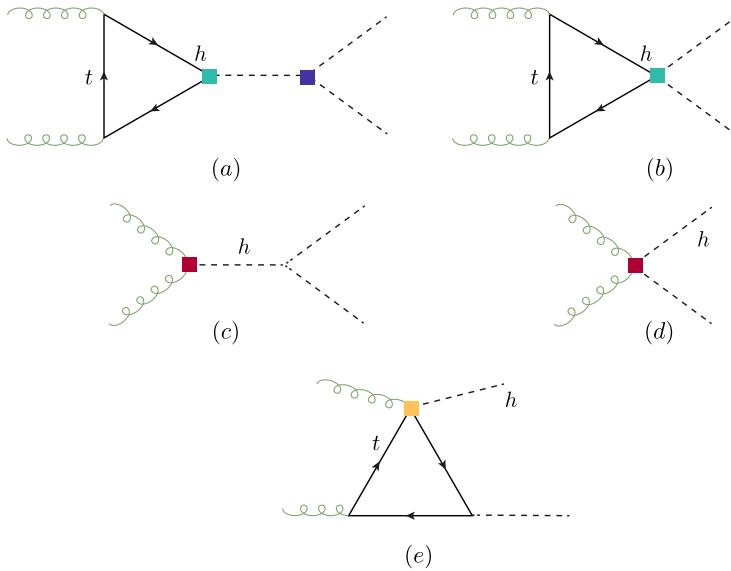
The dependence of single Higgs rates on the SMEFT Wilson coefficients gets more complicated once higher order effects are taken into an account. As shown in the fit results reported from [18], the RGE of these Wilson coefficients introduces mixing with operators that do not appear at LO, also loop corrections to the rates and masses of the EW and Higgs bosons.

A prominent example of an operator appearing only at NLO in single Higgs processes is  $\mathcal{O}_\phi$ , which modifies the Higgs self interactions, namely the trilinear coupling. Typically, in order to probe the Higgs trilinear self-coupling directly, one ought to observe Higgs pair production. However, due to the appearance of Higgs self-interaction and its modifiers, i.e.  $C_\phi$  in SMEFT context, in higher order EW corrections [175, 176] and Higgs observables [148–155], one can extract bounds on the Higgs trilinear coupling from single Higgs and EWPO data. Figure 4.2 illustrates example Feynman diagrams of single Higgs processes of which the trilinear Higgs self-coupling enters via NLO corrections. Using the results from the aforementioned references, a global fit with all operators that



**Figure 4.2.** NLO EW corrections of single Higgs processes, were the Higgs trilinear self-coupling (the red circle) enters. Here the Higgs decay to two photons is shown as an example.

enter at tree-level in addition to the loop effects from the Higgs self-coupling has been preformed in refs. [18, 177]. Additionally, experimental searches for Higgs trilinear self-coupling have been presented by ATLAS [178] and CMS [129].



**Figure 4.3.** Example of diagrams illustrating how the dimension-six SMEFT operators enter in Higgs pair production at hadron colliders.

### 4.1.2 Higgs pair production and SMEFT

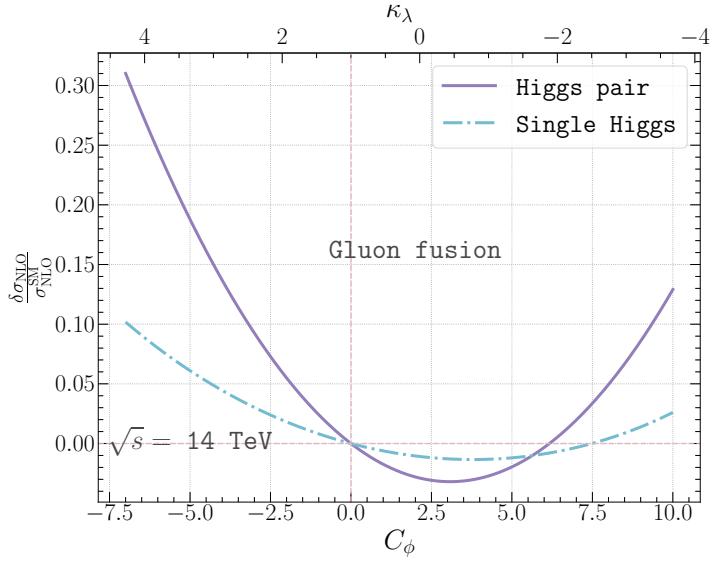
Higgs pair production in hadron colliders is sensitive to six  $\mathcal{CP}$  even SMEFT operators<sup>1</sup>, under the assumption of Minimal Flavour violation (MFV)<sup>2</sup>. These operators are

$$\mathcal{O}_{\phi D}, \mathcal{O}_{\phi \square}, \mathcal{O}_\phi, \mathcal{O}_{t\phi}, \mathcal{O}_{\phi G}, \mathcal{O}_{tG}, \quad (4.11)$$

and their effects, with the corresponding colours are demonstrated in Figure 4.3, except for  $\mathcal{O}_{\phi D}$  and  $\mathcal{O}_{\phi \square}$ , as they modify all SM Higgs vertices. However, MFV is not the only way to approach SMEFT, there exist more complex flavour structures that allow for significant enhancements of the first and second generation Yukawas without being excluded by flavour observables. Such formalisms will be discussed in chapter 9, in addition to the potential for Higgs pair production in probing operators modifying light Yukawa couplings. The main operator to constrain from Higgs pair as mentioned before is  $\mathcal{O}_\phi$ , for two reasons; a) the rest of the operators appearing in di-Higgs are already strongly constrained from single Higgs and top processes. b) The effect of  $\mathcal{O}_\phi$  on Higgs pair production is significantly higher than in single Higgs or EW observables. This is illustrated in Figure 4.4, by comparing the relative change of the gluon fusion cross-sections at NLO QCD for single and di-Higgs production. This is not surprising, since  $C_\phi$  appears at LO in Higgs pair production. Another advantage for Higgs pair production searches is the sensitivity of this process to non-linear couplings, for example diagrams

<sup>1</sup>For or Higgs pair production with  $\mathcal{CP}$  violating operators, see ref. [179].

<sup>2</sup>MFV assumes that new physics operators will follow the same flavour hierarchies as the SM.



**Figure 4.4.** The relative change of the NLO QCD cross-section of gluon fusion production of single Higgs (dashed line) and Higgs pair (solid line) at a  $pp$  collider with  $\sqrt{s} = 14$  TeV as a function of  $C_\phi$  or the corresponding  $\kappa_\lambda$ .

(b) and (d) of [Figure 4.3](#). Although in SMEFT these diagrams correspond to the same operators in (a) and (c), respectively, in another EFT this is not necessarily the case.

## 4.2 The chiral Lagrangian

Given the strong bounds on the  $\rho$  parameter, it would be plausible to assume that the NP maintains the custodial symmetry  $SU(2)_V$ , and treat the chiral symmetry breaking pattern  $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$  the same way the QCD chiral symmetry breaking is treated, in terms of considering the pions as pseudo-Nambu Goldstone bosons in order to describe their properties and couplings. In the pion case, this is known as **chiral perturbation theory** [180, 181]. The same mathematical description could be applied for the case of EW symmetry breaking by constructing the EW chiral Lagrangian (EWChL). In this formalism, the Goldstone bosons  $\pi^a(x)$  of the SM are considered the generators of  $SU(2)_L$  unitary transformation

$$\mathcal{U}(x) = e^{i\pi^a(x)\sigma_a/v}, \quad (4.12)$$

which implies that the Goldstone fields transform non-linearly under  $SU(2)_L \otimes SU(2)_R$ . As for the Higgs boson  $h$ , it is added as an  $SU(2)_L \otimes U(1)_Y$  singlet, and can appear in the EWChL at any power. Contrary to the SMEFT power counting in the NP scale  $\Lambda$ , in the EWChL, terms are ordered according to their *chiral dimension*  $\chi$ , defined for

spacetime derivatives  $\partial_\mu$ , bosonic  $\phi, X_\mu$  and  $\psi$  fermionic generic fields as [182, 183]

$$[\phi]_\chi = 0, [X]_\chi = 0, [\partial_\mu]_\chi = 1, [\psi]_\chi = 2. \quad (4.13)$$

The zeroth order term of the EWChL possesses a chiral dimension of  $\chi = 2$ , while higher order terms could be considered as terms generated perturbatively from  $L$  loop interactions, with chiral dimension  $\chi = 2L + 2$ . The expansion of the EWChL is in the chiral order in addition to the powers of  $h(x)/v$ . This power-counting causes some SMEFT dimension-six operators to be considered of a higher order in HEFT. A prominent example of this is the chromomagnetic operator  $\mathcal{O}_{tG}$  being of chiral dimension 5 in EWChL. The relevant terms for single- and di-Higgs production of the EWChL are given in the Unitary gauge by [177, 184]

$$\begin{aligned} \mathcal{L}_{\text{HEFT}} = & \frac{h}{v} \left[ \left( \delta c_W m_W^2 W_\mu^+ W^{-\mu} + \delta c_Z \frac{m_Z^2}{2} Z_\mu Z^\mu \right) \right. \\ & + c_{ww} \frac{g_2^2}{2} W_{\mu\nu}^+ W^{-\mu\nu} + c_{w\square} g_2^2 \left( W_\mu^- \partial_\nu W^{+\mu\nu} + \text{h.c.} \right) + c_{\gamma\gamma} \frac{\alpha}{8\pi} A_{\mu\nu} A^{\mu\nu} \\ & + c_{zz} \frac{g_2^2 + g_1^2}{4} Z_{\mu\nu} Z^{\mu\nu} + c_{z\gamma} \frac{eg_1}{16\pi^2} Z_{\mu\nu} A^{\mu\nu} + c_{z\square} g_2^2 Z_\mu \partial_\nu Z^{\mu\nu} + c_{\gamma\square} g_2 g_1 Z_\mu \partial_\nu A^{\mu\nu} \Big] \\ & + \frac{\alpha_s}{8\pi} \left( c_{gg} \frac{h}{v} + c_{gg}^{(2)} \frac{h^2}{2v^2} \right) \text{Tr}[G_{\mu\nu} G^{\mu\nu}] - \sum_f \left[ m_f \left( c_f \frac{h}{v} + c_{ff} \frac{h^2}{2v^2} \right) \bar{f}_R f_L + \text{h.c.} \right] \\ & - c_{hh} \frac{m_h^2}{2v} h^3 + \dots, \end{aligned} \quad (4.14)$$

I have omitted here the kinetic and mass terms of the Higgs,  $\mathcal{CP}$  violating terms, as well as couplings not relevant to LHC phenomenology and higher chiral order operators.

In addition to NP effects, this Lagrangian also includes the LO and NLO SM vertices, for example the parameter  $\delta c_V = 1$  corresponds to the tree-level coupling between the Higgs field and the EW bosons  $V = W, Z$ . While the coupling  $c_{gg} = 4/3$  corresponds to the SM effective coupling at NLO if the heavy top limit (HTL)  $m_t \rightarrow \infty$ .

In contrast to the SMEFT, the couplings of one and two Higgs bosons to fermions or gluons become de-correlated. Giving this Lagrangian a richer phenomenology for Higgs pair production.

The HEFT coefficients modifying the Higgs pair production via gluon fusion are

$$c_{hh}, \textcolor{teal}{c}_t(a), \textcolor{teal}{c}_{tt}(b), \textcolor{red}{c}_{gg}(c), \textcolor{red}{c}_{gg}^{(2)}(d), \quad (4.15)$$

with the same colours highlighted in the operator insertions of Figure 4.3 and the letter next to the coefficient indicates the diagram, in which the coefficient appears. Full parametrisation of the Higgs pair cross-section at NLO (inclusive and differential) and NNLO (inclusive) can be found in refs. [185–187] and implemented at NLO in POWHEG-BOX [188].

UV-complete models that yield in the EWChL are composite Higgs models [170, 171, 189], dilaton theories [190], techni-dilaton models [191], technicolour models [192] and other models with induced EW symmetry breaking [193, 194].

### 4.2.1 Translation between SMEFT and HEFT

In order to facilitate the translation between SMEFT and HEFT or to the  $\kappa$ -formalism, one needs to put the SMEFT Lagrangian into the canonical form, that is to convert the operators with covariant derivatives acting on the Higgs to canonically normalised Higgs kinetic term. This is done done by the field redefinition.

$$\phi = \begin{pmatrix} 0 \\ h(1 + c_{h,kin}) + v \end{pmatrix} \quad (4.16)$$

with

$$c_{h,kin} = \left( C_{\phi,\square} - \frac{1}{4} C_{\phi D} \right) \frac{v^2}{\Lambda^2}. \quad (4.17)$$

This field redefinition will generate derivative interactions of the form  $h(\partial_\mu h)^2$  and  $h^2(\partial_\mu h)^2$ . In order to remove these terms, and for sake of simplicity, I use a gauge-dependent field redefinition<sup>3</sup>

$$h \rightarrow h + c_{h,kin} \left( h + \frac{h^2}{v} + \frac{h^3}{3v^2} \right). \quad (4.18)$$

This field redefinition leads to  $c_{h,kin}$  modifying all Higgs couplings.

Before we discuss the translation between SMEFT and HEFT, some words of caution are in order: First, HEFT is less restrictive than SMEFT therefore it contains more degrees of freedom. This makes some points of the HEFT parameter space unmappable to SMEFT. In addition, the power counting is different in both formalisms, as mentioned before there will be some operators present in SMEFT that are absent in HEFT and vice-versa. In Table 4.2, the translation between the HEFT and SMEFT Wilson coefficients of the operators relevant to Higgs pair production at LO is shown. More general translation between SMEFT in Warsaw and SILH basis and HEFT can be done automatically using `Rosetta` package [196]

### 4.2.2 EFT and $\kappa$ -formalism

The  $\kappa$  formalism provides an experimentally accessible and well-defined QFT-wise approach to studying the Higgs properties. The  $\kappa$  parameters are part of more generalised formalism called the Higgs **Pseudo-observables** [197] If the new physics contributions do not generate new Lorentz structures there is a possible translation between the Wilson coefficients in the SMEFT Warsaw basis, and the  $\kappa$  formalism. In particular, taking

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<sup>3</sup>For gauge-independent formalism cf. [195].

HEFT	SMEFT (Warsaw)
$c_{hhh}$	$1 - 2 \frac{v^4}{m_h^2} C_\phi + 3 c_{h,kin}$
$c_f$	$1 + c_{h,kin} - C_{f\phi} \frac{v^3}{\sqrt{2} m_f}$
$c_{ff}$	$-C_{f\phi} \frac{3v^3}{2\sqrt{2} m_f} + c_{h,kin}$
$c_{gg}$	$8\pi/\alpha_s v^2 C_{\phi G}$
$c_{gg}^{(2)}$	$4\pi/\alpha_s v^2 C_{\phi G}$

**Table 4.2.** Translation between the Wilson coefficients of HEFT and SMEFT for the operators relevant to Higgs pair production.

the rescaling of the trilinear coupling,  $\kappa_\lambda$ , the translation is given by

$$\kappa_\lambda = 1 - \frac{v^4}{m_h^2} \frac{C_\phi}{\Lambda^2} + 3 c_{h,kin}, \quad (4.19)$$

A similar relation exists for the rescaling of the quark Yukawa couplings  $\kappa_q$

$$\kappa_q = 1 + c_{h,kin} - \frac{v^3}{\sqrt{2} m_q} \frac{C_{q\phi}}{\Lambda^2}. \quad (4.20)$$

One can see the similarities between  $\kappa$ -formalism and HEFT in these two examples, but this is not always the case. Other translations could be obtained by comparing how SMEFT operators modify the Higgs couplings with the SM, and matching it with the corresponding  $\kappa$  or other Higgs pseudo-observable.

However, one should be careful while interpreting results quoted in terms of Wilson coefficients in the SMEFT framework extracted from multi-Higgs or multi-vector bosons searches, as these results include couplings that are not present in the SM. For example, the  $hhq\bar{q}$  coupling, though being linearly related to the quark Yukawa coupling  $hq\bar{q}$ , is not a rescaling of any SM Higgs coupling. With this in mind, one can strictly remain within a linear EFT and link the rescaling of the quark Yukawa,  $\kappa_q$ , to the  $hhq\bar{q}$  coupling through

$$g_{hhq\bar{q}}^{\text{linear-EFT}} = -\frac{3}{2} \frac{1 - \kappa_q}{v} g_{hq\bar{q}}^{\text{SM}}. \quad (4.21)$$

This relation will no longer hold once a non-linear EFT, like HEFT, is used. Hence, the  $\kappa$ -formalism, in a strict sense, is not applicable generally to multi-Higgs studies.

### 4.3 Conclusions

Effective field theories provide a systematic yet simplified approach for NP searches by simplifying its complex interaction structures. This can be viewed as a dimensionality reduction approach, by collapsing all the NP interaction into their effective ones as they

would be observed at colliders with energy reaches below the NP scale  $\Lambda$ . The linear approach to EFT is called the SMEFT, which preserves the SM fields and symmetries and the Higgs boson is a part of an  $SU(2)_L$  doublet  $\phi$  like the SM case. While non-linear approaches such as HEFT treats the Higgs boson as an added singlet. The latter approach is more general and introduces independent parameters involving multiple Higgs bosons. For example, the couplings  $f\bar{f}h$  and  $f\bar{f}hh$  will be both generated in SMEFT and HEFT, but in SMEFT both are related by the Wilson coefficient  $C_{\phi f}$ , while in HEFT they have independent Wilson coefficients  $c_f$  and  $c_{ff}$ , respectively. Most of the Wilson coefficients involving Higgs interactions are strongly constrained by EWPO's, as well as Higgs and top data. However, the bounds on the Wilson coefficient modifying Higgs self-couplings  $C_\phi$  remains dominated by theoretical constraints from perturbative unitarity [15, 90]. This can be improved by the searches for Higgs pair production at the HL-LHC, as this process is more sensitive to the trilinear Higgs self-coupling, than EWPO and single-Higgs data.

## Part II

# Single Higgs Processes at the LHC



## 5 Overview of Higgs production at colliders

The production of the Higgs boson at the LHC is mediated by four distinct processes: gluon fusion (ggF), vector boson fusion (VBF), vector bosons Higgsstrahlung ( $Vh$ ) and the production with top ( and anti-top) pair ( $t\bar{t}h/t\bar{t}\bar{t}$ ). It should be noted that sometimes the ggF category will include the quark anti-quark annihilation, but this is negligible in the SM, but becomes important for significant modifications of light Yukawa couplings. These process are illustrated in Figure 5.1, and their details were summarised in Table 5.1. All of these four channels have been observed at the LHC with  $> 5\sigma$  precision. Since the experimental measurements of this Higgs were discussed previously in section 3.3, the aim of this chapter is to provide an overview of the current theoretical status of these channels.

Process	Cross-section 13 TeV (pb)	Theo. accuracy	Exp. uncertainty (%)	Contribution (%)
ggF	48.51	N3LO QCD & NLO EW	6.5	88
$t\bar{t}h$ & $t\bar{t}\bar{t}$	0.58	NLO QCD & NLO EW	20.0	1
VBF	3.78	NNLO QCD* & NLO EW	10.0	7
$Vh$	2.25	NNLO QCD & NLO EW	15.0	4

Table 5.1. Summary of the Higgs production processes at the LHC.

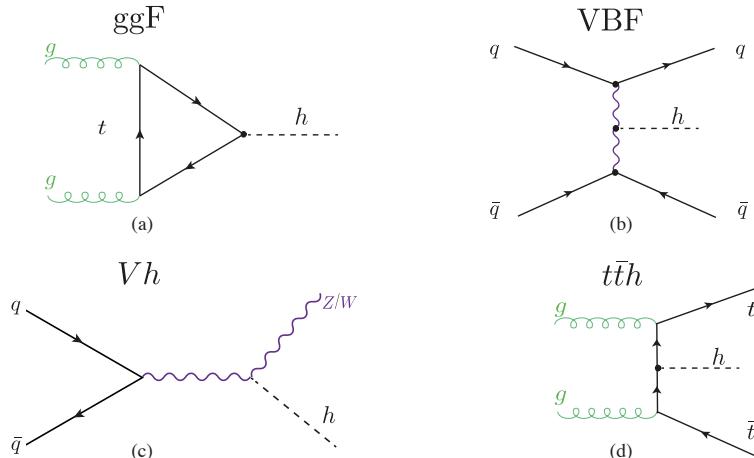
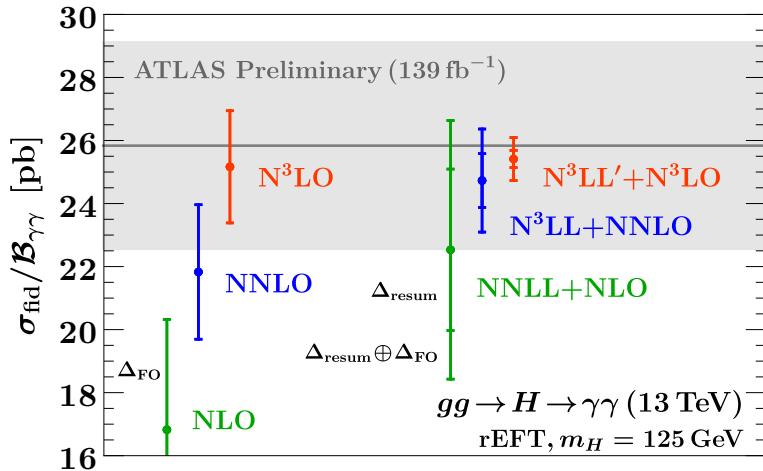


Figure 5.1. Feynman-diagram examples of the leading Higgs production processes at the LHC.

## 5.1 Current status of the Higgs production channels

### 5.1.1 Gluon fusion process

The Higgs production via gluon fusion (ggf) has the highest cross-section amongst the Higgs production channels, and consequently has the lowest experimental uncertainty. The current state-of-the-art theoretical computation for the Higgs inclusive cross-section is N<sup>3</sup>LO in QCD and NLO in EW [198]. A full differential cross-section for the final state  $gg \rightarrow h \rightarrow \gamma\gamma$  has been computed recently to N<sup>3</sup>LO in QCD, also for the kinematic variables  $y_h$ ,  $y_{\gamma_1}$ ,  $y_{\gamma_2}$ ,  $\Delta y_{1,2}$  using the projection-to-born method [199]. In addition to the fiducial differential cross-section in  $p_T$  with experimental cuts has been computed up to third re-summed and fixed order, i.e. N<sup>3</sup>LL' N<sup>3</sup>LO dependence [200]. The theoretical computation of this fiducial cross-section with different orders compared to the experimental measurement by ATLAS [123] is shown in Figure 5.2, we can see that the resummed result has significantly smaller theoretical uncertainties. The current total theoretical uncertainty with this order calculation is 5.4%, with only 2.7% of it coming from the perturbation order cut-off of the calculation, while the rest comes from the branching fraction, PDF+ $\alpha_s$ , EW corrections and mass uncertainties. When compared to Table 3.1, the projected experimental uncertainty of this final states at the HL-LHC is 4.2%, we see that the uncertainties will become comparable, and if the PDF uncertainties are reduced. The predictions can be further improved by the computation of mixed



**Figure 5.2.** The total fiducial cross-section for the final state  $gg \rightarrow h \rightarrow \gamma\gamma$  at both fixed and resummed third order compared to the experimental ATLAS measurement [123] this figure is taken from [200].

QCD-EW effects. These computations invokes three-loop integrals with both gluons and EW bosons, computed in [201] or two-loop ones with two particle final states appearing in the real corrections with the process  $gg \rightarrow hg$  computed in [202] using differential equations. The computation was completed by inclusion of light quark initial states

for the real corrections in [203] with exact quark mass dependence, reducing the EW uncertainty from 2% to  $\sim 0.6\%$ .

The computation of the three-loop form-factors with full top-mass dependence were carried out by [204, 205]. However, there remains an intricate interplay between the mass effects of  $gg$ ,  $qg$  and  $qq$  initial states for the real matrix elements that cannot be fully controlled due to the light quark mass effects.

NLO corrections to the  $h + j$  and  $h + 2j$  processes were computed by [206] in the FT approximation, which used exact born and real correction amplitudes, and approximates the two-loop virtuals by

$$|\mathcal{A}^{2\text{-loop}}(m_t, \mu_R^2)|^2 \approx |\mathcal{A}^{1\text{-loop}}(m_t \rightarrow \infty, \mu_R^2)|^2 \frac{|\mathcal{A}^{1\text{-loop}}(m_t)|^2}{A^{(0)}(m_t) \rightarrow \infty|^2}. \quad (5.1)$$

This approximation works superbly even for  $p_T \gg m_t$ . Later, the full top mass effects computations have been carried out in [207–209] using the high energy (HE) expansion technique.

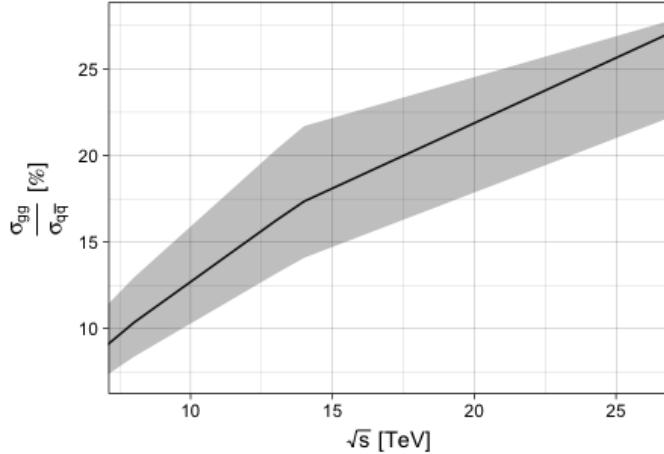
### 5.1.2 Vector boson fusion

The VBF channel has a very distinctive signature, making it a bona fide channel for Higgs signal extraction. The suppressed colour exchange between the quarks result in a little jet activity in the central rapidity region, and the quarks will be scattered into two forward jets, such that the decay products of the Higgs are found in the region between them. These features allows for excellent measurement of Higgs couplings, observation of challenging decays, and  $\mathcal{CP}$  properties determination. Some of these features are also shared with the  $Vh$  production channel. Both of these channels contain the  $VVh$  vertex which could be written generally as [184]

$$T^{\mu\nu}(p_1, p_2) = a_1 g^{\mu\nu} + a_2 \left( g^{\mu\nu} - 2 \frac{p_2^\mu p_2^\nu}{p_1 \cdot p_2} \right) + a_3 \frac{p_1^\alpha p_2^\beta}{p_1 \cdot p_2} \epsilon^{\mu\nu\alpha\beta}. \quad (5.2)$$

In the SM only  $a_1 \neq 0$ , while the rest of the coefficients represent the anomalous coupling, for example if  $a_3 \neq 0$  then the Higgs is  $\mathcal{CP}$  odd. The study of the azimuthal angle distribution  $d\sigma_{VBF}/d\Delta\phi_{jj}$  allows for the determination of these coefficients, with very little dependence on the Higher order corrections [210].

The NLO QCD inclusive cross-section is known since the 90's [211], and later these corrections were made for the differential distributions cf. [212, 213]. Unlike the ggF channel, that has an NLO K-factor of 1.6 at 13 TeV [214], the VBF NLO corrections are small  $\sim 10\%$ . The two-loop NNLO QCD cross-section has been also computed, the most recent results of the two-loop computation were calculated via the structure function approach [215] in addition to STXS level 1.2 bins with EW corrections [216]. These calculation are implemented in the MC event generator **HAWK**. Despite these corrections being small, they are non-negligible and their inclusion is important for uncertainties reduction.



**Figure 5.3.** The ratio of the LO gluon fusion production cross-section  $gg \rightarrow Zh$  ( $\sigma_{gg}$ ) with respect to the NLO Drell-Yan process  $q\bar{q} \rightarrow Zh$  cross-section ( $\sigma_{q\bar{q}}$ ) at a  $pp$  collider with centre-of-mass energy  $\sqrt{s}$ . The error band captures the total theoretical uncertainties on both cross-sections dominated by  $\sigma_{gg}$ .

### 5.1.3 Associated production with EW bosons

The vector boson Higgsstrahlung channels  $pp \rightarrow Wh/Zh$  at LO are quark-initiated tree-level processes interpreted as **Drell-Yan process** [217, 218]. They have been computed up to NNLO in QCD ( $\sim \alpha_s^2$ ), and NLO EW ( $\sim \alpha^2$ ) [219].

Despite arising for the first time at NLO, the gluon fusion channel  $gg \rightarrow Zh$  has a non-negligible contribution to the total hadronic cross-section  $pp \rightarrow Zh$ , which could reach  $> 16\%$  of the total cross-section contribution at 14 TeV [220], see Figure 5.3. The contribution becomes more significant when looking at large invariant mass bins of the differential cross-section. This is due to the significant abundance of gluonic PDF's at the LHC and the extra enhancement coming from the top quark initiated contribution near the  $t\bar{t}$  threshold to boot [221], but also it has a higher scale uncertainty than the quark anti-quark annihilation  $q\bar{q}A$  channel. With these points in mind, and the absence of gluon fusion channel for  $Wh$  channel, the  $Zh$  channel has higher theoretical uncertainties than  $Wh$ . This motivates the need for calculation of the  $gg \rightarrow Zh$  channel to higher orders in perturbation theory in order to reduce these uncertainties. The inclusion of the two-loop calculations for the ggF part would facilitate the precision measurement of the  $Zh$  channel at the future LHC runs, which in turn provides better constraints on several observables, such as sign and magnitude of the top Yukawa and  $ZZh$  couplings amongst others [222].

The leading order (LO) contribution to the  $gg \rightarrow Zh$  amplitude, given by one-loop diagrams, was computed exactly in refs.[223, 224]. For the NLO computations, the virtual corrections contain multi-scale two-loop integrals some of which are still not known analytically, making these computations a formidable task to accomplish. The

first computation of the NLO terms has been done by [225], where the HTL asymptotic expansion, and setting  $m_b = 0$ . The HTL NLO computations pointed to a large  $K$ -factor of about  $\sim 2$ . Later, the computation has been improved via soft gluon resummation, including NLL terms found in ref.[226]. The NLL matching to the fixed-order NLO computation can be found in ref. [225]. Top quark mass effects were first implemented using a combination of HTL and Padé approximants [227]. A data-driven approach to extract the gluon fusion dominated non-Drell-Yan part of  $Zh$  production using the known relation between  $Wh$  and  $Zh$  associated production when only the Drell-Yan component of the two processes is considered has been investigated in ref.[228]. The differential distributions of  $gg \rightarrow Zh$  at NLO was studied in ref.[229] via LO matrix element matching.

More recent studies of the NLO virtual corrections to this process were based on the high-energy (HE) expansion improved by Padé approximants with the LME, which extended the validity range of the HE expansion [230]. However, this expansion is only valid for in the invariant mass region  $\sqrt{\hat{s}} \gtrsim 750 \text{ GeV}$  and  $\sqrt{\hat{s}} \lesssim 350 \text{ GeV}$ , which only covers  $\sim 32\%$  of the hadronic cross-section. Additionally, numerical computation of the two-loop virtual corrections, though implemented exactly in [231], are rather slow for practical use in MC simulations. This highlights the importance of an analytical method that can cover the remaining region of the cross-section and can be merged with the HE expansion via Padé approximants. Fortunately, the two-loop corrections to the triangle diagrams can be computed exactly. And the loop integrals appearing in the box correction having no analytic expression can be expanded in small  $Z$  (or Higgs) transverse momentum,  $p_T$ . This method was first used for Higgs pair production in [11], to compute the NLO virtual corrections to the box diagrams in the forward kinematics. In chapter 6, I will discuss the calculation preformed by my collaborators and myself and published in [232], which includes the full top mass dependence of the virtual two-loop correction to  $gg \rightarrow Zh$  in an analytic form using the same  $p_T$  expansion technique.

#### 5.1.4 Associated production with top quarks

The largest part of the  $t\bar{t}h/th$  uncertainty budget comes from the theoretical modelling of this process's backgrounds, mainly  $t\bar{t}b\bar{b}$ ,  $t\bar{t}W$  as backgrounds for  $t\bar{t}(h \rightarrow b\bar{b})$  and  $t\bar{t}(h \rightarrow \text{multileptons})$ , respectively. There have been several theoretical developments regarding these backgrounds. Starting with  $t\bar{t}W$ , the differential cross-section at NNLL+ NLO QCD calculation of this channel has been done in [233, 234] including EW corrections. The fully decayed final state at NLO QCD [235–237] and at NLO-EW [238] have been computed. Additionally, these calculations were implemented in **POWHEG-BOX** [239]. The comparison between the NLO-QCD with parton showering vs on-shell can be found in [240]. As for  $t\bar{t}b\bar{b}$ , the progress in obtaining higher order corrections is faced with challenges posed by the complexity of this channel. However, progress has been made, for instance the off-shell effects in the fully decayed  $pp \rightarrow 2\ell 2\nu 4b$  with NLO corrections was studied in [241, 242]. Further discussion of the theoretical developments of these channels is beyond the scope of this thesis.

Regarding the higher order corrections to the  $t\bar{t}h/th$  channel itself, the NLO QCD+EW

effects on the off-shell multileptons final state were studied in [243], while the NLO corrections including SMEFT operators were calculated in[244]. The NLO QCD+EW with parton showering is available in all event generators, and the SMEFT operators at NLO are available in `MadGraph5_aMC@NLO` . As of the time of writing this thesis, there is no NNLO calculation of  $t\bar{t}h/th$  available.

## 5.2 Concluding remarks

The precision-era of Higgs measurements requires developments on both experimental and theoretical levels. Though the experimental precision can be improved with Higher luminosities and energies, better detectors and improved analysis techniques. Theoretical uncertainties require higher-order calculations, inclusion of mixed EW and QCD terms, inclusion of mass effects and suitable parton distribution functions with Higher order in QCD. A lot of effort is being put into improving the theoretical predictions of Higgs production channels. Moreover, a plethora of computer tools have been made available to facilitate the computation of these cross-sections, for example `iHixs2` [245] or to generate full events, like `POWHEG` [246–252] and `MadGraph5_aMC@NLO` [253], and many others can be found with greater detail in the LHC-HXWG Twiki page[254].

Sometimes, in order to improve the measurement of the process, it is not sufficient to include Higher order terms of the channel itself, but also its backgrounds, which is particularly important for  $t\bar{t}h$ . Hence, higher order calculations of processes like  $t\bar{t}W$  with parton-shower effects as well as improved analysis to distinguish  $t\bar{t}(h \rightarrow b\bar{b})$  have a significant impact on  $t\bar{t}h$  measurements. Event generator tools with SMEFT implementation for Higgs processes with patron showing interface capabilities have been implemented in a `MadGraph5_aMC@NLO` model `SMEFTatNLO` [255] which enabled loop computations with SMEFT operators and consequently fits of the SMEFT Wilson coefficients with Higgs data at NLO as we have seen in chapter 4.

There is plenty of room for future improvements in the reduction of theory uncertainty budget, and providing better theoretical prediction of the Higgs processes in the SM and beyond. From inclusion of patron shower matching , merging and validation to inclusion of two-loop calculations of gluon fusion  $Zh$  and EW NLO effects of  $t\bar{t}h$ , all in preparation to the HL-LHC Higgs precision era !

## 6 Virtual two-loop calculation of $Zh$ production via gluon fusion

Higgs couplings to the weak vector bosons are approaching the precision level. For their measurements both VBF and  $Vh$  channels are used, the associated Higgs production with the vector bosons are not only important for measuring the  $VVh$  coupling amongst others, as discussed in subsection 5.1.3. The most notable example emphasising the importance of this channel is the measurement of the Higgs decaying to beauty quarks  $h \rightarrow b\bar{b}$  by both ATLAS and CMS [256, 257]. Hence, the  $Vh$  Higgs production has significant potential for the future runs of the LHC. The statistical and systematic uncertainties coming from the experimental setup of the LHC will be eventually reduced in the future runs, due to higher integrated luminosity, upgraded detectors and improved analysis techniques. There is an exigency to reduce theoretical uncertainties emerging from the perturbative calculations of cross-sections, to achieve that, one should include higher order terms. Since  $Wh$  production has no gluon fusion channel, and the main source of  $Zh$  uncertainties stem from its gluon fusion part. Higher order corrections to the  $gg \rightarrow Zh$  is therefore essential for improving the theoretical modelling for this process.

It should be noted that the  $Zh$  channel can receive contributions from new particles [258], also as we shall see in chapter 10; particularly at the large invariant-mass region where the gluon fusion contribution becomes more important, and HTL approximation would typically fail. Therefore, better understanding of the SM prediction of the  $Zh$  gluon fusion channel is crucial for both the SM precision measurements of Higgs production within the SM and for testing NP in this channel.

This chapter aims to demonstrate the use of  $p_T$ -expansion technique, developed in [11] as an approach to compute the two-loop virtual corrections to  $gg \rightarrow Zh$  analytically, including top mass effects. This method also allows for the use of Padé approximants, in order to extend the range of validity of this calculation [341].

This chapter is structured as follows: section 6.1 contains the general notation we have used for the gluon fusion  $Zh$  process calculation. Then in subsection 6.1.1, the transverse momentum expansion method is discussed. Calculation of the LO form-factors in the transverse momentum expansion is illustrated in section 6.2 as a proof of concept for the  $p_T$ -expansion technique. Outline of the two-loop calculation is discussed in section 6.3. Finally, in section 6.4, the results of our calculation are shown with concluding remarks at the end. This chapter is based on the work my collaborators and I published in [232].

## 6.1 General notation

The amplitude  $g_a^\mu(p_1)g_b^\nu(p_2) \rightarrow Z^\rho(p_3)h(p_4)$  can be written as

$$\mathcal{A} = i\sqrt{2}\frac{m_Z G_F \frac{\alpha_s^0}{4\pi}(\mu_R)}{\pi} \delta_{ab} \epsilon_\mu^a(p_1) \epsilon_\nu^b(p_2) \epsilon_\rho(p_3) \hat{\mathcal{A}}^{\mu\nu\rho}(p_1, p_2, p_3), \quad (6.1)$$

$$\hat{\mathcal{A}}^{\mu\nu\rho}(p_1, p_2, p_3) = \sum_{i=1}^6 \mathcal{P}_i^{\mu\nu\rho}(p_1, p_2, p_3) \mathcal{A}_i(\hat{s}, \hat{t}, \hat{u}, m_t, m_h, m_Z), \quad (6.2)$$

where  $\mu_R$  is the renormalisation scale and  $\epsilon_\mu^a(p_1) \epsilon_\nu^b(p_2) \epsilon_\rho(p_3)$  are the polarization vectors of the gluons and the  $Z$  boson, respectively. It is possible to decompose the amplitude into a maximum of 6 Lorentz structures encapsulated by the tensors  $\mathcal{P}_i^{\mu\nu\rho}$ . Due to the presence of the  $\gamma_5$  these projectors are proportional to the Levi-Civita total anti-symmetric tensor  $\epsilon^{\alpha\beta\gamma\delta}$ . One can choose to an orthogonal basis explicitly shown in section A.1, such that

$$\mathcal{P}_i^{\mu\nu\rho} \mathcal{P}_j^{\mu\nu\rho} = 0, \quad \text{for } i \neq j. \quad (6.3)$$

By this choice one obtains unique form factors corresponding to each projector

$$\mathcal{A}_i(\hat{s}, \hat{t}, \hat{u}, m_t, m_h, m_Z), \quad (6.4)$$

that are multivariate complex functions of the top ( $m_t$ ), Higgs ( $m_h$ ) and  $Z$  ( $m_Z$ ) bosons masses, and of the partonic Mandelstam variables

$$\hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 + p_3)^2, \quad \hat{u} = (p_2 + p_3)^2, \quad (6.5)$$

where  $\hat{s} + \hat{t} + \hat{u} = m_Z^2 + m_h^2$  and all the momenta are considered to be incoming. The form-factors  $\mathcal{A}_i$  can be perturbatively expanded in orders of  $\alpha_s$ ,

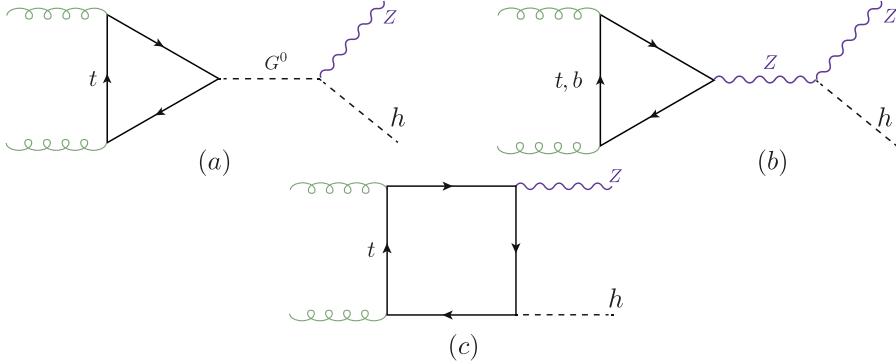
$$\mathcal{A}_i = \sum_{k=0} \left( \frac{\alpha_s}{\pi} \right)^k \mathcal{A}_i^{(k)}. \quad (6.6)$$

Where  $\mathcal{A}_i^{(0)}$  and  $\mathcal{A}_i^{(1)}$  are the LO and NLO terms, respectively. Using Fermi's Golden Rule, we can write the Born partonic cross-section as

$$\hat{\sigma}^{(0)}(\hat{s}) = \frac{m_Z^2 G_F^2 \alpha_s(\mu_R)^2}{64\hat{s}^2(2\pi)^3} \int_{\hat{t}-}^{\hat{t}+} d\hat{t} \sum_i |\mathcal{A}_i^{(0)}|^2, \quad (6.7)$$

where  $\hat{t}^\pm = [-\hat{s} + m_h^2 + m_Z^2 \pm \sqrt{(\hat{s} - m_h^2 - m_Z^2)^2 - 4m_h^2 m_Z^2}] / 2$ .

The LO has two sets of diagrams, the triangle, and box diagrams shown in Figure 6.1. In (a), the triangle diagrams contain a neutral Goldstone boson  $G^0$ , instead in (b) the  $Z$  boson is mediated. The interplay between these two diagram types depends on the  $\xi$  gauge. Moreover, the  $Z$  boson is strictly off-shell, due to Furry's theorem. In the Landau gauge the  $Z$ -mediated diagrams will also vanish, this can be seen by considering the



**Figure 6.1.** Example Feynman diagrams for the LO  $gg \rightarrow Z h$  process. The triangle diagrams in a general  $\xi$  gauge involve  $Z$  and the neutral Goldstone  $G^0$  propagators.

sub-amplitude  $ggZ^*$  which in the Landau gauge can be related to the decay of a massive vector boson with mass  $\sqrt{\hat{s}}$  into two massless ones, a process that is forbidden by the Landau-Yang theorem [259, 260]. The triangle diagrams are also proportional to the mass difference between the up and down type quarks. In this calculation, only the top quark is considered massive, hence light quarks loops do not contribute to this process. It should be noted that, triangle diagrams with  $b-$  quark loops contribute to  $\sim 1\%$  of the total amplitude, computed in the limit  $m_b \rightarrow 0$ .

### 6.1.1 The transverse momentum expansion

Choosing to expand in small  $p_T$  of the  $Z$  boson, the first step is expressing  $p_T$  in terms of the Mandelstam variables and masses

$$p_T^2 = \frac{\hat{t}\hat{u} - m_Z^2 m_h^2}{\hat{s}}. \quad (6.8)$$

From eq.(6.8), together with the relation between the Mandelstam variables, one finds

$$p_T^2 + \frac{m_h^2 + m_Z^2}{2} \leq \frac{\hat{s}}{4} + \frac{\Delta_m^2}{\hat{s}}, \quad (6.9)$$

where  $\Delta_m = (m_h^2 - m_Z^2)/2$ . Eq.(6.9) implies  $p_T^2/\hat{s} < 1$  that, together with the kinematical constraints  $m_h^2/\hat{s} < 1$  and  $m_Z^2/\hat{s} < 1$ . With these relations in mind, one can expand the amplitudes in terms of small  $p_T^2/\hat{s}$ ,  $m_h^2/\hat{s}$  and  $m_Z^2/\hat{s}$ , which is technically valid throughout the whole phase space, contrary to the HTL and HE limits. The caveat for this expansion is that, the amplitude does not depend on  $p_T$  explicitly. Instead, one would expand in the reduced Mandelstam variables  $t'/s' \ll 1$  or  $u'/s' \ll 1$ , defined as

$$s' = p_1 \cdot p_2 = \frac{\hat{s}}{2}, \quad t' = p_1 \cdot p_3 = \frac{\hat{t} - m_Z^2}{2}, \quad u' = p_2 \cdot p_3 = \frac{\hat{u} - m_Z^2}{2} \quad (6.10)$$

and satisfy

$$s' + t' + u' = \Delta_m. \quad (6.11)$$

The choice of the expansion parameter  $t'$  or  $u'$  depends whether one expands in the forward or backwards kinematics. Because the process  $gg \rightarrow Zh$ , has two particles in the final states with different masses, the amplitude is not symmetric under the their exchange. One therefore cannot compute the cross-section by integrating only the forward-expanded amplitude, contrary what has been done for the Higgs pair [11]. In order to overcome this issue, one could further examine the projectors in section A.1 and observe that they can be split into symmetric and anti-symmetric parts with respect to the exchange  $t' \leftrightarrow u'$ . Then, expand the symmetric part in the forward kinematics, like the Higgs pair case. Regarding the anti-symmetric part, the antisymmetric factor is simply extracted by multiplying the form-factors by  $1/(\hat{t} - \hat{u})$ , written as  $1/(2s' - 4t' - 2\Delta_m)$ , then perform the expansion in the forward kinematics and finally multiply back by  $(\hat{t} - \hat{u})$ .

In order to implement the  $p_T$ -expansion at the Feynman diagrams level we start by splitting the momenta into longitudinal and transverse components with respect to the beam direction, by introducing the vector [11],

$$r^\mu = p_1^\mu + p_3^\mu, \quad (6.12)$$

which satisfies

$$r^2 = \hat{t}, \quad r \cdot p_1 = \frac{\hat{t} - m_Z^2}{2}, \quad r \cdot p_2 = -\frac{\hat{t} - m_h^2}{2}, \quad (6.13)$$

and hence can be also written as

$$r^\mu = -\frac{\hat{t} - m_h^2}{\hat{s}} p_1^\mu + \frac{\hat{t} - m_Z^2}{\hat{s}} p_2^\mu + r_\perp^\mu = \frac{t'}{s'} (p_2^\mu - p_1^\mu) - \frac{\Delta_m}{s'} p_1^\mu + r_\perp^\mu, \quad (6.14)$$

where

$$r_\perp^2 = -p_T^2. \quad (6.15)$$

substituting the definition of  $p_T$  from eq.(6.8) one obtains

$$t' = -\frac{s'}{2} \left\{ 1 - \frac{\Delta_m}{s'} \pm \sqrt{\left(1 - \frac{\Delta_m}{s'}\right)^2 - 2\frac{p_T^2 + m_Z^2}{s'}} \right\} \quad (6.16)$$

implying that the expansion in small  $p_T$  (the minus sign case in eq.(6.16)) can be realised at the level of Feynman diagrams, by expanding the propagators in terms of the vector  $r^\mu$  around  $r^\mu \sim 0$  or, equivalently,  $p_3^\mu \sim -p_1^\mu$ , see eq.(6.14).

## 6.2 Born cross-section in the $p_T$ -expansion

As a baseline test for the validity and convergence behaviour of the  $p_T$ -expansion we start by computing the LO amplitude, and consequently the Born partonic cross-section

in the  $p_T$  expansion then compare it with the exact results found in [223, 224].

Starting by defining the one-loop functions appearing in the similar calculation of the Born cross-section for  $gg \rightarrow hh$  in the same expansion carried out in ref. [11]

$$B_0[\hat{s}, m_t^2, m_t^2] \equiv B_0^+, \quad B_0[-\hat{s}, m_t^2, m_t^2] \equiv B_0^-, \quad (6.17)$$

$$C_0[0, 0, \hat{s}, m_t^2, m_t^2, m_t^2] \equiv C_0^+, \quad C_0[0, 0, -\hat{s}, m_t^2, m_t^2, m_t^2] \equiv C_0^- \quad (6.18)$$

$$B_0[q^2, m_1^2, m_2^2] = \frac{1}{i\pi^2} \int \frac{d^n k}{\mu^{n-4}} \frac{1}{(k^2 - m_1^2)((k+q)^2 - m_2^2)}, \quad (6.19)$$

$$\begin{aligned} C_0[q_a^2, q_b^2, (q_a + q_b)^2, m_1^2, m_2^2, m_3^2] = \\ \frac{1}{i\pi^2} \int \frac{d^d k}{\mu^{d-4}} \frac{1}{[k^2 - m_1^2][(k+q_a)^2 - m_2^2][(k-q_b)^2 - m_3^2]} \end{aligned} \quad (6.20)$$

are the Passarino-Veltman functions [261], with  $d$  being the spacetime dimension and  $\mu$  the 't Hooft mass. There are only two non-vanishing form-factors at LO, one is symmetric  $\mathcal{A}_2$ , and the other is antisymmetric  $\mathcal{A}_6$ , in the  $p_T$ -expansion, these form-

factors are give by, up to order  $\mathcal{O}(p_T^2)$

$$\begin{aligned}
\mathcal{A}_2^{(0,\Delta)} &= -\frac{p_T}{\sqrt{2}(m_Z^2 + p_T^2)}(\hat{s} - \Delta_m)m_t^2 C_0^+, \\
\mathcal{A}_2^{(0,\square)} &= \frac{p_T}{\sqrt{2}(m_Z^2 + p_T^2)} \left\{ \right. \\
&\quad \left( m_t^2 - m_z^2 \frac{\hat{s} - 6m_t^2}{4\hat{s}} - p_T^2 \frac{12m_t^4 - 16m_t^2\hat{s} + \hat{s}^2}{12\hat{s}^2} \right) B_0^+ \\
&- \left( m_t^2 - \Delta_m \frac{m_t^2}{(4m_t^2 + \hat{s})} + m_z^2 \frac{24m_t^4 - 6m_t^2\hat{s} - \hat{s}^2}{4\hat{s}(4m_t^2 + \hat{s})} - \right. \\
&\quad \left. p_T^2 \frac{48m_t^6 - 68m_t^4\hat{s} - 4m_t^2\hat{s}^2 + \hat{s}^3}{12\hat{s}^2(4m_t^2 + \hat{s})} \right) B_0^- \\
&+ \left( 2m_t^2 - \Delta_m + m_z^2 \frac{3m_t^2 - \hat{s}}{\hat{s}} + p_T^2 \frac{3m_t^2\hat{s} - 2m_t^4}{\hat{s}^2} \right) m_t^2 C_0^- \\
&+ \left( \hat{s} - 2m_t^2 + m_z^2 \frac{\hat{s} - 3m_t^2}{\hat{s}} + p_T^2 \frac{2m_t^4 - 3m_t^2\hat{s} + \hat{s}^2}{\hat{s}^2} \right) m_t^2 C_0^+ \\
&+ \log \left( \frac{m_t^2}{\mu^2} \right) \frac{m_t^2}{(4m_t^2 + \hat{s})} \left( \Delta_m + 2m_z^2 + p_T^2 \frac{2\hat{s} - 2m_t^2}{3\hat{s}} \right) \\
&- \left. \Delta_m \frac{2m_t^2}{(4m_t^2 + \hat{s})} + m_z^2 \frac{\hat{s} - 12m_t^2}{4(4m_t^2 + \hat{s})} + p_T^2 \frac{8m_t^4 - 2m_t^2\hat{s} + \hat{s}^2}{4\hat{s}(4m_t^2 + \hat{s})} \right\}, 
\end{aligned} \tag{6.22}$$

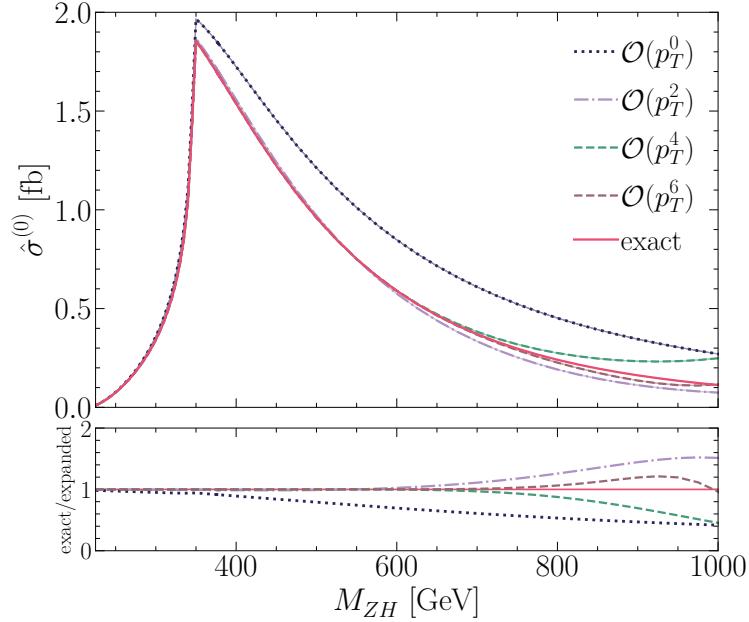
and

$$\mathcal{A}_6^{(0,\Delta)} = 0, \tag{6.23}$$

$$\begin{aligned}
\mathcal{A}_6^{(0,\square)} &= \frac{\hat{t} - \hat{u}}{\hat{s}^2} p_T \left[ \frac{m_t^2}{2} \left( B_0^- - B_0^+ \right) - \frac{\hat{s}}{4} \right. \\
&- \left. \frac{2m_t^2 + \hat{s}}{2} m_t^2 C_0^- + \frac{2m_t^2 - \hat{s}}{2} m_t^2 C_0^+ \right], 
\end{aligned} \tag{6.24}$$

where these form-factors were divided into triangle ( $\Delta$ ) and box ( $\square$ ) contributions, and  $B_0$  functions are understood as the finite part of the integrals on the right hand side of eq.(6.19).

Using several truncations of the  $p_T$ -expansion, and comparing it to the exact LO result, one can see in Figure 6.2 the exact Born partonic LO cross section (red line) as a function of the invariant mass of the  $Zh$  system,  $M_{Zh}$ , in comparison to the  $p_T$ -expansions. For the numerical evaluation of the cross-section here and in the following,



**Figure 6.2.** The Born partonic cross-section as a function of the invariant mass  $M_{Zh}$ . The exact (red line) is plotted together with results at different orders in the  $p_T$ -expansion (dashed lines). In the bottom part, the ratio of the full result over the  $p_T$ -expanded one at various orders is shown. This plot has been published in [232]

the SM parameters were used

$$\begin{aligned} m_Z &= 91.1876 \text{ GeV}, & m_h &= 125.1 \text{ GeV}, & m_t &= 173.21 \text{ GeV}, \\ m_b &= 0 \text{ GeV}, & G_F &= 1.16637 \text{ GeV}^{-2}, & \alpha_s(m_Z) &= 0.118. \end{aligned}$$

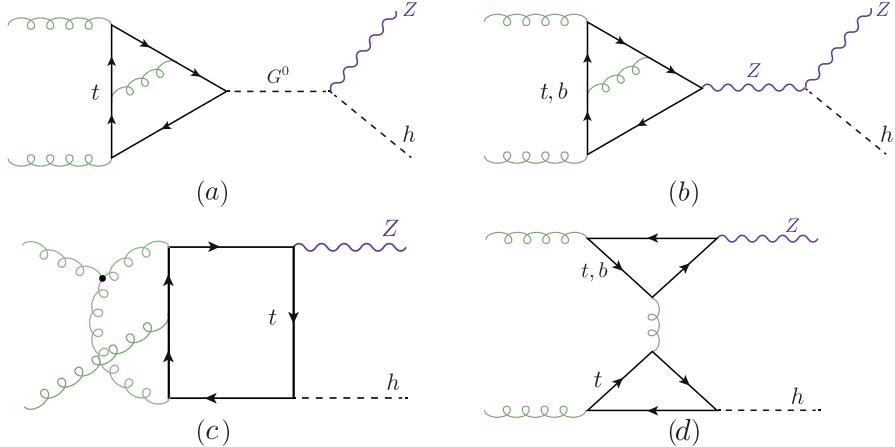
From the ratio plotted in the lower panel of Figure 6.2, we observe that the  $\mathcal{O}(p_T^0)$  expansion is in good agreement with the exact result when  $M_{Zh} \lesssim 2m_t$ . Inclusion of higher order terms up to  $\mathcal{O}(p_T^6)$  extended the validity of the expansion to reach  $M_{Zh} \lesssim 750$  GeV. This is the similar behaviour seen in [11] for Higgs pair. Therefore, one would expect the  $p_T$ -expanded two-loop virtual correction to be an accurate approximation with the exact (numerical) result for the region of the invariant mass of  $M_{Zh} \sim 700 - 750$  GeV. Similar conclusions can be seen more explicitly in Table 6.1, where it is shown that the partonic cross-section at  $\mathcal{O}(p_T^4)$  agrees with the full result for  $M_{ZH} \lesssim 600$  GeV on the per-mille level and the agreement further improves when  $\mathcal{O}(p_T^6)$  terms are included.

$M_{Zh}$ [GeV]	$\mathcal{O}(p_T^0)$	$\mathcal{O}(p_T^2)$	$\mathcal{O}(p_T^4)$	$\mathcal{O}(p_T^6)$	full
300	0.3547	0.3393	0.3373	0.3371	0.3371
350	1.9385	1.8413	1.8292	1.8279	1.8278
400	1.6990	1.5347	1.5161	1.5143	1.5142
600	0.8328	0.5653	0.5804	0.5792	0.5794
750	0.5129	0.2482	0.3129	0.2841	0.2919

**Table 6.1.** The partonic cross section  $\hat{\sigma}^{(0)}$  at various orders in  $p_T$  and the full computation for several values of  $M_{Zh}$ . This table has been published in [232].

### 6.3 NLO calculation

The virtual two-loop corrections to  $gg \rightarrow Zh$  are shown in Figure 6.3, which involve corrections to the triangle topology in (a) and (b). The corrections to the box topology in (c) and a new topology , dented by double triangle in (d). Both two-loop corrections to the triangles, and the double triangle diagrams can be computed exactly analytically. However, the two-loop box diagrams contain master-integrals (MI's) that have no analytic solutions, so far. The two-loop box diagrams will be computed in the  $p_T$ -expansion.



**Figure 6.3.** Feynman diagrams types for the virtual NLO corrections to the  $gg \rightarrow Zh$  process.

#### 6.3.1 Renormalisation

The two-loop corrections to the triangle and box diagrams contain both UV and IR divergences. The first emerges from UV divergent sub-diagrams, such as top mass renormalisation and QCD vertex correction. While the IR divergences come from massless

loops. In order to remove these divergences, one introduces adequate counter-terms. On the other hand, the double triangle is both UV and IR finite.

We start by the gluon wavefunction renormalisation of the incoming gluons (external legs) such that the amplitude is renormalised by  $Z_A^{1/2}$  for each gluon.

$$Z_A = 1 + \frac{\alpha_s^0}{4\pi} \frac{2}{3\epsilon} \left( \frac{\mu_R^2}{m_t^2} \right)^\epsilon. \quad (6.25)$$

The on-shell scheme for the top mass renormalisation has been used, in which the bare mass is replaced by the renormalised one  $m_0 = Z_m m$  in the propagators this gives the  $\overline{\text{MS}}$  renormalised mass.

$$Z_m = 1 + C_F \frac{3}{\epsilon}. \quad (6.26)$$

In order to convert the mass definition to the on-shell scheme we add the finite renormalisation term

$$Z_m^{OS} = 1 - 2C_F, \quad (6.27)$$

here  $C_F = (N_c^2 - 1)/2N_c$  is one of the two Casimir invariants of QCD along with  $C_A = N_c$ . The  $q\bar{q}g$  vertex correction involves a renormalisation of the strong couplings constant  $\alpha_s$  which is done via replacing the bare constant  $\alpha_s^0$  with the renormalised one, hence it becomes  $\alpha_s^0 = \frac{\mu_R^{2\epsilon}}{S_\epsilon} Z_{\alpha_s} \alpha_s$ , where

$$Z_{\alpha_s} = 1 - \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} \left( \beta_0 - \frac{2}{3} \right) \left( \frac{\mu_R^2}{m_t^2} \right)^\epsilon, \quad (6.28)$$

and the constant  $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}N_f$ , where  $N_f$  is the number of “active” flavours. The 5-flavour scheme  $N_f = 5$  is adopted here.

The loop integrals were evaluated via dimensional regularisation in  $d = 4 - 2\epsilon$  dimensions. Which requires some caution when  $\gamma_5$  is present in the amplitude. We let  $\gamma_5$  naively anti-commute with all  $d$ -dimensional  $\gamma_\mu$ 's and then correct that with the finite renormalisation constant known as **Larin counter-term** [262]

$$Z_5 = 1 - 2C_F. \quad (6.29)$$

The renormalised amplitude is written as

$$\mathcal{M}(\alpha_s, m, \mu_R) = Z_A \mathcal{M}(\alpha_s^0, m^0). \quad (6.30)$$

Putting all the above substitutions together, we get the renormalised two-loop form-

factor:

$$(\mathcal{A}^{(1)})^R = \mathcal{A}^{(1)} - \mathcal{A}_{UV}^{(0)} - \mathcal{A}_{UV,m}^{(0)} + \mathcal{A}_{\text{Larin}}^{(0)} \quad (6.31)$$

$$\mathcal{A}_{UV}^{(0)} = \frac{\alpha_s}{4\pi} \frac{\beta_0}{\epsilon} \left( \frac{\mu_R^2}{\hat{s}} \right)^{-\epsilon} \mathcal{A}^{(0)}.$$

$$\mathcal{A}_{UV,m}^{(0)} = \frac{\alpha_s}{4\pi} \left( \frac{3}{\epsilon} - 2 \right) C_F \left( \frac{\mu_R^2}{\hat{s}} \right)^{-\epsilon} m^0 \partial_m \mathcal{A}^{(0)}. \quad (6.32)$$

$$\mathcal{A}_{\text{Larin}}^{(0)} = -\frac{\alpha_s}{4\pi} C_F \mathcal{A}^{(0)}.$$

The following IR-counter-term is used in order to cancel the IR divergences.

$$\mathcal{A}_{IR}^{(0)} = \frac{e^{\gamma_E \epsilon}}{\Gamma(1-\epsilon)} \frac{\alpha_s}{4\pi} \left( \frac{\beta_0}{\epsilon} + \frac{C_A}{\epsilon^2} \right) \left( \frac{\mu_R^2}{\hat{s}} \right)^{2\epsilon} \mathcal{A}^{(0)} \quad (6.33)$$

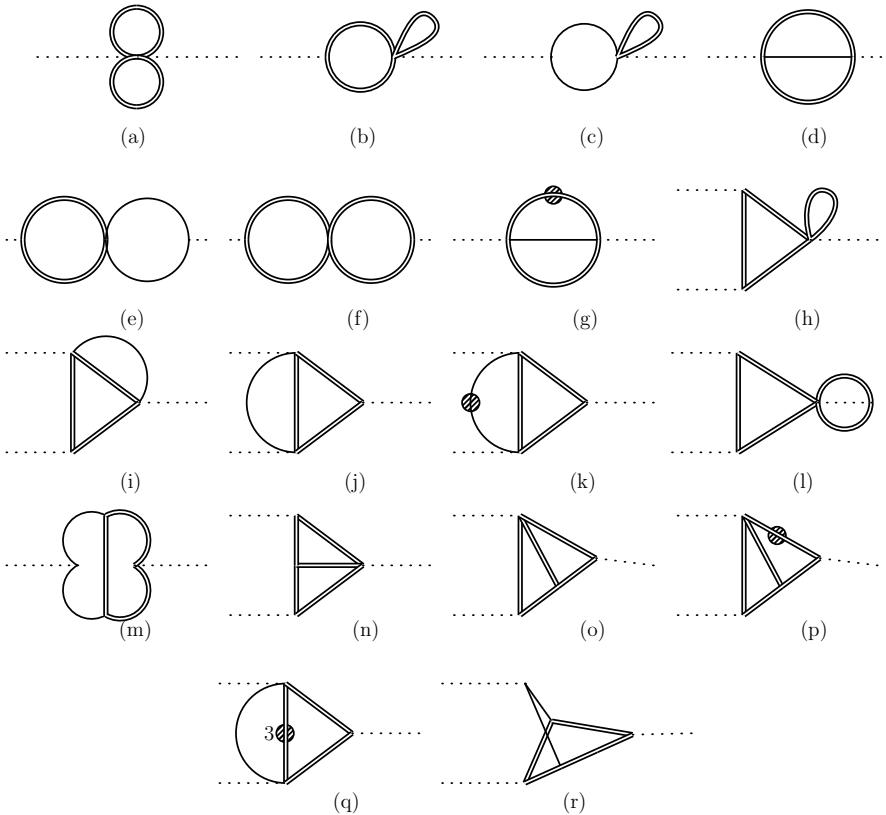
The one-loop form-factors, need to be expanded up to order  $\mathcal{O}(\epsilon^2)$ , for the UV and IR counter-terms.

### 6.3.2 Calculation of the exact virtual corrections

The two-loop calculations of the triangle diagrams involves the diagrams of with virtual  $Z^*$  and  $G^0$ , depending on the gauge of choice. Observations found in ref.[225] shows that due to Landau-Yang theorem in the Landau gauge the diagrams with the  $Z^*$  exchange vanishes. Therefore, the part of the top triangle diagrams can be obtained from the decay amplitude of a pseudoscalar boson into two gluons which is known in the literature in the full mass dependence up to NLO terms [263, 264]. On the contrary, in the unitary gauge, the NLO calculation needs to be done with the  $Z^*$  exchange diagrams only. The calculations result in apparently different Lorentz structures, that are linked via the Schouten identity

$$q^\alpha \epsilon^{\beta\gamma\delta\phi} + q^\beta \epsilon^{\gamma\delta\phi\alpha} + q^\gamma \epsilon^{\delta\phi\alpha\beta} + q^\gamma \epsilon^{\delta\phi\alpha\beta} + q^\delta \epsilon^{\phi\alpha\beta\gamma} + q^\phi \epsilon^{\alpha\beta\gamma\delta} = 0 \quad (6.34)$$

A cross-check has been preformed in order to ensure that the NLO calculation introduces no new Lorentz structures, and gives the same result in a general  $R_\xi$  gauge as the results in [263, 264]. The two-loop calculation has been carried out in  $R_\xi$  gauge. The amplitudes have been automatically generated by **FeynArts** [265] and contracted with the projectors as defined in section A.1 using **FeynCalc** [266, 267] and **Package X** [268] and in-house Mathematica routines. The two-loop integrals were reduced to a set of master integrals MI, illustrated graphically in Figure 6.4 using **Kira** [269]. These MI's are either products of one-loop functions (a)-(c), (e),(f),(h) and (l) or can be found in the literature [264, 270]. Their implementation in our calculation has been validated numerically using **SecDec** [271, 272]. The virtual correction for the triangle diagrams



**Figure 6.4.** The list of two-loop master integrals (MI's) resulting from the reduction of the two-loop triangle corrections, and the product of one-loop MI's appearing in this list also appear in the calculation of the double-triangle diagrams. A single line denotes a massless propagator, while a double line denotes a massive one. The dot denotes a squared propagator, unless the number of the exponent is indicated, here only 3 appears in diagram (q).

can be separated according to their colour factors into

$$\mathcal{A}^{(1)} = C_F \mathcal{A}_{CF}^{(1)} + C_A \mathcal{A}_{CA}^{(1)}, \quad (6.35)$$

The  $C_A$  part contains a double pole  $\mathcal{O}(1/\epsilon^2)$  and a single pole  $\mathcal{O}(1/\epsilon)$ , both coming from the IR divergence. Whilst the  $C_F$  part contains a UV divergent pole that needs to be cured via mass renormalisation. The poles do not have a dependence on the renormalisation scale  $\mu_R$ . However, there is a dependence on that scale in the finite part, as well. No new Lorentz structures appeared, and the final result in  $R_\xi$  matched the one found in [263, 264] for the Landau gauge. The explicit results are shown in [Appendix B](#)

The calculation of the double triangle diagrams (d) of [Figure 6.3](#) is fairly straightforward, all of the integrals can be rewritten in terms of products of one-loop functions. All of the Lorentz structures appear in the double triangle except for  $\mathcal{P}_6$ , analogous to the triangle case. The explicit forms of form-factors corresponding to these structures are presented in [Appendix B](#). Although we write the amplitude using a different tensorial structure with respect to ref.[230] we have checked, using the relations between the two tensorial structures reported in [section A.1](#), that our result is in agreement with the one presented in ref.[227].

### 6.3.3 Calculation of the $p_T$ -expanded virtual corrections

The two-loop triangle diagrams can also be interpreted as an expansion in  $p_T$ , but this expansion terminates at  $\mathcal{O}(p_T^2)$ , rather being an infinite series. Hence, in this section we concentrate on the two-loop box diagrams  $p_T$ -expansion<sup>1</sup>.

Similar to the two-loop triangle diagrams, the box diagrams amplitudes were generated projected through the same pipeline. After the contraction of the epsilon tensors the diagrams were expanded as described in [subsection 6.1.1](#), keeping only  $\mathcal{O}(p_T^4)$  terms. They were reduced to MI's using **FIRE** [273] and **LiteRed** [274]. The resulting MI's were identical to the one for Higgs pair production [11]. Nearly all of them are expressed in terms of generalised harmonic polylogarithms with the exception of two elliptic integrals [275, 276]. The renormalisation and IR pole subtraction procedure was carried out like prescribed [subsection 6.3.1](#).

As a control, the two-loop box diagrams were also computed in the HTL up to  $\mathcal{O}(1/m_t^6)$ . Since this expansion should be included within the  $p_T$ -expansion. We have retained the HTL analytic expression by further expanding the  $p_T$ -expanded amplitude in small  $\hat{s}/m_t^2$ . Providing an additional cross-check for the validity of the  $p_T$ -expansion.

## 6.4 Results and conclusions

The virtual corrections to the gluon fusion  $Zh$  production have been implemented in a **FORTRAN** code using **handyG** [277], for the evaluation of generalised harmonic polylogarithms, **Chaplin** [278] for the harmonic polylogarithms appearing in the triangle two-loop

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<sup>1</sup>The calculation of the box diagrams has been done mainly by my collaborators, the co-authors of [232]

$\hat{s}/m_t^2$	$\hat{t}/m_t^2$	ref.[231]	$\mathcal{O}(p_T^6)$
1.707133657190554	-0.441203767016323	35.429092(6)	35.430479
3.876056604162662	-1.616287256345735	4339.045(1)	4340.754
4.130574250302561	-1.750372271104745	6912.361(3)	6915.797
4.130574250302561	-2.595461551488002	6981.09(2)	6984.20

**Table 6.2.** Comparison of  $\mathcal{V}_{fin}4/(\alpha_s^2 \alpha^2)$  with the numerical results of ref.[231]. This plot has been published in [232].

functions while the elliptic integrals are evaluated using the routines of ref.[276]. Since the result is analytic, the code is significantly faster than the numerical evaluation of the two-loop amplitude [231], with evaluation time of ca. 0.5 min per one phase space point on a personal laptop.

In order to facilitate the comparison of our results with the ones presented in the literature, we define the finite part of the virtual corrections as in ref.[230]<sup>2</sup>

$$\begin{aligned} \mathcal{V}_{fin} = & \frac{G_F^2 m_Z^2}{16} \left( \frac{\alpha_s}{\pi} \right)^2 \left[ \sum_i \left| \mathcal{A}_i^{(0)} \right|^2 \frac{C_A}{2} \left( \pi^2 - \log^2 \left( \frac{\mu_R^2}{\hat{s}} \right) \right) \right. \\ & \left. + 2 \sum_i \text{Re} \left[ \mathcal{A}_i^{(0)} \left( \mathcal{A}_i^{(1)} \right)^* \right] \right] \end{aligned} \quad (6.36)$$

and in the numerical evaluation of eq.(6.36) we fixed  $\mu_R = \sqrt{\hat{s}}$ . Triangle and HTL box topologies were validated against the results of refs. [227, 230] finding perfect agreement at the form-factor level, i.e.  $\mathcal{A}_i^{(1)}$ .

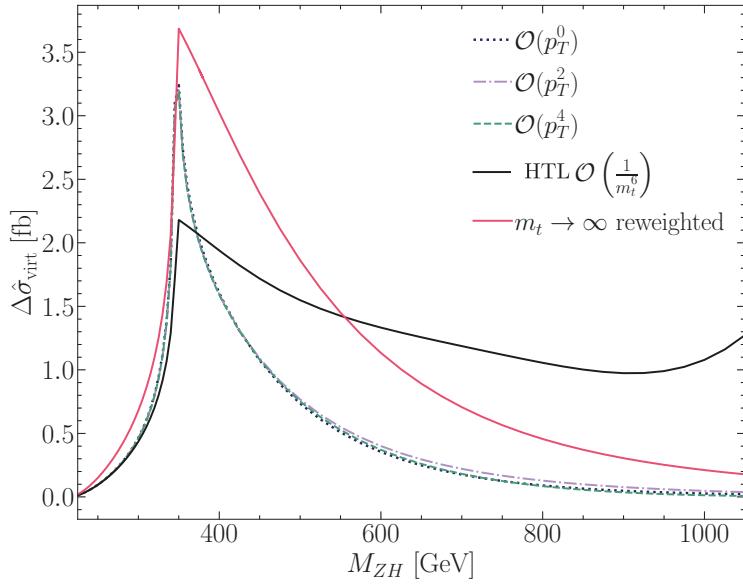
The virtual part of the partonic cross-section from the finite part of the virtual corrections in eq. (6.36) is defined by

$$\Delta \hat{\sigma}_{virt} = \int_{\hat{t}^-}^{\hat{t}^+} d\hat{t} \frac{\alpha_s}{16\pi^2} \frac{1}{\hat{s}^2} \mathcal{V}_{fin} \quad (6.37)$$

This function is used to confront  $p_T$ -expanded results. Starting with low  $M_{Zh}$  we have compared the  $p_T$ -expanded with the HTL  $\mathcal{V}_{fin}$ , finding a good numerical agreement. It is important to note that, at the same order in the expansion, the  $p_T$ -expanded terms are more accurate than the HTL ones, although computationally more demanding. Additional checks have been done using the numerical evaluation of the NLO amplitude by [231], where they have evaluated the exact two-loop MI's using `pySecDec` [279, 280]. Table 6.2 shows a comparison between our  $p_T$ -expanded  $\mathcal{V}_{fin}4/(\frac{\alpha_s^0}{4\pi} \alpha^2)$  versus the exact numerical result of [231] for several phase space points. As can be seen from the table the relative difference between the two results is less than half a per-mille.

In Figure 6.5, the dashed lines show the different orders of the expansion. For all

<sup>2</sup>The definition of the matrix elements here differs by a factor of  $\frac{1}{\hat{s}}$  from ref.[230], cf. also section A.1.



**Figure 6.5.**  $\Delta\hat{\sigma}_{\text{virt}}$  defined by eq.(6.37), shown as a function of  $M_{Zh}$ . The various orders of the  $p_T$ -expansion are plotted as dashed lines, while the black and red continuous lines stand for the HTL and reweighted  $m_t \rightarrow \infty$  results, respectively. This plot has been published in [232].

parts of the matrix elements the best results available, i.e. both  $\mathcal{A}^{(0)}$  were used and the double-triangle contribution are evaluated exactly, while for  $\mathcal{A}^{(1)}$  we use the various orders in the  $p_T$ -expansion. For comparison, the results are shown were  $\mathcal{A}^{(1)}$  is replaced by the one computed in HTL up to  $\mathcal{O}(1/m_t^6)$  (full black line), which as mentioned before is valid up to  $M_{ZhH} < 2m_t$ . We observe that within the validity of the HTL our results agree well with it. Furthermore, the results in the infinite top mass limit reweighted by the full amplitudes squared can be seen as the full red line in the plot, corresponding to the approach of ref.[225], keeping though the double triangle contribution in full top mass dependence. Differently from the HTL line, the  $m_t \rightarrow \infty$  reweighted one shows a behaviour, for  $M_{Zh} \gtrsim 400$  GeV, similar to the behaviour of the  $p_T$  lines. Still, the difference between the reweighted result and the  $p_T$ -expanded ones is significant. The  $p_T$ -expanded results show very good convergence. The zero order in our expansion agrees extremely well with the higher orders in the expansion, and all the three results are very close up to  $M_{Zh} \sim 500$  GeV.

The calculation of the virtual two-loop corrections to the  $gg \rightarrow Zh$  is done using exact results for the triangle and double-triangle topologies, and in the  $p_T$ -expansion for the box one. The result of the calculation showed that we get the exact same MI's that was found for Higgs pair production [11], mostly these MI's are expressed in terms of generalised harmonic polylogarithms with the exception of two elliptic integrals. Using the LO calculation, we have shown the validity of the  $p_T$ -expansion covering the invariant mass interval  $M_{Zh} \lesssim 750$  GeV which covers  $\sim 98\%$  of the total phase space for 13 – 14

TeV energies.

The  $p_T$ -expansion agrees with per mill level with the numerical results found in [231]. However, it allows for fast computation of the amplitude with circa one second per phase space point using a modern laptop with mid-range specifications. Additionally, the integration over the  $\hat{t}$  variable in eq.(6.37) converges superbly. The flexibility of our analytic results, an application to beyond-the-Standard Model is certainly possible.

Finally, it should be noted that this calculation complements nicely the results obtained in ref. [230] using a HE expansion, that according to the authors provides precise results for  $p_T \gtrsim 200 \text{ GeV}$ . The merging of the two analyses is going to provide a result that covers the whole phase space, can be easily implemented into a Monte Carlo code using, the virtual correction results of this bridging procedure is published in [341].



## 7 Four top operators in Higgs production and decay

In chapter 4, the SMEFT has been portrayed as a pragmatic yet robust parametrisation of potential NP degrees of freedom for LHC searches, with the ansatz that these degrees of freedom have masses that are higher than the LHC reach. From the discussion and overview of Higgs-related SMEFT operators in that chapter, the operator  $\mathcal{O}_\phi$  stood out as one of the weakly constrained amongst them. This is due to the current low experimental sensitivity on the Higgs self-coupling as shown in ??.

Though many of top operators are strongly constrained from top observables, some of them remain as weakly constrained as the trilinear Higgs self-coupling, particularly four-fermion operators involving the third generation quarks. They would be constrained directly from the production of four tops observation. However, this process has small cross-section at the LHC  $\sim 12 \text{ fb}$  [281], which is more or less comparable to the Higgs pair production. Experimental searches for the production of four top quarks has been first made by CMS [282] combining different LHC runs, followed by ATLAS [283], the latter reporting a  $4.3\sigma$  observation of this processes with cross-section of  $24^{+7}_{-6} \text{ fb}$ . When the whole third generation quarks is included, one sees the same story with  $t\bar{t}b\bar{b}$  contact interaction which requires the observation of  $t\bar{t}b\bar{b}$  production for a direct constraint, see [284, 285] for experimental searches and [286, 287] for SMEFT fits. It should be noted that for the production of four tops, or two tops two beauty quarks in SMEFT, the contact terms do not interfere with the SM process, and only appear proportional to  $\mathcal{O}(1/\Lambda^4)$ . This makes the SMEFT global analysis of these operators is highly dependent on the EFT truncation scheme used, i.e. whether to keep quadratic terms or not.

Intriguingly, these four-fermion operators enter in single Higgs processes at NLO, in a similar manner as the Higgs self-coupling. Since the four-fermions operators are weakly constrained, they should be included in fits that includes Higgs data. We shall demonstrate that, there is a significant correlation amongst the Higgs self-coupling and the four-fermion operators.

The chapter is based on a the paper [288] and structured as follows: in ?? the full NLO calculation of Higgs rates due to the four-fermion operators is shown. Afterwards, in section 7.2, a fit from Higgs data combining the Higgs trilinear coupling and the four-fermion operators is presented, for both Run-II and HL-LHC, with more elaborate results for the latter is found in ???. The results are further discussed in section 7.3.

## 7.1 Contribution of four-fermion operators to Higgs rates at NLO

We will consider the following dimension-six SMEFT operators:

Four-Heavy-quark SMEFT operators modifying Higgs rates at NLO

Operators with homogenous chiral structure, i.e. (RR)(RR) or (LL)(LL)

$$\mathcal{O}_{tt}, \mathcal{O}_{bb}, \mathcal{O}_{tb}^{(1)}, \mathcal{O}_{tb}^{(8)}, \mathcal{O}_{QQ}^{(1)}, \mathcal{O}_{QQ}^{(8)}. \quad (7.1)$$

Operators with heterogeneous chiral structure, i.e. (LR)(LR) or (LL)(RR)

$$\mathcal{O}_{Qt}^{(1)}, \mathcal{O}_{Qt}^{(8)}, \mathcal{O}_{Qb}^{(1)}, \mathcal{O}_{Qb}^{(8)}, \mathcal{O}_{QtQb}^{(1)}, \mathcal{O}_{QtQb}^{(8)} \quad (7.2)$$

The explicit definition of these operators can be found in [Table 4.1](#). Here, the notation is slightly modified from the standard Warsaw basis. The flavour indices were suppressed since only the the third generation is considered throughout this chapter. Adopting the same notation from previous chapters,  $Q$  denotes the (heavy) left-handed  $SU(2)_L$  doublet quarks while  $t$  and  $b$  refer to the right-handed singlets. In studies involving SMEFT fits, such as [173] the  $SU(3)_C$  singlet and octet left-handed operators  $\mathcal{O}_{QQ}^{(1),SU(3)}$ ,  $\mathcal{O}_{QQ}^{(8)}$  are used instead of the singlet and triplet of  $SU(2)_L$  appearing in the standard Warsaw basis. The two conventions are related via the relations

$$\begin{aligned} C_{QQ}^{(1),SU(3)} &= 2C_{QQ}^{(1)} - \frac{2}{3}C_{QQ}^{(3)}, \\ C_{QQ}^{(8)} &= 8C_{QQ}^{(3)}. \end{aligned} \quad (7.3)$$

Additionally, all of these Wilson coefficients are assumed to be real.

We will consider operators that induce sizeable NLO correction to Higgs processes are taken into account. These operators turns out to be the ones that introduce loop corrections to the top or beauty Yukawa, their masses and finite corrections from top loops. Such corrections will be proportional to the top mass. On the contrary, corrections from beauty loops are highly suppressed by  $m_b$ . Also, operators that have chiral structure that does not enable them to enter in the Yukawa RGE's will not be constrained from Higgs data as they would only contribute through small finite terms, as we shall see later. Hence, only four top and the  $\mathcal{O}_{QtQb}^{(1),(8)}$  operators will be considered.

This section will demonstrate the calculation of NLO Higgs production and decay rates from the four-heavy-quarks operators discussed above. For the production of Higgs via gluon fusion or Higgs decay to gluon, photons and beauty quarks, the results were computed fully analytically and presented in this section. However, for the associated production of the Higgs with top pair  $t\bar{t}h$ , the corrections were computed numerically, due to the length of the the analytic expressions if the result.

### 7.1.1 Analytic calculations

The NLO corrections to gluon fusion,  $h \rightarrow gg$ ,  $h \rightarrow \gamma\gamma$  and  $h \rightarrow b\bar{b}$  all come from the sub-diagrams listed in [Table 7.1](#), with top loops entering in the mass renormalisation or to/beauty Yukawa vertex correction. Where  $N_c = 3$  the number of colours, and  $c_F = (N_c^2 - 1)/(2N_c) = 4/3$  the eigenvalues of the Casimir operator of  $SU(3)_c$  in the fundamental representation. The effect of beauty loops coming from for  $C_{QtQb}^{(1/8)}$ , can be

Diagram	colour factor		mass/coupling
	singlet	octet	
	$2N_c + 1$	$c_F$	$y_t m_b m_t^2$
	1	$c_F$	$y_t m_t^3$
	$2N_c + 1$	$c_F$	$m_t^3$
	1	$c_F$	$m_t^3$

**Table 7.1.** Sub-diagrams contributing to the NLO corrections of gluon fusion Higgs production and its decay to gluons, photons and beauty quarks.

easily read from this table by exchanging  $t \leftrightarrow b$ , which is significantly smaller than the corrections coming from top loops.

We see that these corrections correspond to the Wilson coefficients appearing in the RGE's, and operators with (LL)(LL) or (RR(RR)) chiral structures do not contribute to these processes.

By considering the two-loop corrections to the ggF illustrated in [Figure 7.1](#) we find that such correction contain the sub-diagrams shown in [Table 7.1](#), except for diagram (e), which is found to be vanishing for on-shell gluons. Additionally, these diagrams indicated that the two-loop corrections will be reduced to products of two one-loop functions after the integral reduction.

Following the Feynman rules derived in ref. [289] for the four-fermion operators of interest here, the  $gg \rightarrow h$  two-loop amplitude was calculated, then Dirac algebra and further algebraic manipulations were preformed in Mathematica using `PackageX` [290]. Reduc-

tion of the resulting two-loop to Master integrals has been preformed using **KIRA** [291]. The computation has been cross-checked independently by my collaborators, using a different pipeline : **FeynArts** [265], for amplitude generation then **FeynRules** [292] and **Fire** [293] for algebraic manipulation and loop-integral reduction.

The sub-diagrams appearing in the two-loop calculation, correspond to mass and vertex renormalisation, which require counter-terms for pole cancellation. A mixture of on-shell (OS) and  $\overline{\text{MS}}$  – schemes has been used for the mass and  $hq\bar{q}$  coupling renormalisation, respectively. The renormalisation of SM quantities in the OS and NP ones in the  $\overline{\text{MS}}$  scheme was proposed by [294].

The top/beauty mass renormalisation can be expressed as

$$m_{t/b}^{\text{OS}} = m_{t/b}^{(0)} - \delta m_{t/b}, \quad (7.4)$$

with the corresponding counter-terms

$$\delta m_t = \frac{1}{16\pi^2} \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} m_t^3 \left[ \frac{2}{\bar{\epsilon}} + 2 \log \left( \frac{\mu_R^2}{m_t^2} \right) + 1 \right] \quad (7.5)$$

$$+ \frac{1}{16\pi^2} \frac{(2N_c + 1)C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}}{\Lambda^2} \left[ \frac{1}{\bar{\epsilon}} + \log \left( \frac{\mu_R^2}{m_b^2} \right) + 1 \right] m_b^3,$$

$$\delta m_b = \frac{1}{16\pi^2} \frac{(2N_c + 1)C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}}{\Lambda^2} \left[ \frac{1}{\bar{\epsilon}} + \log \left( \frac{\mu_R^2}{m_t^2} \right) + 1 \right] m_t^3, \quad (7.6)$$

with  $\bar{\epsilon}^{-1} = \epsilon^{-1} - \gamma_E + \log(4\pi)$ , in dimensional regularization with  $d = 4 - 2\epsilon$ . It is possible to convert from OS to the  $\overline{\text{MS}}$  – scheme for mass counter-terms via the following relations

$$\delta m_t^{\overline{\text{MS}}} = \frac{1}{8\pi^2} \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} m_t^3 \frac{1}{\bar{\epsilon}} + \frac{1}{16\pi^2} \frac{(2N_c + 1)C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}}{\Lambda^2} \frac{1}{\bar{\epsilon}} m_b^3, \quad (7.7)$$

$$\delta m_b^{\overline{\text{MS}}} = \frac{1}{16\pi^2} \frac{(2N_c + 1)C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}}{\Lambda^2} \frac{1}{\bar{\epsilon}} m_t^3. \quad (7.8)$$

The effect of changing to the mass renormalisation scheme is small for the top mass but significant, up to 100% for the beauty mass.

The top/beauty Higgs coupling in SMEFT, is written as

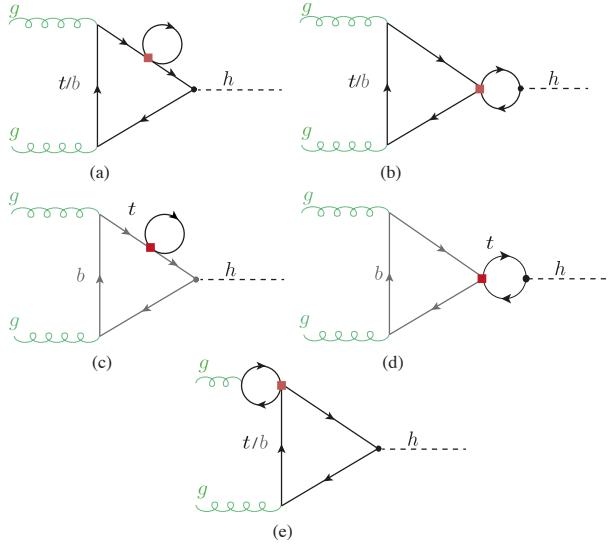
$$g_{ht\bar{t}/hb\bar{b}} = \frac{m_{t/b}}{v} - \frac{v^2}{\Lambda^2} \frac{C_{t\phi/b\phi}}{\sqrt{2}}. \quad (7.9)$$

Hence, a modification of the Higgs couplings to beauty and top quarks is generated by operator mixing, even if  $C_{t\phi/b\phi}$  are set to zero at  $\Lambda$ . From this, the  $\overline{\text{MS}}$  counter-term

should take the form

$$\delta g_{ht\bar{t}/hb\bar{b}} = \frac{m_{t/b}}{v} \delta m_{t/b} - \frac{v^2 \delta C_{t\phi/b\phi}}{\sqrt{2}}, \quad (7.10)$$

where  $\delta C_{t\phi/b\phi}$  is directly read from the anomalous dimension, see ref. [14]



**Figure 7.1.** Example Feynman diagrams for four-fermion-operator contributions to the Higgs production via gluon fusion. The red box indicates the four-fermion operator.

#### Correction to gluon fusion and $h \rightarrow gg$

The modification of the Higgs production via gluon fusion can be written as

$$\frac{\sigma_{ggF}}{\sigma_{ggF}^{\text{SM}}} = 1 + \frac{\sum_{i=t,b} 2\text{Re}(F_{\text{LO}}^i F_{\text{NLO}}^{*i})}{|F_{\text{LO}}^t + F_{\text{LO}}^b|^2} \quad (7.11)$$

with

$$F_{\text{LO}}^i = -\frac{8m_i^2}{m_h^2} \left[ 1 - \frac{1}{4} \log^2(x_i) \left( 1 - \frac{4m_i^2}{m_h^2} \right) \right] \quad (7.12)$$

and the NLO form-factors are given by

$$\begin{aligned}
 F_{\text{NLO}} = & \frac{1}{4\pi^2 \Lambda^2} (C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}) F_{\text{LO}}^t \left[ 2m_t^2 + \frac{1}{4}(m_h^2 - 4m_t^2) \left( 3 + 2\sqrt{1 - \frac{4m_t^2}{m_h^2}} \log(x_t) \right) \right. \\
 & \left. + \frac{1}{2}(m_h^2 - 4m_t^2) \log \left( \frac{\mu_R^2}{m_t^2} \right) \right] \\
 & + \frac{1}{32\pi^2 \Lambda^2} ((2N_c + 1) C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}) \left[ F_{\text{LO}}^b \frac{m_t}{m_b} \left( 4m_t^2 - 2m_h^2 \right. \right. \\
 & \left. \left. - (m_h^2 - 4m_t^2) \sqrt{1 - \frac{4m_t^2}{m_h^2}} \log(x_t) - (m_h^2 - 4m_t^2) \log \left( \frac{\mu_R^2}{m_t^2} \right) \right) + (t \leftrightarrow b) \right]. \tag{7.13}
 \end{aligned}$$

Only top quark loops contribute to the parts proportional to  $C_{Qt}^{(1),(8)}$ . The variable  $x_i$  for a loop particle with mass  $m_i$  is given by

$$x_i = \frac{-1 + \sqrt{1 - \frac{4m_i^2}{m_h^2}}}{1 + \sqrt{1 - \frac{4m_i^2}{m_h^2}}}. \tag{7.14}$$

Using the same amplitudes, the  $h \rightarrow gg$  partial width modification can be written as

$$\frac{\Gamma_{h \rightarrow gg}}{\Gamma_{h \rightarrow gg}^{\text{SM}}} = 1 + \frac{\sum_{i=t,b} 2\text{Re}(F_{\text{LO}}^i F_{\text{NLO}}^*)}{|F_{\text{LO}}^t + F_{\text{LO}}^b|^2} \tag{7.15}$$

### Correction to Higgs decay to photons

Analogously, since the decay  $h \rightarrow \gamma\gamma$  contains the same topologies as gluon fusion, we could use the result from the above calculation to obtain the NLO correction to the partial width for this decay

$$\frac{\Gamma_{h \rightarrow \gamma\gamma}}{\Gamma_{h \rightarrow \gamma\gamma}^{\text{SM}}} = 1 + \frac{2\text{Re}(F_{\text{LO},\gamma} F_{\text{NLO},\gamma}^*)}{|F_{\text{LO},\gamma}|^2}. \tag{7.16}$$

However, one should pay attention to the change in the prefactors, and the extra EW contributions for  $h \rightarrow \gamma\gamma$

$$F_{\text{LO},\gamma} = N_C Q_t^2 F_{\text{LO}}^t + N_C Q_b^2 F_{\text{LO}}^b + F_{\text{LO}}^W + F_{\text{LO}}^G, \tag{7.17}$$

and  $F_{\text{NLO},\gamma}$  is obtained from  $F_{\text{NLO}}$  by replacing the LO form factor that appears inside of it by  $F_{\text{LO}}^i \rightarrow N_c Q_i^2 F_{\text{LO}}^i$ , with the charges  $Q_t = 2/3$  and  $Q_b = -1/3$ . The

$W$  boson contribution

$$F_{\text{LO}}^W = 2 \left( 1 + 6 \frac{m_W^2}{m_h^2} \right) - 6 \frac{m_W^2}{m_h^2} \left( 1 - 2 \frac{m_W^2}{m_h^2} \right) \log^2(x_W), \quad (7.18)$$

with  $m_W$  the  $W$  mass, and the Goldstone contribution

$$F_{\text{LO}}^G = 4 \frac{m_W^2}{m_h^2} \left( 1 + \frac{m_W^2}{m_h^2} \log^2(x_W) \right). \quad (7.19)$$

These operators also affect the  $h \rightarrow Z\gamma$  partial width. However, as in the diphoton case, the effect is expected to be small due to the dominance of the  $W$  boson loop. Because of this, and given the smallness of the  $h \rightarrow Z\gamma$  branching ratio and the relatively low precision expected in this channel at the LHC, the effects of four-fermion interactions in this decay are neglected.

#### Correction to Higgs decays to $b\bar{b}$

The dominant four-fermion contributions to decay channel  $h \rightarrow b\bar{b}$  come from the operators  $\mathcal{O}_{QtQb}^{(1),(8)}$ . The corresponding diagram at NLO is shown in fig 7.2. Adopting the same renormalisation procedure as described earlier, we obtain the following expression for the correction to the  $h \rightarrow b\bar{b}$  decay rate in the presence of  $\mathcal{O}_{QtQb}^{(1),(8)}$ ,

$$\begin{aligned} \frac{\Gamma_{h \rightarrow b\bar{b}}}{\Gamma_{h \rightarrow b\bar{b}}^{\text{SM}}} = & 1 + \frac{1}{16\pi^2} \frac{m_t}{m_b} (m_h^2 - 4m_t^2) \frac{(2N_c + 1)C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}}{\Lambda^2} \\ & \times \left[ 2 + \sqrt{1 - \frac{4m_t^2}{m_h^2} \log(x_t)} - \log\left(\frac{m_t^2}{\mu_R^2}\right) \right], \end{aligned} \quad (7.20)$$

which carries an enhancement factor of  $m_t/m_b$  and is hence expected to be rather large.

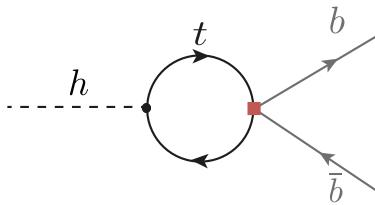


Figure 7.2. Feynman diagram contributing to the NLO  $h \rightarrow b\bar{b}$  process.

The results of the NLO effects from the four-fermion operators reported above, do not take into account the running of the Wilson coefficients. This would be based on the assumption that these coefficients are defined at the process scale. Nevertheless,

when we want to compare different process or assume that the four-fermion operators are defined at the UV scale  $\Lambda$ . One has take into an account the running of these Wilson coefficients from  $\Lambda$  down to the process scale.

These running effects can be included via the RGE for the operators with Wilson coefficient  $C_{t\phi}$  and  $C_{b\phi}$  [295, 296], that leads approximatively to

$$C_{t\phi}(\mu_R) - C_{t\phi}(\Lambda) = \frac{1}{16\pi^2 v^2} \left[ -2y_t(m_h^2 - 4m_t^2)(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}) \log\left(\frac{\mu_R^2}{\Lambda^2}\right) + \frac{y_b}{2}(m_h^2 - 4m_b^2) ((2N_c + 1)C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}) \log\left(\frac{\mu_R^2}{\Lambda^2}\right) \right] \quad (7.21)$$

and

$$C_{b\phi}(\mu_R) - C_{b\phi}(\Lambda) = \frac{y_t}{32\pi^2 v^2} \left[ (m_h^2 - 4m_t^2) ((2N_c + 1)C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}) \log\left(\frac{\mu_R^2}{\Lambda^2}\right) \right], \quad (7.22)$$

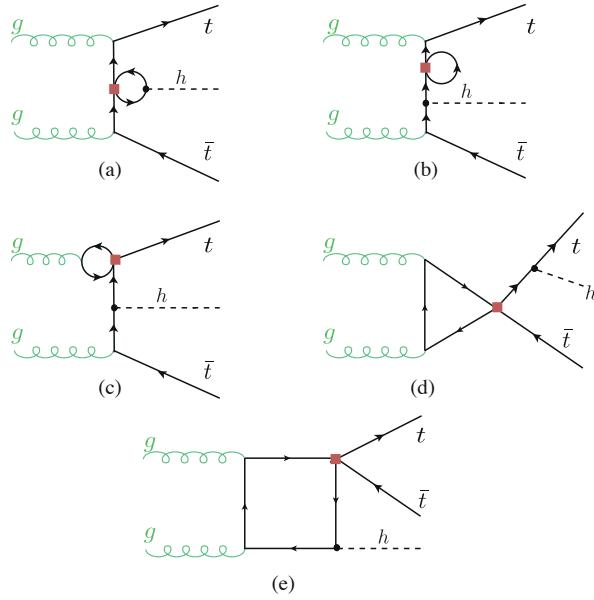
where  $y_{t/b} = \sqrt{2}m_{t/b}/v$ . Note that the combinations of Wilson coefficients appearing in (7.21) and (7.22) are the same as in  $F_{NLO}$  in (7.13). Effectively, we can then obtain the result under the assumption that the four-fermion operators are the only non-zero ones at the high scale by replacing in (7.13)  $\mu_R \rightarrow \Lambda$ , noting that we have renormalised the top and beauty quark masses in the OS scheme. Including the leading logarithmic running of  $C_{b\phi}$  of (7.22) from the high scale  $\Lambda$  to the electroweak scale is achieved by setting in (7.20)  $\mu_R \rightarrow \Lambda$ . The expression in (7.20) agrees with results obtained from the full calculation of the NLO effects in the dimension-six SMEFT, computed in [297].

### 7.1.2 SMEFT-NLO calculation of $t\bar{t}h$

Unlike the previous processes, the associated production of the Higgs with top quark pair involves new topologies not limited to Yukawa vertex or mass renormalisation. At the LHC, there are two sub-processes responsible for the  $t\bar{t}h$  production: gluon-initiated process illustrated in Figure 7.3 and quark-initiated in Figure 7.4. We see the new *finite* topologies induced by the four-fermion operator corrections in (d) triangle and (e) box topologies in Figure 7.3 and (b) triangle topology in Figure 7.4. Additionally, the  $t\bar{t}g$  vertex correction in the quark-initiated process (diagram (c)) of Figure 7.4 is non-vanishing as the gluon is off-shell. This vertex correction has a UV pole that requires a counter-term for its cancellation

$$\text{Diagram: } \text{Gluon} \rightarrow t\bar{t} \rightarrow h\gamma = \frac{ig_s}{12\pi^2 \Lambda^2} T_{ij}^A p_g^2 \gamma^\mu \left( C_{tt} P_R + (C_{QQ}^{(1)} + C_{QQ}^{(3)}) P_L + \frac{C_{Qt}^{(8)}}{4} \right) \left( \frac{1}{\epsilon} - 1 \right). \quad (7.23)$$

Another difference between  $t\bar{t}h$  and the other Higgs processes studies in this chapter comes from the fact that this channel has a non-trivial colour structured. This mani-

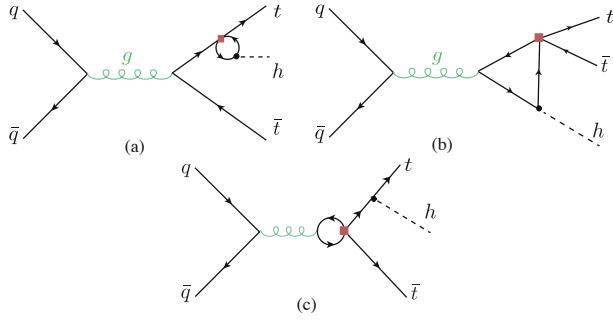


**Figure 7.3.** Feynman diagrams including the four-fermion loop contributions to the  $gg \rightarrow t\bar{t}h$  subprocess.

fects in the presence of multiple colour projectors, because the quark anti quark triplets or the gluon pairs do not have to recombine to only a singlet state rather to both a singlet and an octet, according to the expansion of product  $\mathbf{3} \otimes \overline{\mathbf{3}} \rightarrow \mathbf{1} + \mathbf{8}$ . This breaks the degeneracy between the singlet and octet Wilson coefficients. Lastly, due to the new topologies and  $t\bar{t}g$  vertex correction, operators with single chirality will contribute to NLO corrections, namely  $C_{ttt}$  and  $C_{QQ}^{(1,3)}$ .

All of the four-fermion operators are implemented in the loop-capable UFO model SMEFTatNLO [255] computed via Madgraph\_aMCNLO [253] with some tweaking to remove the NLO QCD corrections. This is done via a user-defined loop filter function in Madgraph. The results were reproduced by an analytic computation based on the reduction of one-loop amplitudes via the method developed by G. Ossola, C.G. Papadopoulos and R. Pittau (OPP reduction) [298], implemented in the FORTRAN code CutTools programme [299]. This programme takes the full one-loop amplitude and then reduces it to terms with 1,2,3 and 4-point loop functions in four dimensions, keeping spurious terms from the  $\epsilon$  part of the amplitude. To correct for such terms, one needs to compute the divergent UV counter-terms as well as a finite rational terms, denoted by  $R_2$  as in ref. [300].<sup>1</sup> The amplitudes were generated in the same way as for ggF. The UV and  $R_2$  counter-terms, that need to be supplemented to CutTools, were computed manually following the method detailed in [300]. For both codes, the NNPDF23 parton distribution functions set at NLO [301] was used.

<sup>1</sup>Another rational term  $R_1$  appears due to the mismatch between the four and  $d$  dimensional amplitudes, but this is computed automatically in CutTools.



**Figure 7.4.** Feynman diagrams including the four-fermion loop contributions to the  $q\bar{q} \rightarrow t\bar{t}h$  subprocess.

The singlet and octet operators  $\mathcal{O}_{QtQb}^{(1),(8)}$  contribute to  $t\bar{t}h$  only via beauty loops and in principle, could be directly dismissed like the other beauty quark operators mentioned above. However, it is instructive to investigate their effect albeit it is expected to be small. Since the **SMEFTatNLO** model does not have these operators, it was needed to implement them manually in that model. This is simply done by include the vertices generated by these operators as well as their UV and  $R_2$  counter-terms. The calculation of the NLO correction by these operators was done both in Madgraph using a modified UFO model and with the code based on **CutTools**. The effects were comparable to the leading log effects computed using **SMEFTsim** package [302] of  $\sim 10^{-6}$ . Hence confirming the expectation that beauty quark loops have a negligible effect.

In order to include the effects of Wilson coefficients' running, the relevant contribution for the gluon-initiated process is the same as the stated for the gluon fusion in (7.21). While for the quark-initiated process, one needs to consider the operator mixing in the running, particularly between operators that contain second and third generation quarks mixed together. These corrections can be obtained from the RGEs in refs. [14, 295, 296].

### 7.1.3 Results

The NLO correction from the four-fermion operators of the third generation quarks on the Higgs rates i.e., partial width  $\Gamma$  or cross-section  $\sigma$ , is extracted from the above computation using the formula

$$\delta R(C_i) = R/R^{\text{SM}} - 1, \quad (7.24)$$

here the dependence of the Higgs rate  $R$  on the Wilson coefficient  $C_i$  is denoted by  $\delta R(C_i)$ . Only contributions linear in the Wilson coefficients are considered. In order to isolate the finite terms from the ones coming from the RGE leading log approximation, the correction is further expanded to finite  $\delta R_{C_i}^{fin}$  and leading log terms  $\delta R_{C_i}^{log}$  as follows

$$\delta R(C_i) = \frac{C_i}{\Lambda^2} \left( \delta R_{C_i}^{fin} + \delta R_{C_i}^{log} \log \left( \frac{\mu_R^2}{\Lambda^2} \right) \right). \quad (7.25)$$

Operator	Process	$\mu_R$	$\delta R_{C_i}^{fin}$ [TeV $^2$ ]	$\delta R_{C_i}^{log}$ [TeV $^2$ ]
$\mathcal{O}_{Qt}^{(1)}$	ggF	$\frac{m_h}{2}$	$9.91 \cdot 10^{-3}$	$2.76 \cdot 10^{-3}$
	$h \rightarrow gg$	$m_h$	$6.08 \cdot 10^{-3}$	$2.76 \cdot 10^{-3}$
	$h \rightarrow \gamma\gamma$		$-1.76 \cdot 10^{-3}$	$-0.80 \cdot 10^{-3}$
	$t\bar{t}h$ 13 TeV		$-4.20 \cdot 10^{-1}$	$-2.78 \cdot 10^{-3}$
	$t\bar{t}h$ 14 TeV	$m_t + \frac{m_h}{2}$	$-4.30 \cdot 10^{-1}$	$-2.78 \cdot 10^{-3}$
$\mathcal{O}_{Qt}^{(8)}$	ggF	$\frac{m_h}{2}$	$1.32 \cdot 10^{-2}$	$3.68 \cdot 10^{-3}$
	$h \rightarrow gg$	$m_h$	$8.11 \cdot 10^{-3}$	$3.68 \cdot 10^{-3}$
	$h \rightarrow \gamma\gamma$		$-2.09 \cdot 10^{-3}$	$-1.07 \cdot 10^{-3}$
	$t\bar{t}h$ 13 TeV		$6.81 \cdot 10^{-2}$	$-2.40 \cdot 10^{-3}$
	$t\bar{t}h$ 14 TeV	$m_t + \frac{m_h}{2}$	$7.29 \cdot 10^{-2}$	$-2.48 \cdot 10^{-3}$
$\mathcal{O}_{QtQb}^{(1)}$	ggF	$\frac{m_h}{2}$	$2.84 \cdot 10^{-2}$	$9.21 \cdot 10^{-3}$
	$h \rightarrow gg$		$1.57 \cdot 10^{-2}$	$9.21 \cdot 10^{-3}$
	$h \rightarrow \gamma\gamma$	$m_h$	$-1.30 \cdot 10^{-3}$	$-0.78 \cdot 10^{-3}$
	$h \rightarrow b\bar{b}$		$9.25 \cdot 10^{-2}$	$1.68 \cdot 10^{-1}$
$\mathcal{O}_{QtQb}^{(8)}$	ggF	$\frac{m_h}{2}$	$5.41 \cdot 10^{-3}$	$1.76 \cdot 10^{-3}$
	$h \rightarrow gg$		$2.98 \cdot 10^{-3}$	$1.76 \cdot 10^{-3}$
	$h \rightarrow \gamma\gamma$	$m_h$	$-0.25 \cdot 10^{-3}$	$-0.15 \cdot 10^{-3}$
	$h \rightarrow b\bar{b}$		$1.76 \cdot 10^{-2}$	$3.20 \cdot 10^{-2}$
$\mathcal{O}_{QQ}^{(1)}$	$t\bar{t}h$ 13 TeV		$1.75 \cdot 10^{-3}$	$1.84 \cdot 10^{-3}$
	$t\bar{t}h$ 14 TeV	$m_t + \frac{m_h}{2}$	$1.65 \cdot 10^{-3}$	$1.76 \cdot 10^{-3}$
$\mathcal{O}_{QQ}^{(3)}$	$t\bar{t}h$ 13 TeV		$1.32 \cdot 10^{-2}$	$5.48 \cdot 10^{-3}$
	$t\bar{t}h$ 14 TeV	$m_t + \frac{m_h}{2}$	$1.24 \cdot 10^{-2}$	$5.30 \cdot 10^{-3}$
$\mathcal{O}_{tt}$	$t\bar{t}h$ 13 TeV		$4.60 \cdot 10^{-3}$	$1.82 \cdot 10^{-3}$
	$t\bar{t}h$ 14 TeV	$m_t + \frac{m_h}{2}$	$4.57 \cdot 10^{-3}$	$1.74 \cdot 10^{-3}$

**Table 7.2.** The NLO effects of the four heavy-quarks operators on the Higgs rates. The effects are separated into finite  $\delta R_{C_i}^{fin}$  and leading log parts, in correspondence with (7.25). This table has been published in [288].

Using this formula, one can obtain the correction at any NP scale  $\Lambda$ , though in the remainder of this chapter this scale is set to 1 TeV. In Table 7.2, the finite and logarithmic corrections for the operators considered in this study is reported. Using this table in filling the formula (7.25) will give the correction to Higgs rates. However, since some of the rates are Higgs partial widths, the Higgs total width  $\Gamma_h$  will be affected and therefore all of Higgs rates are changed. An important observation from Table 7.2 is that the finite terms, are either larger or at the same order than the leading-log ones, except for  $h \rightarrow b\bar{b}$  corrections from  $\mathcal{O}_{QtQb}^{(1),(8)}$ . This highlights the importance of the full NLO calculation for these corrections in constraining these four-fermion operators, in particular  $\mathcal{O}_{Qt}^{(1),(8)}$ .

As mentioned earlier, there is a degeneracy amongst the singlet and octet operators, seen clearly in the analytic result for gluon fusion and Higgs decays considered. This

degeneracy is though broken for  $\mathcal{O}_{Qt}^{(1),(8)}$  due to  $t\bar{t}h$ . Since, the effect of  $\mathcal{O}_{QtQb}^{(1),(8)}$  is negligible for this process, thee true degree of freedom for these operators' Wilson coefficients is the linear combination

$$C_{QtQb}^+ = (2N_c + 1)C_{QtQb}^{(1)} + c_F C_{QtQb}^{(8)}. \quad (7.26)$$

## 7.2 Fit to Higgs observables

Using the results from the NLO calculations discussed above, and combining them with the calculations of NLO Higgs rates from the trilinear Higgs self-coupling  $\lambda_3$ , preformed in refs. [149–152, 154] expnading on the previous fits for  $\lambda_3$  from Higgs data, by the inclusion of the four-fermion SMEFT Wilson coefficients. In order to examine the true sensitivity of single Higgs observables to  $\lambda_3$ . Although combined fits from Higgs data including  $\lambda_3$  and SMEFT operators modifying Higgs rates at LO have been preformed [177]. Such fits would not be sufficient to determine the actual sensitivity for  $\lambda_3$ , in particular when the SMEFT operators are weakly constrained and induce significant modifications to Higgs rates, that can be seen in Table 7.2. This chapter does not include a global SMEFT fit, rather merely motivates it, by illustrating how the sensitivity for probing the Higgs-self coupling from single Higgs data gets mitigated when the four-fermion operators are included in the fit.

In the antecedent studies, the modification to Higgs self coupling was reported in terms of the  $\kappa$ -formalism, for the consistency of this analysis, the NLO corrections from the trilinear self-coupling will be converted to the SMEFT notation, in terms of the Wilson coefficient  $C_\phi$ , for more details on the conversion between SMEFT and  $\kappa$ -formalism see subsection 4.2.2. In order to keep track of he SMEFT power-counting, we rewrite the results of [150] after converting it to SMEFT

$$\delta R_{\lambda_3} \equiv \frac{R_{\text{NLO}}(\lambda_3) - R_{\text{NLO}}(\lambda_3^{\text{SM}})}{R_{\text{LO}}} = -2 \frac{C_\phi v^4}{\Lambda^2 m_h^2} C_1 + \left( -4 \frac{C_\phi v^4}{\Lambda^2 m_h^2} + 4 \frac{C_\phi^2 v^8}{m_h^4 \Lambda^4} \right) C_2. \quad (7.27)$$

In (7.27), the coefficient  $C_1$  corresponds to the contribution of the trilinear coupling to the single Higgs processes at one loop, adopting the same notation as [150]. The values of  $C_1$  for the different processes of interest for this paper are given in Table 7.3. The coefficient  $C_2$  describes universal corrections and is given by

$$C_2 = \frac{\delta Z_h}{1 - \left( 1 - \frac{2C_\phi v^4}{\Lambda^2 m_h^2} \right)^2 \delta Z_h}, \quad (7.28)$$

where the constant  $\delta Z_h$  is the SM contribution from the Higgs loops to the wave function renormalisation of the Higgs boson,

$$\delta Z_h = -\frac{9}{16} \frac{G_F m_h^2}{\sqrt{2}\pi^2} \left( \frac{2\pi}{3\sqrt{3}} - 1 \right). \quad (7.29)$$

The coefficient  $C_2$  thus introduces additional  $\mathcal{O}(1/\Lambda^4)$  (and higher order) terms in  $\delta R_{\lambda_3}$ . In ref. [150] considering the  $\kappa$ -formalism the full expression of (7.28) is kept, while we define two different descriptions: one in which we expand  $\delta R_{\lambda_3}$  up to linear order in  $C_\phi$  and an alternative scheme in which we keep also terms up to  $\mathcal{O}(1/\Lambda^4)$  in the EFT expansion. Keeping the full expression in (7.28) and including terms up to  $\mathcal{O}(1/\Lambda^4)$  in  $C_2$  lead to nearly the same results as the simple  $\mathcal{O}(1/\Lambda^4)$  fit.

Process	$C_1$	$\delta R_{C_\phi}^{fin}$
ggF/ $gg \rightarrow h$	$6.60 \cdot 10^{-3}$	$-3.10 \cdot 10^{-3}$
$t\bar{t}h$ 13 TeV	$3.51 \cdot 10^{-2}$	$-1.64 \cdot 10^{-2}$
$t\bar{t}h$ 14 TeV	$3.47 \cdot 10^{-2}$	$-1.62 \cdot 10^{-2}$
$h \rightarrow \gamma\gamma$	$4.90 \cdot 10^{-3}$	$-2.30 \cdot 10^{-3}$
$h \rightarrow b\bar{b}$	0.00	0.00
$h \rightarrow W^+W^-$	$7.30 \cdot 10^{-3}$	$-3.40 \cdot 10^{-3}$
$h \rightarrow ZZ$	$8.30 \cdot 10^{-3}$	$-3.90 \cdot 10^{-3}$
$pp \rightarrow Zh$ 13 TeV	$1.19 \cdot 10^{-2}$	$-5.60 \cdot 10^{-3}$
$pp \rightarrow Zh$ 14 TeV	$1.18 \cdot 10^{-2}$	$-5.50 \cdot 10^{-3}$
$pp \rightarrow W^\pm h$	$1.03 \cdot 10^{-2}$	$-4.80 \cdot 10^{-3}$
VBF	$6.50 \cdot 10^{-3}$	$-3.00 \cdot 10^{-3}$
$h \rightarrow 4\ell$	$8.20 \cdot 10^{-3}$	$-3.80 \cdot 10^{-3}$

**Table 7.3.** The NLO dependence of single Higgs rates on  $C_\phi$ , these results were computed in [154]. The  $C_1$  coefficients are to be used in eq. (7.27), while for a direct comparison with the effect of the four-fermion operators, we quote the translated effect  $\delta R_{C_\phi}^{fin}$ , which can be used directly in eq. (7.25). If the value of  $\sqrt{s}$  is not indicated the effect is the same for both 13 and 14 TeV. This table has been published in [288].

A Bayesian fit was preformed using Markov-chain Monte Carlo (MCMC) method. Using a flat prior s  $\pi(C_i) = const.$  and a log likelihood of a Gaussian distribution

$$\log(L) = -\frac{1}{2} \left[ (\vec{\mu}_{\text{Exp}} - \vec{\mu})^T \cdot \mathbf{V}^{-1} \cdot (\vec{\mu}_{\text{Exp}} - \vec{\mu}) \right]. \quad (7.30)$$

Constructed as follows:

**Experimental inputs**  $\vec{\mu}_{\text{Exp}}$  The signal strengths from experimental measurements of single Higgs rates defined as

$$\mu_{\text{Exp}} \equiv \sigma_{\text{obs}} / \sigma_{\text{SM}}. \quad (7.31)$$

These measurements as taken from LHC Run II for centre-of-mass energy of  $\sqrt{s} = 13$  TeV and integrated luminosity of  $139 \text{ fb}^{-1}$  for ATLAS and  $137 \text{ fb}^{-1}$  for CMS. In addition to HL-LHC projections by CMS for  $\sqrt{s} = 14$  TeV and integrated luminosity of  $3000 \text{ fb}^{-1}$ . Both of these inputs have been already discussed in chapter 3 and summarised in Table 3.1.

**Theoretical predictions**  $\vec{\mu}$  The corresponding theoretical predictions for each of the ex-

perimental measurement /projection have been built using the modification to the cross-sections and branching ratios coming from the SMEFT four-fermion operators and  $C_\phi$ . To keep with the power-counting, the signal strength is also expanded in powers of  $\Lambda$ , keeping only  $\Lambda^{-2}$  terms.

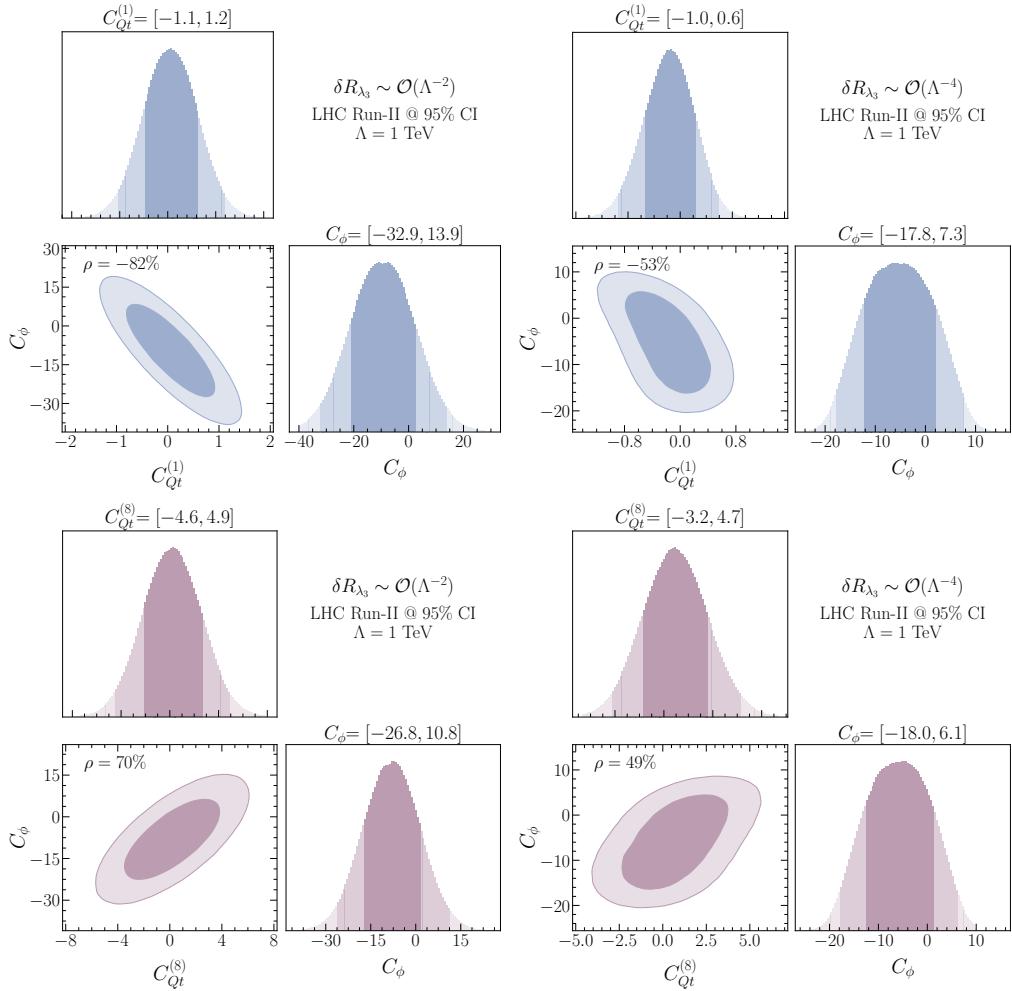
$$\mu(C_\phi, C_i) = \frac{\sigma_{\text{Prod}}(C_\phi, C_i) \times \text{BR}(C_\phi, C_i)}{\sigma_{\text{Prod,SM}} \times \text{BR}_{\text{SM}}} \approx 1 + \delta\sigma(C_\phi, C_i) + \delta\Gamma(C_\phi, C_i) - \delta\Gamma_h(C_\phi, C_i). \quad (7.32)$$

**Uncertainties and correlations**  $\mathbf{V}$  The variance matrix  $\mathbf{V}$  is build from thee experimental uncertainties found in Table 3.1. For Run-II data, only ATLAS collaboration reported the correlation amongst different channels, and only correlations  $> 10\%$  are considered. While for the HL-LHC, the whole correlation matrix found on the webpage [303]. The HL-LHC projections for the S2 scenario explained in [220] were used. These assume the improvement on the systematics that is expected to be attained by the end of the HL-LHC physics programme, and that theory uncertainties are improved by a factor of two with respect to current values. Theoretical uncertainties were not considered in this fit.

The python package `pymc3` [304] was used to construct the posterior distribution. I have used the `Arviz` Bayesian analysis package [305] to extract the credible intervals (CIs) from the highest density posterior intervals (HDPI) of the posterior distributions, where the intervals covering 95% (68%) of the posterior distribution are considered the 95% (68%) CIs. In the Gaussian limit, these 95% (68%) CIs should be interpreted as equivalent to the 95% (68%) Frequentist Confidence Level (CL) two-sided bounds. `HEPfit` [306] code was used to validate the fits. Given that current bounds on these operators are rather weak, one may wonder about the uncertainty in our fits associated to the truncation of the EFT. Note that, since the four-quark operators only enter into the virtual corrections at NLO, Higgs production and decay contain only linear terms in  $1/\Lambda^2$  in the corresponding Wilson coefficients, i.e. the quadratic terms coming from squaring the amplitudes are technically NNLO. Hence, the quadratic effects in the signal strengths come from not linearising the corrections to the product  $\sigma_{\text{Prod}} \times \text{BR}$ . These effects have been investigated, and found to have a negligible effect on the fit. The operators of single chirality  $\mathcal{O}_{tt}^{(1)}$  and  $\mathcal{O}_{QQ}^{(1)/(3)}$  were not included in the fit, as their effect on Higgs rates is limited to small  $\delta R$  for  $t\bar{t}h$ . Thus, they cannot be contained simultaneously with  $C_\phi$  using single Higgs data.

### 7.2.1 Fit results

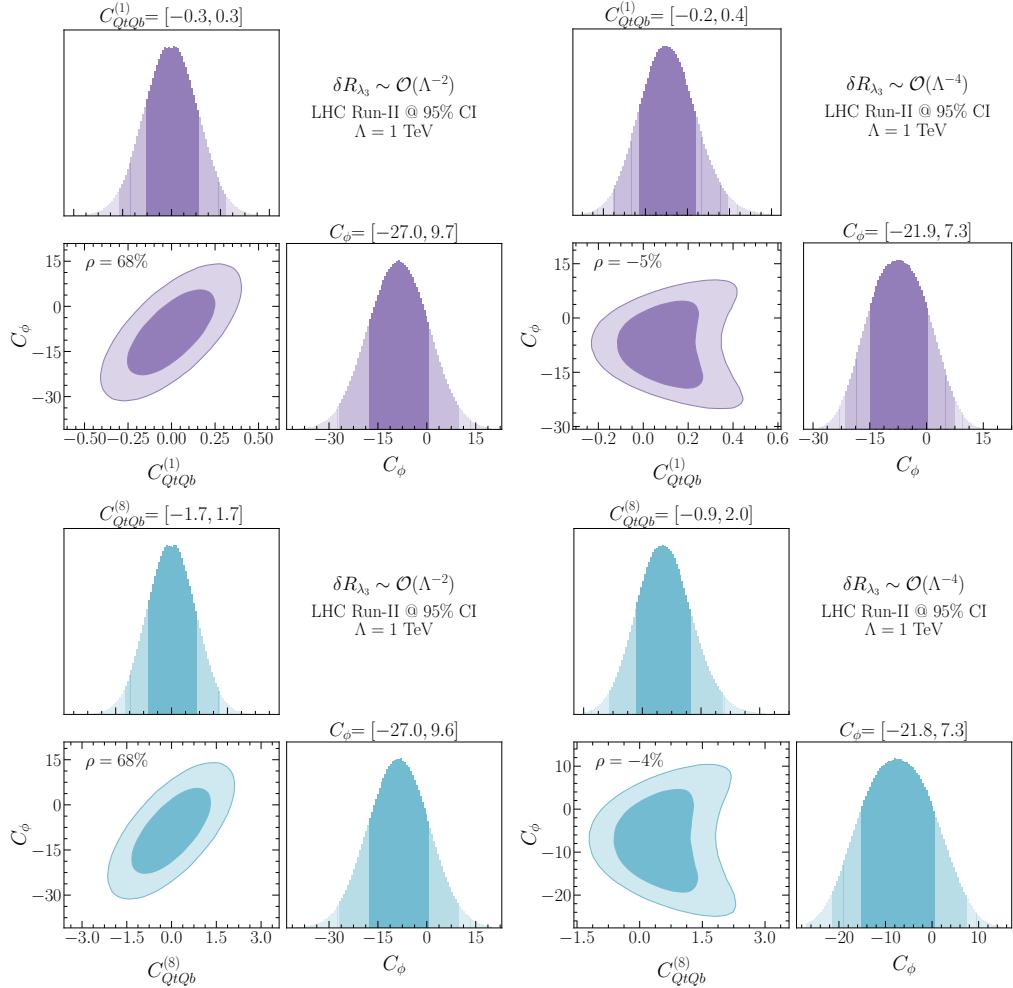
In Figure 7.5 and Figure 7.6 I show the 68% and 95% CIs of the two-parameter posterior distributions and their marginalisation for the two-parameter fits involving  $C_\phi$  and one of the four-heavy quark Wilson coefficients, evaluated at the scale  $\Lambda = 1$  TeV for Run-II LHC measurements . Both linearised and quadratically truncated  $\delta R_{\lambda_3}$  fits are shown, and one can observe that the 95% CI bounds (shown on top of the panels) and correlations depends on the truncation.



**Figure 7.5.** The posterior distributions of the Run-II data fits for  $C_\phi$  with  $C_{Qt}^{(1)}$  (up) and  $C_\phi$  with  $C_{Qt}^{(8)}$  (down). The 68% and 95% highest density posterior contours indicated. The limits shown on top of the plots indicate the 95% CIs. Plots on the left are made for the fully linearised  $\delta R_{\lambda_3}$ , while the ones on the right include the quadratic effects. This figure has been published in [288].

We observe that the four-fermion operators are strongly correlated with Higgs self-coupling modifier  $\mathcal{O}_\phi$ , in the linear fit, with Pearson's correlation of  $\gtrsim 0.7$  and  $p$ -value  $< 10^{-4}$ . In the case of quadratic  $\delta R_{\lambda_3}$  fit, we observe diminished Pearson correlation, but in this scenario Pearson's correlation test is not particularly applicable, as we have non-linear relation between the variables.

The two-parameter fit results for the four-fermion Wilson coefficients are summarised in the forest plots in Figure 7.7 marginalising the posteriors distributions over  $C_\phi$ . The finite effects were isolated by preforming fits with  $\delta R^{fin}$  only. The finite effects are small for  $O_{QtQb}^{(1)/(8)}$  but dominant for the four-top operators  $O_{Qt}^{(1)/(8)}$  mainly coming from  $t\bar{t}h$ .

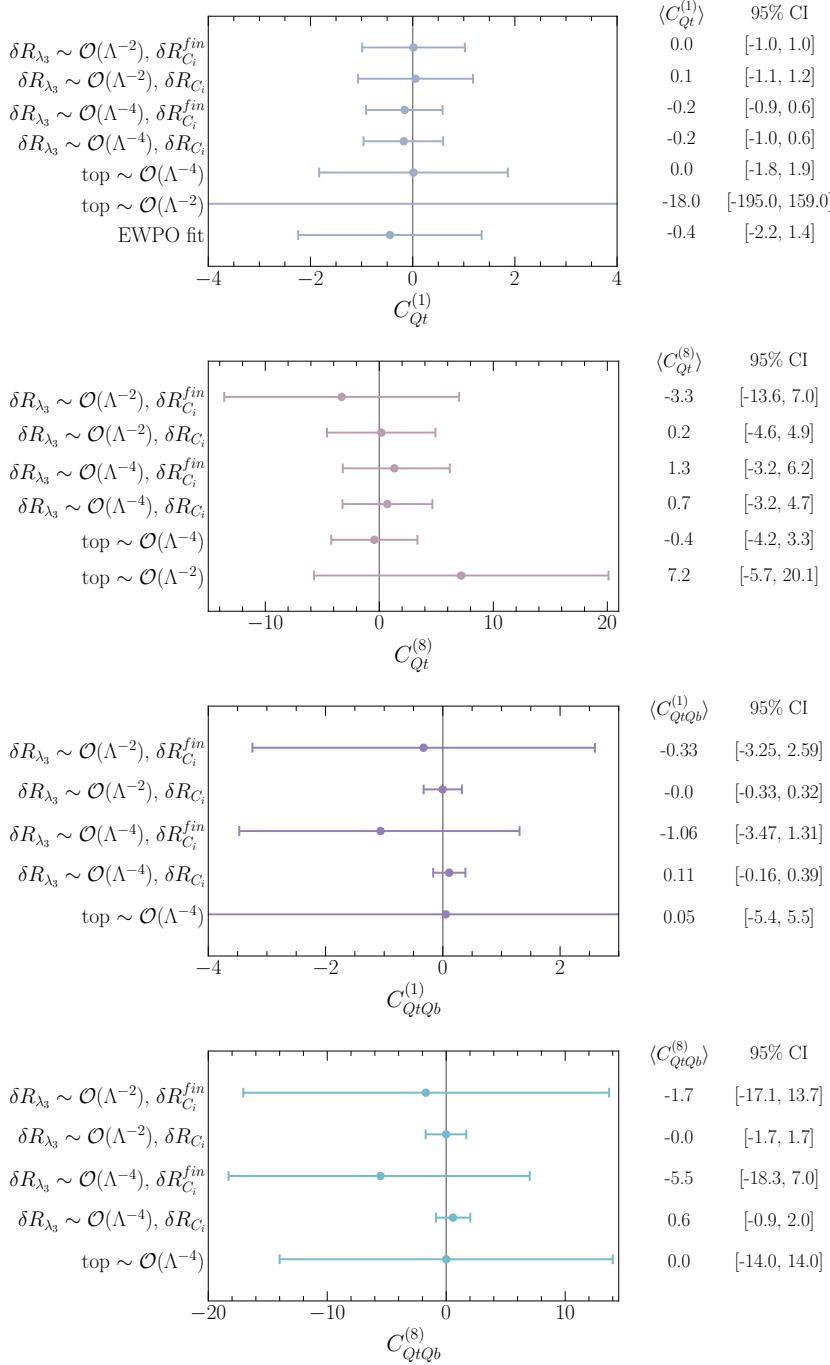


**Figure 7.6.** The posterior distributions of the Run-II data fits for  $C_\phi$  with  $C_{QtQb}^{(1)}$  (up) and  $C_\phi$  with  $C_{QtQb}^{(8)}$  (down). With the same annotations as in Figure 7.5. This figure has been published in [288].

The effect of EFT truncations of  $\delta R_{\lambda_3}$  can also be observed as shifts in the mean value for the Wilson coefficients, but the 95% CIs themselves are not significantly affected. In these plots, the fits results from this study are also confronted with the limits obtained from fits to top data [17, 173, 286, 287, 307, 308] and EWPO fits from [174]. Showing that when the Wilson coefficient running is taken into an account, the 95% CI bounds obtained from Higgs data are consistently stronger than the ones from top data.

We see in Figure 7.8 the fit results for  $C_\phi$  after marginalising over the four-fermion Wilson coefficients in both EFT truncations schemes of  $\delta R_{\lambda_3}$ , as well as a single parameter fit for  $C_\phi$ . Additionally the current 95 % CL bound on  $C_\phi$  extracted from Higgs pair

## 7.2 Fit to Higgs observables



**Figure 7.7.** Forest plots illustrating the means and 95% CIs constraints on the four-heavy-quark Wilson coefficients  $C_i$  from Run-II data. These bounds are obtained from two-parameter fits including the aforementioned coefficients along with  $C_\phi$ , then marginalising over the latter. The different fits with only the finite part of the NLO correction included vs the full results, as well as the EFT truncation scheme for the trilinear coupling, linear vs quadratic. Fits from top data [173] for  $C_{Qt}^{(1),(8)}$  and [287] for  $C_{QtQb}^{(1),(8)}$  as well as EWPO fits from [174] were included for comparison. This figure has been published in [288].

production search using the final state  $b\bar{b}\gamma\gamma$  performed by ATLAS using Run-II data [309], translated from  $\kappa$  formalism.

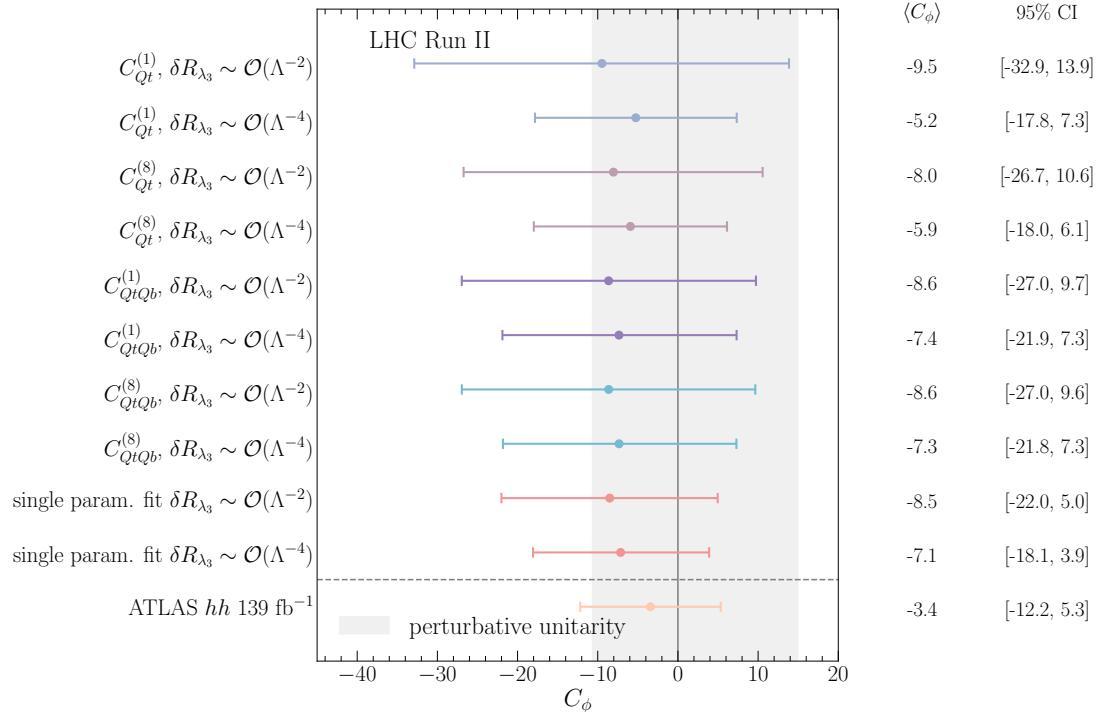
The mean values and the 95% CIs change depending on the four-fermion Wilson coefficient that was paired with  $C_\phi$  in the two-parameter fit. As expected, the single parameter fits for  $C_\phi$  yield stronger bound on  $C_\phi$  than the two-parameter fits, thus the inclusion of the four-fermion operators in single Higgs data dilutes  $C_\phi$  bounds. Additionally, the truncation order of  $\delta R_{\lambda_3}$  appears to have a significant effect on the length of the CIs, with quadratic fits giving more stringent constraints on  $C_\phi$ . Instead, for Higgs pair production it makes only a negligible effect if linear or up to quadratic terms in the EFT expansion are kept for the  $C_\phi > 0$  bound, while the bound weakens at linear order in  $1/\Lambda^2$  for  $C_\phi < 0$  [310]. For instance, the quadratic single parameter fit for  $C_\phi$  is comparable to the direct bound from Higgs pair production. However, this changes dramatically, when one includes the four-fermion operators in a combined fit, and the single-Higgs data constraints on  $C_\phi$  become less significant compared to the direct  $hh$  bounds.

It should be noted that the strongest bound on the Higgs self-coupling currently comes from the perturbative unitarity bound of ref. [90].

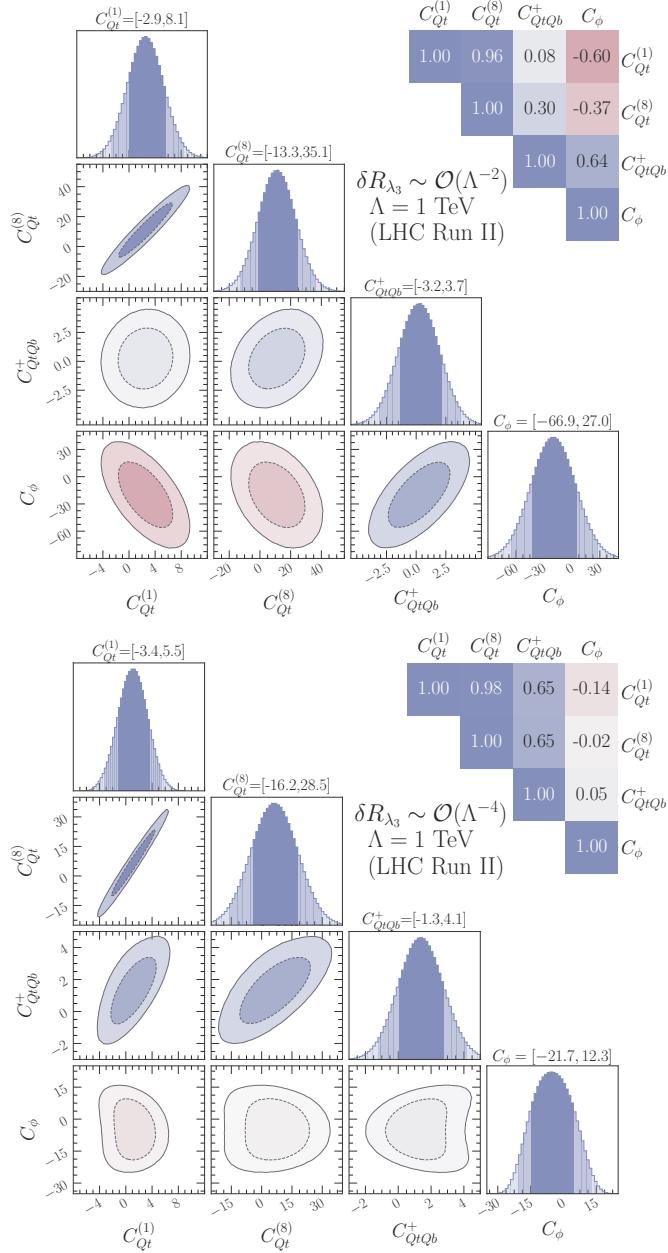
One of the important aspects of multivariate studies is the correlation among variables. Apart from the two-parameter fits discussed above, here we also consider a four-parameter fit to  $C_\phi$  plus the three directions in the four heavy-quark operator parameter space that the Higgs rates are mostly sensitive too, i.e. neglecting  $C_{QQ}^{(1),(3)}$  and  $C_{tt}$ , and trading  $C_{QtQb}^{(1)}$  and  $C_{QtQb}^{(8)}$  by  $C_{QtQb}^+$ . When considering two- or four-parameter fits of  $C_\phi$  and the four-heavy-quark Wilson coefficients, we observe a non-trivial correlation patterns emerging amongst these coefficients. Figure 7.9 illustrates these correlation patterns for the four-parameter fit. We observe that the Wilson coefficients  $C_{Qt}^{(1),(8)}$  are strongly correlated because, in analogy to  $C_{QtQb}^{(1),(8)}$ , they only appear in certain linear combination whenever correcting the Yukawa coupling. However, unlike  $C_{QtQb}^{(1),(8)}$  they are not completely degenerate because the main part of the NLO correction to  $t\bar{t}h$  does not contain the aforementioned linear combination. The four-parameter fit also reveals that the Wilson coefficients  $C_{Qt}^{(1),(8)}$  have a large correlation with  $C_{QtQb}^+$  because all of the four Wilson coefficients appear in a linear combination in the NLO corrections except for  $h \rightarrow b\bar{b}$  and  $t\bar{t}h$ . However, this correlation is not as strong due to the large NLO correction of the Higgs decay  $h \rightarrow b\bar{b}$  from  $C_{QtQb}^{(1),(8)}$ . Moreover, the correlation between the four-heavy-quark Wilson coefficients and  $C_\phi$  depends on the  $\delta R_{\lambda_3}$  truncation.

## 7.2.2 Prospects for HL-LHC

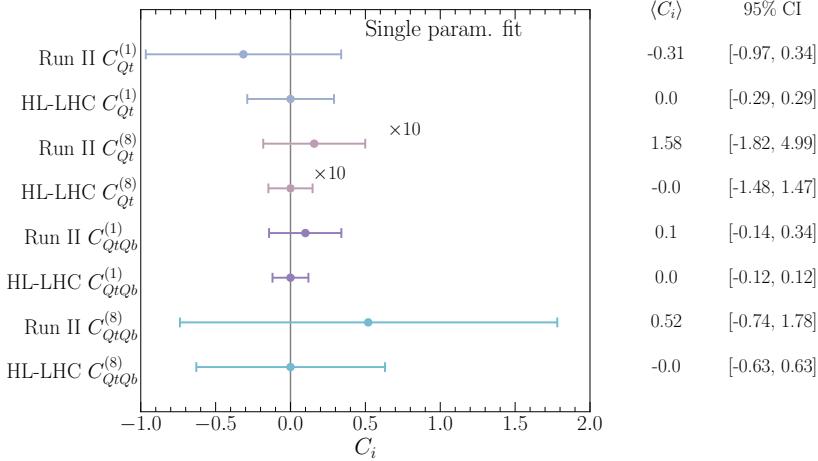
Using the CMS Higgs signal strength projections for the HL-LHC in refs. [128, 303] for a centre-of-mass energy of  $\sqrt{s} = 14$  TeV and integrated luminosity of  $3 \text{ ab}^{-1}$ , it is possible to repeat the fits done for Run-II. The projections for the S2 scenario explained in [220] were used. In Figure 7.10, I show the comparison between the fit results of Run-II data and the projections for the HL-LHC for single parameter fits. For the operators



**Figure 7.8.** A forest plot illustrating the means and 95% CIs bounds for  $C_\phi$  from the two-parameter fit, with the four-fermion operators marginalised. The fits results for  $C_\phi$  from full run-II Higgs data keeping terms up to  $\mathcal{O}(1/\Lambda^2)$  or  $\mathcal{O}(1/\Lambda^4)$  in  $\delta R_{\lambda_3}$  are shown. For comparison, also the 95% CI and means for the single parameter fit for  $C_\phi$  with the same single Higgs data is shown as well as the bounds on  $C_\phi$  from the  $139 \text{ fb}^{-1}$  search for Higgs pair production [309]. The horizontal grey band illustrates the perturbative unitarity bound [90]. This figure has been published in [288].



**Figure 7.9.** The marginalised 68% and 95% Highest density posterior contours for the four-parameter fits including the different four-quark Wilson coefficients and  $C_\phi$ . The numbers above the plots show the 95% CI bounds while the correlations are given on the top-right side. The correlation between each pair of the Wilson coefficients is highlighted as a heatmap. The upper panel shows the fit including up to  $\mathcal{O}(1/\Lambda^2)$  in  $\delta R_{\lambda_3}$  while the lower one shows the fit with including also  $\mathcal{O}(1/\Lambda^4)$ . This figure has been published in [288].

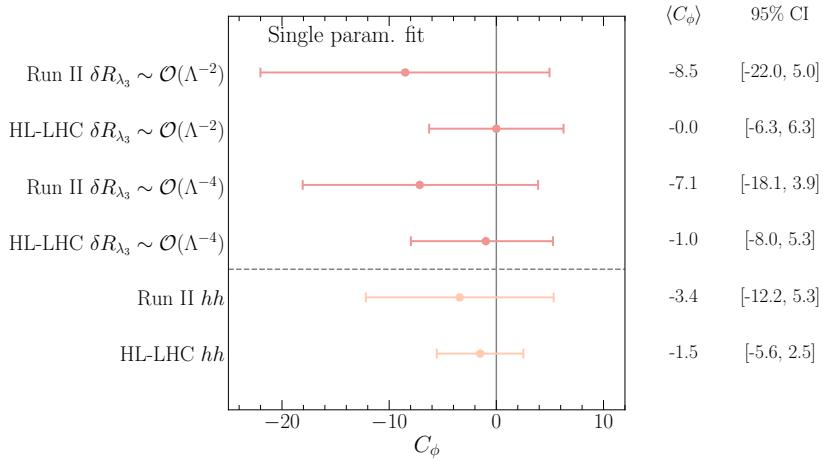


**Figure 7.10.** Results of single parameter fit showing the improvement in the constraining power of the HL-LHC over the current bounds from Run-2 data. This figure has been published in [288].

$\mathcal{O}_{Qt}^{(1),(8)}$  the constraining power of the HL-LHC is roughly a factor two better as the current bounds on could set from single Higgs data, while for the operators  $\mathcal{O}_{QtQb}^{(1),(8)}$  the improvement is a little less prominent. While in Figure 7.11 the limits on  $C_\phi$  in a single parameter fit for Run-2 and the projections for the HL-LHC are shown, including in  $\delta R_{\lambda_3}$  up to order  $\mathcal{O}(1/\Lambda^2)$  or  $\mathcal{O}(1/\Lambda^4)$ . While for Run-II data the inclusion of  $\mathcal{O}(1/\Lambda^4)$  made a significant difference, this is less pronounced for the HL-LHC projections. These results are similar to the projections presented in a  $\kappa_\lambda$  fit in [16]. The results were also confronted with data from searches for Higgs pair production  $139 \text{ fb}^{-1}$  [309] and HL-LHC projections [311] on Higgs pair production, showing that Higgs pair production would still allow to set stronger limits on  $C_\phi$ .

### 7.3 Conclusion

In this chapter, the NLO corrections emanating from the SMEFT four-heavy-quark operators to single-Higgs rates have been calculated. We have seen that both four-fermion operators' classes involving homogenous and heterogeneous chirality structures contribute to Higgs rates at NLO. Though, the operators with heterogeneous chirality structure have more sizeable effects as they would contribute to  $hf\bar{f}$  vertex renormalisation in SMEFT and therefore appear in more channels compared to the operators baring homogenous chirality structure. The results of these calculations were utilised in fits on the Wilson coefficients associated with these operators using single-Higgs data. The operators with the same chirality structure will not be constrained strongly by these fits, and hence their results were not included. This applies to the operators that contribute only via beauty quarks loops, like  $\mathcal{O}_{Qb}^{(1),(8)}$ .



**Figure 7.11.** A forest plot illustrating the means and 95% CI's of the posteriors built from the  $C_\phi$  in a single-parameter fit, showing also the differences in including terms of  $\mathcal{O}(1/\Lambda^2)$  or up to  $\mathcal{O}(1/\Lambda^4)$  in the definition of  $\delta R_{\lambda_3}$ . For comparison, also the limits and projections from searches for Higgs pair production are shown. This figure has been published in [288].

Two processes stood out in this calculation in terms of their sensitivity to these operators. The decay of the Higgs to beauty quarks, which had strong sensitivity to  $\mathcal{O}_{QtQb}^{(1),(8)}$ , Moreover, the associated production of the Higgs with top pair  $t\bar{t}h$ , which had large finite corrections coming from  $\mathcal{O}_{Qt}^{(1),(8)}$ . These corrections were depending on the colour factor and thus broke the degeneracy between the singlet and octet operators.

Bayesian analysis combining the four-fermion operators with the SMEFT operator modifying the Higgs self-coupling  $C_\phi$  has been performed. Motivated by the fact that both operators are weakly constrained, and only appears at NLO in single Higgs rates. The fits results showed that the constraints on  $C_\phi$  from single Higgs data will become significantly diluted compared to the fits performed with this operator alone, or even with ones that enter at LO [149–152, 154]. This is due to the strong correlation patterns amongst  $C_\phi$  and the four-fermion operators considered in this study. On the other hand, the fits yielded overall stronger bounds on the four-heavy-quark operators than the ones obtained from top data [173, 287]. Comparable bounds can be also seen when EWPO data is considered [174], which these operators also enter at NLO in these observables. Additionally, the authors of ref. [312] have shown that these operators could also be constrained from flavour observables involving  $\Delta F = 2$ , in particular  $B_s - \overline{B}_s$  mixing. Although these bounds depend on the flavour ansatz of the New Physics, and not completely model independent.

The results of these calculations and consequent fits further emphasize the interconnectivity of SMEFT operators and experimental observables, which was discussed in chapter 4. Then remains the question: *How this interconnectivity would manifest in a NP model?* Particularly, one might wonder if the strong correlation between these four-fermion

operators and  $\mathcal{O}_\phi$  could appear in a UV complete model. In fact, large effective couplings involving four top quarks are expected in many NP models, for example partial compositeness [313]. These models would also generate sizeable modifications to the Higgs self-interaction. Similar effects could be obtained from models containing new scalars such as an additional Higgs doublet  $\varphi \sim (1, 2)_\frac{1}{2}$ , or other scalars with non-singlet representation under  $SU(3)_c$  like  $(6, 1)_\frac{1}{3}$  and  $(8, 2)_\frac{2}{3}$ . For further details on these models and their matching see [314], for the NLO matching to SMEFT see [315].



## **Part III**

# **Higgs Pair Production**



# 8 Overview of Higgs pair production at colliders

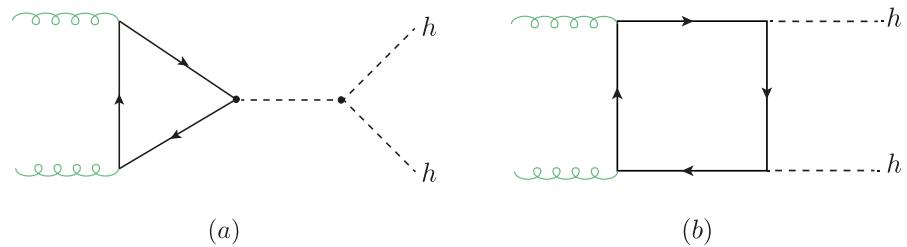
The determination of the shape of the Higgs potential is an essential part of the LHC physics programme. Unlike the determination of most properties of the Higgs and its couplings to heavy particles, the light Yukawa and Higgs-self couplings are exceptionally hard to probe. This is particularly evident from the conclusion of [chapter 7](#). When we have seen that the effectiveness of the utilisation of single Higgs signals in order to probe the Higgs trilinear coupling is challenged with the fact that other weakly constrained operators also affect these signals. Thus, Higgs pair production remains as the only direct way to access this elusive interaction.

The production of Higgs in pairs has roughly  $10^{-4}$  the signal of producing a single Higgs at the LHC. The Higgs pair production with Higgs pair decays considered have a cross-section of  $\sim 1\text{fb}$ , in the SM. This would make it inaccessible from Run-II or Run-III data, but should be accessed using the whole luminosity of the HL-LHC [101, 220, 316]. As for the quartic coupling, which would require NLO corrections to Higgs pair, which are currently unknown, or triple Higgs production, both of which are beyond the sensitivity of the LHC [317]. The measurement potentials for the light Yukawa couplings shall be discussed in the Next chapter. The main advantages for Higgs pair production in determining the Higgs trilinear self-coupling comes from the dependence of the cross-section of  $\lambda_3$  at the LO level, as well as the fact that the rest of SMEFT operators entering in this process (see eq (4.11)) can be strongly constraint from other processes, breaking any potential correlations that might appear between them and the trilinear coupling using only di-Higgs data. However, the inclusion of light quark Yukawa couplings modifiers e.g.  $C_{u\phi}$  and  $C_{d\phi}$  would complicate things as we shall see in ??.

This chapter will start by reviewing the theoretical status of the dominant process for Higgs pair production, the gluon fusion, in [section 8.1](#). Then, the other subdominant channels will be briefly reviewed in [section 8.2](#). I will afterwards overview the experimental efforts in probing this rare yet fascinating processes in [section 8.3](#). Finally, I will present in [section 8.4](#) a summary of the trilinear Higgs-self coupling constraints.

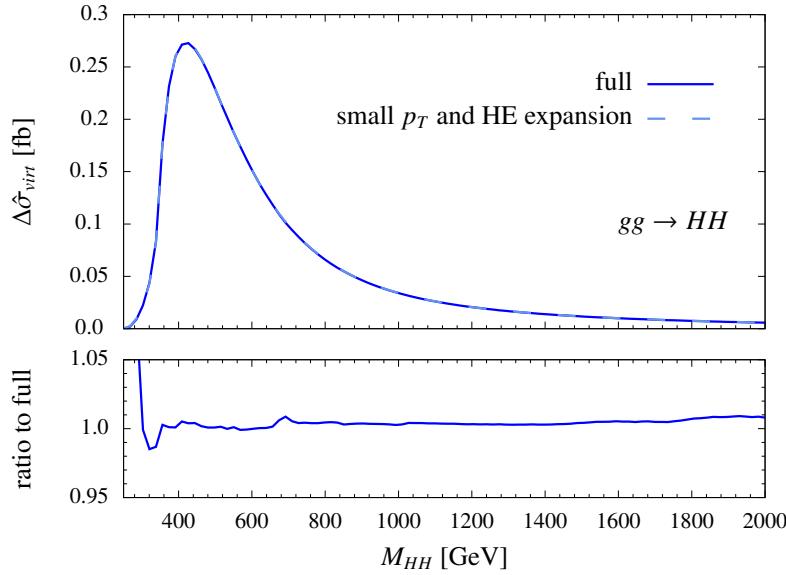
## 8.1 Higgs pair production by gluon fusion

The dominant process for Higgs pair production at the LHC (and hadron colliders in general) is the gluon gluon fusion (ggF) via a heavy quark loop  $Q$ , mainly the top and beauty quark, with the latter contributing only to about 1%, as shown in [Figure 8.1](#). This process is well-studied at leading order (LO) analytically [318–321]. The higher or-



**Figure 8.1.** Feynman diagrams for the ggF process of Higgs pair production in the SM.

der computations are significantly more complicated to perform compared to the gluon fusion production of a single Higgs. This is due to the fact that multi-scale amplitudes at two-loops (and more) cannot be always computed analytically using the current computational techniques. The first attempt to compute the NLO corrections to di-Higgs were via the infinite top limit (HTL/LME) approximation [146, 225, 322] and implemented in `HPAIR` [321]. These corrections were found to be large, with a K-factor of  $\sim 2$ . This prompted more calculations with inclusion of top mass effects [206, 323–326], which improved the stability of the LME expansion as well as corrected the cross-section by  $\sim 10\%$ . In addition, the threshold resummation effects of the LME has been included in [327]. This approach, however, is not sufficient to produce corrections to the differential cross-section, as the LME fails for  $m_{hh}^2/4m_t^2 \lesssim 1$ . Using numerical evaluation of the two-loop integrals, it is possible to obtain exact results with full top mass dependence, see refs. [328–330]. But this comes at the cost of computational power required to evaluate the cross-section. Hence, approximation methods were imperative in obtaining more flexible results for use at simulations and BSM Higgs pair production predictions. These approximations methods are analogous, and sometimes connected to the ones used for  $Zh$  production discussed in chapter 6. This includes, small final particle transverse momentum [11], and high energy (HE) expansions [? ]. In addition to a method developed in refs. [331, 332] which considers both  $\hat{s}, \hat{t}$  and  $m_t$  as large quantities while keeping the Higgs mass as small one. This method has a wide coverage of the  $m_{hh}$  spectrum. The use of Padé approximation to improve the  $p_T$ -expanded amplitude coverage as well as to obtain a description for the three-loop (NNLO) form factors was demonstrated in [333]. The NNLO cross section with top mass effects has been computed numerically in [334] and also at differential level [335], and analytically only in the LME [336]. Also, NLO+NNL analytic results have been obtained by [337]. Parton shower matching for NLO Higgs pair production has been computed in [338, 339], which was essential for the `POWHEG` implementation for di-Higgs, with NLO corrections computed from a grid has been made available by [188, 339, 340]. Figure 8.2 shows the Higgs pair virtual partonic cross-section defined in eq.(6.37) vs the  $p_T$  and HE expansions bridged using Padé approximants [341]. The matching between the results across low and high energy intervals of  $m_{hh}$  shows the strength of Padé approximants technique. This is the most recent analytic higher order correction result for Higgs pair production.



**Figure 8.2.** Combination of the HE and  $p_T$  expansions of the virtual two-loop NLO corrections using Padé approximants, confronted with the NLO results from a numerical grid. This plot is taken from [341].

Calculation of LO in addition to Higher order corrections to Higgs pair production in EFT, MSSM and composite Higgs models can be found in [179, 185, 342–344]. The NNLO correction were used according to the Higgs cross section working group recommended values [345, 346]:

$$K = \frac{\sigma_{NNLO}}{\sigma_{LO}}, \quad K_{14\text{TeV}} \approx 1.71. \quad (8.1)$$

### 8.1.1 Theoretical uncertainties

There are four main sources of theoretical uncertainties for Higgs pair production:

1. Scale uncertainty: coming form the arbitrariness of scales choice.
2. PDF uncertainties : coming form the uncertainty in the PDF fitting and model.
3.  $\alpha_s$  running uncertainty: originating from the initial value (i.e.  $\alpha_s(M_Z)$ ).
4. Top mass renormalisation scheme, which involves  $m_t$  appearing in the loop propagators and in the top Yukawa.

The computation of the uncertainties is described in [347, 348]. for PDF and  $\alpha_s$  uncertainties. In order to calculate the scale uncertainties, the cross-section was computed

	$\sigma$ [fb]	Scale [fb]	PDF+ $\alpha_s$ [fb]	Total [fb]
SM HEFT (LO)	18.10	—	—	—
SM running mass (LO)	16.96	—	—	—
SM (LO)	21.45	$+4.29$ $-3.43$	$\pm 1.46$	$+4.53$ $-3.73$
SM (NLO) [352]	33.89	$+6.17$ $-4.98$	$+2.37$ $-2.01$	$+6.61$ $-5.37$
SM (NNLO) [334]	36.69	$+0.77$ $-1.83$	$\pm 1.10$	$+1.66$ $-6.43$ (incl. $m_t$ uncertainty [349])

**Table 8.1.** Gluon fusion (ggF) Higgs pair production cross-section at 14 TeV with theoretical uncertainties, the HTL/LME is computed using (SM HEFT), top running mass, LO, NLO and NNLO QCD corrections. The NLO and NNLO results are taken from the references cited in the table. The LO results are computed via a FORTRAN code.

with different  $\mu_R$  and  $\mu_F$  values ranging between:

$$\frac{M_{hh}}{4} \leq \mu_R/\mu_F \leq M_{hh} \quad (8.2)$$

As for the  $m_t$  renormalisation uncertainty, one uses the  $\overline{\text{MS}}$  running of the top mass formula at N<sup>3</sup>LO [349]

$$\overline{m}_t(m_t^{pole}) = m_t^{pole} \left( 1 + \frac{4}{3\pi} \alpha_s(m_t^{pole}) + 10.9 \frac{\alpha_s^2(m_t^{pole})}{\pi^2} + 107.11 \frac{\alpha_s^3(m_t^{pole})}{\pi^3} \right)^{-3} \quad (8.3)$$

The total 14 TeV ggF  $hh$ , cross-section at different orders in computation with its uncertainties are shown in [Table 8.1](#), which indicates that the uncertainties are dominated by the  $m_t$  renormalisation scheme of  $\sim -18\%$  uncertainty in the lower envelope. This is significant part of the uncertainty budget and needs to be resolved by including N<sup>3</sup>LO corrections to ggF  $hh$ , such corrections are available in the HTL [350, 351].

## 8.2 Other processes

Like the single Higgs production at hadron colliders, the production of Higgs pairs has the same subdominant channels VBF, di-Higgsstrahlung  $Vhh$  and associates production of Higgs pair with tops  $t\bar{h}h/tjhh$ . Their cross-sections and uncertainties at 14 TeV are shown in [Table 8.2](#), while in [Figure 8.3](#) their cross-sections as a function of the centre-of-mass energy  $\sqrt{s}$  is shown [16].

### 8.2.1 VBF $hh$

Vector boson fusion  $hh$  production has the second largest cross-section after ggF  $hh$ , which is calculated up to N<sup>3</sup>LO [352–354] inclusively and differentially at NNLO [355]. The dominant diagrams are analigious to the single Higgs VBF, which involve the  $W/Z$  bosons exchanged in the  $t$ –channel. The process has the same topology as the -off shell-

Process	Cross-section 14 TeV (fb)	Theo. accuracy	Theo. uncertainty (%)	Contribution (%)
1. ggF hh	36.690	NNLO QCD	12.3	90.1
2. VBF hh	2.050	N <sup>3</sup> LO QCD	2.1	5.0
3. Zhh	0.415	NNLO QCD	3.6	1.0
4. W <sup>+</sup> hh	0.369	NNLO QCD	2.1	0.9
5. W <sup>-</sup> hh	0.198	NNLO QCD	3.0	0.5
6. tt hh & tjh	0.986	NLO QCD	5.1	2.4

**Table 8.2.** Summery of the Higgs pair production processes at 14 TeV LHC.

single Higgs VBF, with the off-shell Higgs giving two final states ones via the trilinear self-coupling.

### 8.2.2 Di-Higgsstrahlung

The associated production of Higgs pair with  $W$  and  $Z$  bosons has a small cross-section compared to ggF and VBF, this process is known up to NNLO QCD accuracy, which includes the gluon-fusion component in the full computation [356? , 357].

### 8.2.3 Associated Higgs pair production with $t$ -quarks

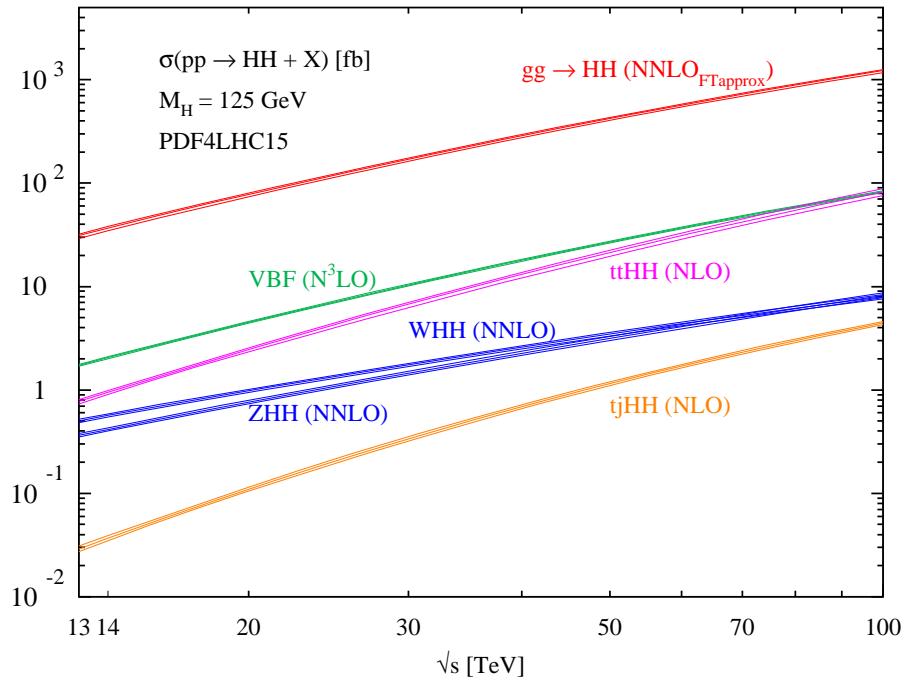
Sometimes called the di-Higgs bremsstrahlung off top quarks [16], this channel has a steeper dependence on  $\sqrt{s}$  than the single Higgs bremsstrahlung  $t\bar{t}h$ . One can see, for example, from Figure 8.3 that its cross-section becomes at roughly the same values as the VBF's. Only NLO computations for this channels have been carried out [358].

All of the three channels have a relatively small NLO correction, compared to gluon fusion. Which ranges from 10-30%.

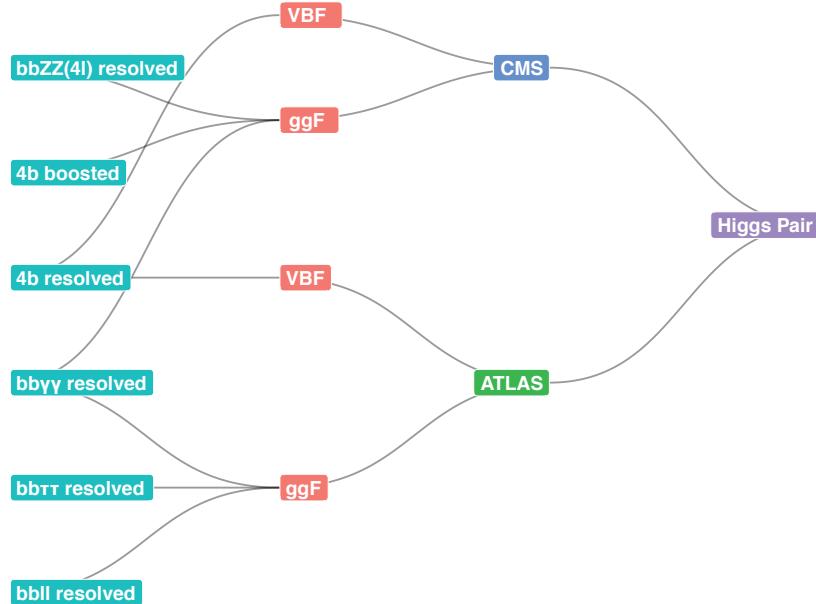
## 8.3 Experimental overview for Higgs pair production

The search for Higgs pair production can be divided into two categories, resonant and non-resonant searches. The first searches for a heavy scalar or spin-2 resonance that decays into a Higgs pair. While the latter is concerned about the SM or if the new particle has a mass beyond the reach of the LHC, i.e. when the EFT limit is valid. In this review, I shall focus on the non-resonant searches, as these are the ones relevant to focus of this thesis, for detailed overview of the resonant searches, and non-resonant ones, see [16].

Figure 8.4 shows the current searches for non-resonant Higgs pair production by both ATLAS and CMS. The searches are summarised according to the final state:



**Figure 8.3.** The cross-section of all di-Higgs processes at the highest available perturbation order as a function of centre-of-mass energy  $\sqrt{s}$ . The bands show the uncertainties without the top-mass renormalisation scheme. This plot is taken from [16].



**Figure 8.4.** The non-resonant Higgs pair searches conducted by ATLAS and CMS using the full Run-II data.

$hh \rightarrow b\bar{b}b\bar{b}$ 

The final state  $hh \rightarrow b\bar{b}b\bar{b}$  has the highest cross-section possible for Higgs pair, but poses a difficulty due to the large QCD background coming from production of 4 b-tagged jets. CMS [359] has used Boosted decision trees (BDT) for studying this final state for ggF and VBF channels, separated. This allowed for sensitivity for the trilinear and  $hhVV$  coupling. This analysis lead to 95% CL bounds on  $\kappa_\lambda \in [-2.3; 9.4]$  and  $\kappa_{2V} \in [-0.1; 2.2]$ . They have also performed boosted analysis for the VBF channel, by defining two large jets with jet radius of  $\Delta R = 0.8$ . This analysis is not sensitive to the trilinear self-coupling, but it is sensitive to both  $\kappa_V$  and  $\kappa_{2V}$ , which leads to the most stringent bound on the latter coupling modifier so far  $\kappa_{2V} \in [0.6; 1.4]$ . The  $\kappa_{2V} = 0$  hypothesis is excluded with  $p < 0.001$  [360]. On the other hand, ATLAS has performed only a resolved analysis for this final state and only for the VBF production channel [361], hence they were able to only report bounds on  $hhVV$  coupling  $\kappa_{2V} \in [-0.43; 2.56]$ .

 $hh \rightarrow b\bar{b}VV$ 

ATLAS has considered the gluon fusion final state  $hh \rightarrow b\bar{b}\ell\ell$ , with the leptons coming from  $WW/ZZ$  decays [362]. This states covers around 90% of the total  $hh \rightarrow b\bar{b}VV$  signal. Their analysis was divided into two categorise, same-flavour and different-flavour leptons. The observed signal strength were higher than the expected one. Hence, no bounds on the self-coupling could be extracted from this search. Similar analysis has been carried out by CMS, but with a requirement to observe four leptons instead of two, hence they searched for the final state  $hh \rightarrow b\bar{b}(ZZ^* \rightarrow 4\ell)$ . The 95% CL upper limit on the signal strength was 30 times the SM one, with bounds on Higgs self-coupling of  $\kappa_\lambda \in [-9; 14]$  [363].

 $hh \rightarrow b\bar{b}\tau\tau$ 

This channel has backgrounds coming from real  $\tau$ 's, such as  $t\bar{t}$  and  $Zj$  with heavy jets. Also, fake  $\tau$ 's coming from QCD multijet process. A neural network has been used by ATLAS [364] for this channel's search, using resolved b jets. The extracted bounds on  $\kappa_\lambda$  are  $[-2.4; 9.2]$ .

 $hh \rightarrow b\bar{b}\gamma\gamma$ 

This final is the most promising for Higgs pair searches and observation. Despite having a lower cross-section than the previous final states with BR of 0.27% in the SM, it has the highest selection efficiency. This is due to the low backgrounds and the ability to fully reconstruct the photons. The dominant non-reducible background is  $b\bar{b}\gamma\gamma$  which has a cross-section of  $\sim 13\text{fb}$  at the 14 TeV LHC, more details about the backgrounds of this final states are stated in [Table 8.3](#).

Both ATLAS and CMS have published searches of this channel using resolved b-jets and BDT and neural networks [309, 365]. With ATLAS reporting the strongest 95% CL

Channel	LO $\sigma$ [fb]	NLO $K$ -fact	$6 \text{ ab}^{-1}$ [#evt @ NLO]
$b\bar{b}h, y_b^2$	0.0648	1.5	583
$b\bar{b}h, y_b y_t$	-0.00829	1.9	-95
$b\bar{b}h, y_t^2$	0.123	2.5	1,840
$Zh$	0.0827	1.3	645
$\sum b\bar{b}h$	0.262	-	2,970
$b\bar{b}\gamma\gamma$	12.9	1.5	116,000
$t\bar{t}h$	1.156	1.2	6,938

**Table 8.3.** SM cross-section for the main background processes at 14 TeV with  $6 \text{ ab}^{-1}$  data at the HL-LHC. For  $b\bar{b}h$  production, the Higgs boson is decayed to a pair of photons. The total production of Higgs associated with  $b\bar{b}$  is denoted by  $\sum b\bar{b}h$  and is the sum of the top four channels.

bound on  $\kappa_\lambda$  yet, which was used in the comparisons in Figure 7.11. While CMS has reported bounds on  $\kappa_\lambda$  and  $\kappa_{2V}$ :  $\kappa_\lambda \in [-3.3; 8.5]$  and  $\kappa_{2V} \in [-1.3; 3.5]$ .

### 8.3.1 Prospects for the HL-LHC

The highlight of the HL-LHC programme is the search for the Higgs pair production. It is projected that the Higgs pair signal to be observed at  $\sim 4 - 4.5\sigma$  level. The use of machine learning techniques in the event analysis of  $hh$  searches will be a key factor in the potential discovery of this process [220]. In ?? the interpretable machine learning technology will be exploited in improving the sensitivity for  $hh$  signals at the HL-LHC. With the main focus on the  $b\bar{b}\gamma\gamma$  final state. As this channel has the highest potential for discovery of di-Higgs production [352, 366–371]. The expected bounds on  $\kappa_\lambda$  at the HL-LHC for combined ATLAS and CMS is  $\kappa_\lambda \in [0.1, 2.3]$  [16, 220]

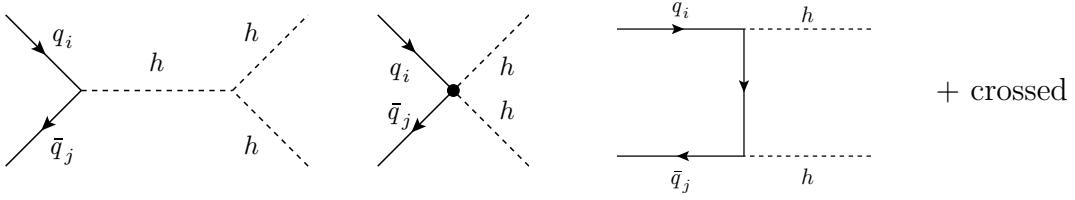
## 8.4 Summary

The Higgs pair production is a missing key measurement of the SM, it is essential for the determination of the Higgs potential by directly constraining the Higgs trilinear self-coupling. Moreover, this channel is sensitive to non-linear couplings with the Higgs, like  $hhVV$  and  $hhff$ . Due to the small cross-section of this channel, current searches obtain rather weak bounds on  $\kappa_\lambda$  that are comparable with the perturbative unitarity bounds [90]. Hence, the need for higher luminosity is imperative. Consequently, the HL-LHC is expected to result in an observation or even discovery of this process. Particularly with the help of advanced machine learning techniques.

The observation of Higgs pair production is expected to provide a direct measurement on one of the two “difficult” couplings of the Higgs, which is the trilinear self-coupling. However, as we shall explore in the upcoming chapters, it could also provide a window for observing the second difficult coupling discussed in the first chapter; the coupling between the Higgs and light quarks.

## 9 Higgs pair as a probe for light Yukawa couplings

The vast hierarchy of quark (and lepton) masses that we have seen in section 2.3 is one of the unsolved mysteries of the SM. One might wonder whether the Higgs is actually responsible for the light quarks masses or there exist other physics that interplays with the Higgs in generating the light quark mass terms. In fact, one of Weinberg’s last papers was exactly addressing this question [372], in this paper he proposed that only the third generation fermions obtain their masses from Yukawa coupling, while the rest acquire theirs via loop-level interactions. Despite his models being only illustrative, his paper is a proof that even the pioneers of the SM theory still reflect upon this mystery. The pragmatic approach to unravelling this puzzle, is to directly measure the Higgs interaction with light fermions. Ideally, this would be via Higgs decay to first and second generation fermions. This is feasible for the muon case [139, 140] and rather challenging for the charm quarks [141–143] but almost impossible with the current technologies for the electron [373], strange and first generation quarks. Although, lepton colliders might have potential for *strange tagging* [374]. The difficulties here is twofold, first, the SM predicts that these couplings to be extremely small effectually making these decay channels vanishing even at few  $\text{ab}^{-1}$  luminosity. Additionally, even if NP would enhance the Higgs coupling to these fermions, the resolution of the LHC, would not be sufficient for reconstructing the Higgs from electron pairs, and it is not possible to distinguish up, down or gluon jets at the LHC form an overwhelming QCD background . This means that the search for these couplings ought to take a non-trivial path. Enhancements of light quark Yukawa couplings would open the tree-level quark anti-quark inhalation Higgs production channel  $q\bar{q}A$ , which is enhanced by the presence of light quarks in the PDF’s. Moreover breaking the degeneracy amongst the strange up and down quarks, by having a *production tagging* coming form the different distributions of the PDF’s amongst quark flavours. For sufficiently large enhancement of the light quark Yukawa couplings, this channel would even become dominant over the loop-induced gluon fusion, as seen in Figure 9.2. Working strictly in the SMEFT paradigm, the  $q\bar{q}A$  channel would contain a  $hhq\bar{q}$  contact interaction illustrated in Figure 9.1, this interaction enhances the Higgs pair production more than the single Higgs  $q\bar{q}A$ , thus making Higgs pair production more sensitive to light quark Yukawa enhancement, as Figure 9.2 indicates. Although the ggF Higgs pair production channel in SMEFT contains diagrams with contact  $hhq\bar{q}$  interaction shown in Figure 9.3, the contribution of this diagram topology is suppressed by the kinematic mass of the quarks appearing inside the loops, hence the ggF channel is not affected by enhanced light quark Yukawa couplings in a significant way.



**Figure 9.1.** Feynman diagrams for the  $q\bar{q}A$  Higgs pair production in the SMEFT paradigm. The middle diagram shows a contact  $hhq\bar{q}$  interaction, that contributes to significant enhancement of this channel compared to its single Higgs counterpart.

This chapter aims to exploit the potential for Higgs pair production as a direct measurement channel for light quark Yukawa. Focusing on the first generation quarks. I will start by introducing the inclusion of light quark couplings to the Higgs in the SMEFT framework in section 9.1. Then the NLO QCD calculation of the  $q\bar{q}A$  channel will be shown in section 9.2. section 9.4 will outline a cut-based analysis of the di-Higgs final state  $b\bar{b}\gamma\gamma$  in order to estimate the sensitivity of this channel for the HL-LHC. Later, in section 9.5 an optimised approach for enhancing the sensitivity based on multi-variant analysis and interpretable machine learning will be showcased. The results of both analysis techniques will be discussed and compared in section 9.6 While in section 9.7 I will overview the other searches for light Yukawa couplings comparing it the Higgs pair production expected sensitivity. This chapter will be concluded in section 9.8. The cut-based analysis has been published in [375], while the interpretable machine-learning one is an undergoing project with R. Gröber, C. Grojean, A. Paul, and Z. Qian, and expected to be published soon [310].

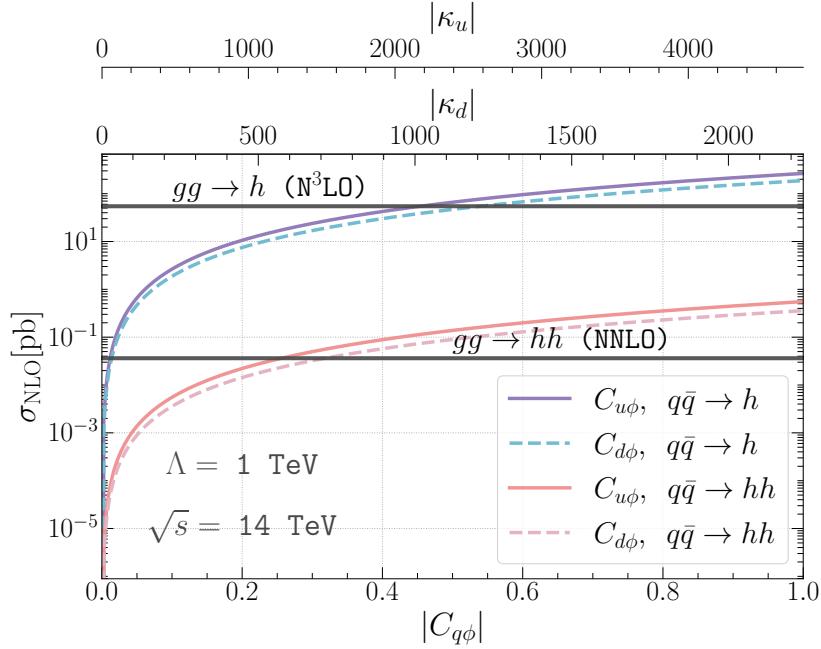
## 9.1 SMEFT and light Yukawa couplings

Including the flavour indices  $ij$  of the SMEFT operators introduced in refs. [13, 165] and chapter 4, we would get light quark -Higgs coupling enhancement from the operators

$$\Delta\mathcal{L}_y = \frac{\phi^\dagger\phi}{\Lambda^2} \left( C_{u\phi}^{ij} \overline{Q}_L^i \tilde{\phi} u_R^j + C_{d\phi}^{ij} \overline{Q}_L^i \phi d_R^j + h.c. \right), \quad (9.1)$$

The mass matrices of the up- and down-type quarks obtained from the Yukawa and the new SMEFT coupling are

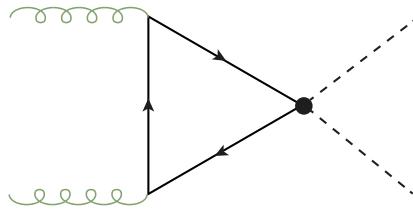
$$\begin{aligned} M_{ij}^u &= \frac{v}{\sqrt{2}} \left( y_{ij}^u - \frac{1}{2} (C_{u\phi})_{ij} \frac{v^2}{\Lambda^2} \right), \\ M_{ij}^d &= \frac{v}{\sqrt{2}} \left( y_{ij}^d - \frac{1}{2} (C_{d\phi})_{ij} \frac{v^2}{\Lambda^2} \right), \end{aligned} \quad (9.2)$$



**Figure 9.2.** The production cross-section of single Higgs and di-Higgs at 14 TeV from the quark anti-quark annihilation  $q\bar{q}A$  as a function of the Wilson coefficients  $C_{u\phi}$  and  $C_{d\phi}$  versus the SM gluon fusion cross-sections (the horizontal solid line for  $gg \rightarrow h$  and the dashed-dotted one for  $gg \rightarrow hh$ ). One can observe that for values of  $C_{u\phi} = 0.22(0.43)$  and  $C_{d\phi} = 0.26(0.47)$  the  $q\bar{q}A$  channel becomes the dominant di-Higgs (single Higgs) production channel. The UV scale is set to  $\Lambda = 1$  TeV.

where  $y_{ij}^q$  are the SM Yukawa matrix elements introduced in eq. (2.22). Since the quark masses are measured quantities, one would naturally rotate to the mass basis using bi-unitary transformation represented by the matrices  $\mathcal{V}_q, \mathcal{U}_q$ , like in the SM. The Wilson coefficients matrix elements in the flavour space in the mass basis can be written as

$$\tilde{C}_{q\phi}^{ij} = (\mathcal{V}_q)_{ni}^* C_{q\phi}^{nm} (\mathcal{U}_q)_{mj}, \quad \text{with} \quad q = u, d. \quad (9.3)$$



**Figure 9.3.** The new diagram for ggF emerging from the  $hhq\bar{q}$  coupling stemming from an effective dim-6 operator.

In order to match these Wilson coefficients to Higgs couplings to quarks, we use the Lagrangian operator describing these couplings

$$\mathcal{L} \supset g_{h\bar{q}_i q_j} \bar{q}_i q_j h + g_{h\bar{q}_i q_j} \bar{q}_i q_j h^2 \quad (9.4)$$

Then the matching results in identifying the SMEFT couplings of Higgs and quarks

$$g_{h\bar{q}_i q_j} := \frac{m_{q_i}}{v} \delta_{ij} - \frac{v^2}{\Lambda^2} \frac{\tilde{C}_{q\phi}^{ij}}{\sqrt{2}}, \quad g_{h\bar{q}_i q_j} := -\frac{3}{2\sqrt{2}} \frac{v}{\Lambda^2} \tilde{C}_{q\phi}^{ij}. \quad (9.5)$$

We observe that, in the general case, we will be having non-diagonal couplings. However, such couplings are strongly constraint by flavour observables, particularly neutral meson mixing [376].

$$|\tilde{C}_{q\phi}^{12}| \lesssim 10^{-5} \Lambda^2/v^2 \quad |\tilde{C}_{d\phi}^{13/23}| \lesssim 10^{-4} \Lambda^2/v^2 \quad (9.6)$$

Due to these strong constraints, it is typical to consider SMEFT with minimal flavour violation (MFV) [377], in which the SM Yukawa matrices  $y_q^{ij}$  are the only spurions breaking the global  $SU(3)_Q \otimes SU(3)_U \otimes SU(3)_D \rightarrow U^6(1)$  flavour symmetry. This implies that the Wilson coefficients matrices in the mass basis are simultaneously diagonalisable with the SM Yukawa matrices. This make the Wilson coefficients maintain the hierarchy of the couplings seen in the SM, thus MFV is not a viable scheme when one wants to consider significant enhancements to the couplings for first and second generations, but keep the third generation couplings unchanged.

In order to bypass the constraints of MFV and yet avoid flavour changing neutral currents (FCNC) that are prohibited by flavour observables, one needs to turn to flavour alignment [378, 379] or its generalisation aligned flavour violation (AFV) [380].

With flavour alignment, the NP flavour parameters (here the Wilson coefficients) are aligned with the SM Yukawa, such that both can be simultaneously diagonalised, hence preventing tree-level FCNCs. But unlike MFV, the constraint on making these new parameters proportional to the SM Yukawas is lifted. This would induce radiative FCNCs, as this formalism is unstable under quantum corrections [381–383]. This alignment breaking would not be seen in the SMEFT, but rather when UV-complete models are considered. AFV resolves this instability, by ensuring that any NP Spurion breaking the flavour symmetry will transform trivially under the quark phases transformations  $U^6(1)$ , keeping the CKM matrix as the only flavour object that has non-trivial transformations. Thereby the CKM will have physical flavour changing currents as well as a  $\mathcal{CP}$ -violating phase. This constraint on the NP flavour spurions  $k_q$ , allows them to be written as a series in powers of the CKM matrix, known as the alignment expansion

$$k_u = K_{0,u} + K_{1,u} V_{CKM}^* K_{2,u} V_{CKM}^T K_{3,u} + \mathcal{O}(V_{CKM}^4) + \dots, \quad (9.7)$$

$$(k_d)^\dagger = K_{0,d} + K_{1,d} V_{CKM}^T K_{2,d} V_{CKM}^* K_{3,d} + \mathcal{O}(V_{CKM}^4) + \dots, \quad (9.8)$$

where  $K_{i,u}$  and  $K_{i,d}$  are complex  $3 \times 3$  diagonal matrices invariant under flavour transformations. This formalism is stable under renormalisation group evolution as any linear

combinations or tensor product of the spurions will remain flavour aligned.

For simplicity, I shall only consider the first term in the alignment expansion, such that only diagonal  $C_{q\phi}$  are investigated, as the other terms are already CKM-suppressed and not of particular phenomenological interest. With this in mind, and using the translation between SMEFT and  $\kappa$ -formalism discussed in subsection 4.2.2, it is possible to identify the couplings in SMEFT with the  $\kappa$ 's

$$g_{h\bar{q}_i q_i} = \kappa_q g_{h\bar{q}_i q_i}^{\text{SM}}, \quad g_{hh\bar{q}_i q_i} = -\frac{3}{2} \frac{1-\kappa_q}{v} g_{h\bar{q}_i q_i}^{\text{SM}}, \quad (9.9)$$

in a slight abuse of language of the  $\kappa$ -framework used often in experimental analyses, as the  $hh\bar{q}\bar{q}$  coupling also depends on the light quarks coupling modifier  $\kappa_q$ .

Higgs pair production offers an extra advantage for probing light Yukawa interactions, as it is particularly sensitive to the  $hh\bar{q}\bar{q}$  interaction, one could also consider the non-linear HEFT, by extending it to include Wilson coefficients  $c_q$  and  $c_{qq}$  for the first and second generation quarks, in analogy to ones defined for the top quark in eq. (4.14) [384]. The analysis preformed on these HEFT parameters is published in [375].

## 9.2 Higgs pair production and Higgs decays with modified light Yukawa couplings

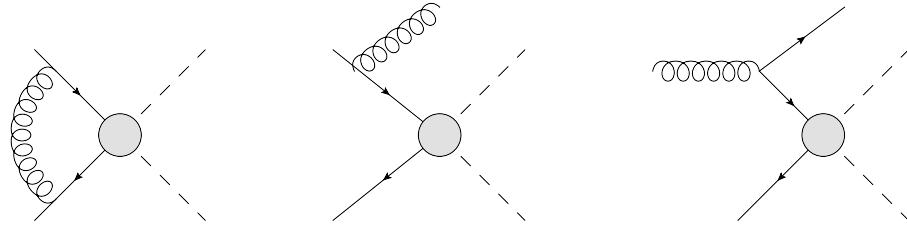
As we have briefly discussed in the introduction, the gluon fusion channel Higgs pair production is affected by enhanced light Yukawa couplings in two ways. First, the inclusion of light quark loops in the triangle and box diagrams. Second, the new diagrams introduced by the contact  $hh\bar{q}\bar{q}$  coupling shown in Figure 9.3. However, these effects are negligible, due to the mass-suppression of these diagrams by the light quark appearing in the loops. Therefore, effectively, one could consider the ggF channel as purely derived by third generation quarks, and only affected by the trilinear coupling  $C_\phi$  as far as this analysis is concerned.

### 9.2.1 Higgs pair production via quark anti-quark annihilation

Contrary to the ggF, the  $q\bar{q}A$  channel does not exist in the SM, except for  $b\bar{b} \rightarrow hh$ , following the assumptions of 4(or 5)-flavour scheme, that the these quarks are massless. This channel contains four-diagrams shown in Figure 9.1, and its differential partonic cross-section is given by

$$\frac{d\hat{\sigma}_{q_i\bar{q}_j}}{d\hat{t}} = \frac{1}{16\pi} \frac{1}{12\hat{s}} \left[ \left| 2g_{hhq_i\bar{q}_j} + \frac{g_{hhh} g_{hq_i\bar{q}_j}}{\hat{s} - m_h^2 - im_h\Gamma_h} \right|^2 + \mathcal{O}(g_{hq_i\bar{q}_j}^4) \right], \quad (9.10)$$

where the  $\mathcal{O}(g_{hq_i\bar{q}_j}^4)$  terms stem from the  $\hat{t}$  and  $\hat{u}$  channel diagrams, and their contribution is typically only  $\sim 0.1\%$  of the total cross-section. The hadronic cross section is then



**Figure 9.4.** Generic form of the QCD corrections of order  $\mathcal{O}(\alpha_s)$  to the  $q\bar{q}A$  Higgs pair production.

obtained by

$$\sigma_{\text{hadronic}} = \int_{\tau_0}^1 d\tau \int_{\hat{t}_-}^{\hat{t}_+} d\hat{t} \sum_{i,j} \frac{d\mathcal{L}^{q_i\bar{q}_j}}{d\tau} \frac{d\hat{\sigma}_{q_i\bar{q}_j}}{d\hat{t}}, \quad (9.11)$$

with  $\tau_0 = 4m_h^2/s$ ,  $\hat{s} = \tau s$  and

$$\hat{t}_\pm = m_h^2 - \frac{\hat{s}(1 \mp \beta)}{2} \quad \text{and} \quad \beta = \sqrt{1 - \frac{4m_h^2}{\hat{s}}}. \quad (9.12)$$

The parton luminosity is given by

$$\frac{d\mathcal{L}^{q_i\bar{q}_j}}{d\tau} = \int_\tau^1 \frac{dx}{x} \left[ f_{q_i}(x/\tau, \mu_F^2) f_{\bar{q}_j}(x, \mu_F^2) + f_{\bar{q}_j}(x/\tau, \mu_F^2) f_{q_i}(x, \mu_F^2) \right]. \quad (9.13)$$

All the kinematic masses were neglected, in accordance with the 5-flavour scheme of the PDF's while the coupling of the Higgs boson to the light quarks (for flavour diagonal couplings) is

$$g_{h q_i \bar{q}_j} = \frac{m_q^{\overline{MS}}(\mu_R)}{v} \kappa_q \delta_{ij}, \quad (9.14)$$

and analogously for the  $g_{h h q_i \bar{q}_j}$  coupling. It is worth noting that there is no inconsistency with such an assumption since in scenarios of modified Yukawa couplings, the masses of the quarks need not to be generated by electroweak symmetry breaking.

### NLO QCD correction

Since the ggF NLO QCD corrections are sizeable, it is reasonable to assume that the same would apply to the  $q\bar{q}A$ . Computing the NLO QCD corrections to this channel is a relatively straight-forward task, as they are only one-loop. More simplifications can be made by neglecting the NLO corrections of the  $\hat{t}$  and  $\hat{u}$  channels because they are strongly suppressed. This enables us to use the NLO QCD corrections results from  $b\bar{b} \rightarrow h$  in the 5-flavour scheme [385–387]<sup>1</sup> by some adjustments taking into account the modified LO cross section and the different kinematics of the process. The Feynman diagrams at NLO QCD are shown in fig. 9.4. For convenience and in order to make our

<sup>1</sup>Note that the NLO and NNLO QCD corrections for  $b\bar{b}hh$  have been given in [388, 389].

adjustments explicit we report here the formulae from [390]

$$\sigma(q\bar{q} \rightarrow h) = \sigma_{LO} + \Delta\sigma_{q\bar{q}} + \Delta\sigma_{qg} \quad (9.15a)$$

$$\Delta\sigma_{q\bar{q}} = \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \sum_q \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau}^1 dz \hat{\sigma}_{LO}(Q^2 = z\tau s) \omega_{q\bar{q}}(z) \quad (9.15b)$$

$$\Delta\sigma_{qg} = \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{qg}}{d\tau} \int_{\tau}^1 dz \hat{\sigma}_{LO}(Q^2 = z\tau s) \omega_{qg}(z) \quad (9.15c)$$

and

$$\hat{\sigma}_{LO}(Q^2) = \int_{\hat{t}_-}^{\hat{t}_+} \frac{d\hat{\sigma}_{q_i\bar{q}_j}}{d\hat{t}} \quad (9.16)$$

with  $z = \tau_0/\tau$ ,  $\sigma_{LO} = \sigma_{\text{hadronic}}$  of eq. (9.11), and the  $\omega$  factors are given by

$$\begin{aligned} \omega_{q\bar{q}}(z) &= -P_{qq}(z) \ln \frac{\mu_F^2}{\tau s} + \frac{4}{3} \left\{ \left( 2\zeta_2 - 1 + \frac{3}{2} \ln \frac{\mu_R^2}{M_{hh}^2} \right) \delta(1-z) \right. \\ &\quad \left. + (1+z^2) \left[ 2\mathcal{D}_1(z) - \frac{\ln z}{1-z} \right] + 1-z \right\}, \end{aligned} \quad (9.17a)$$

$$\omega_{qg}(z) = -\frac{1}{2} P_{qg}(z) \ln \left( \frac{\mu_F^2}{(1-z)^2 \tau s} \right) - \frac{1}{8} (1-z)(3-7z), \quad (9.17b)$$

with  $\zeta_2 = \frac{\pi^2}{6}$ . The Altarelli Parisi splitting functions  $P_{qq}(z)$  and  $P_{qg}(z)$  [391–393] are given by

$$P_{qq}(z) = \frac{4}{3} \left[ 2\mathcal{D}_0(z) - 1 - z + \frac{3}{2} \delta(1-z) \right], \quad (9.18a)$$

$$P_{qg} = \frac{1}{2} \left[ z^2 + (1-z)^2 \right], \quad (9.18b)$$

and the ‘plus’ distribution is

$$\mathcal{D}_n(z) := \left( \frac{\ln(1-z)^n}{1-z} \right)_+. \quad (9.19)$$

The renormalisation scale  $\mu_R = M_{hh}$  and the factorisation scale  $\mu_F = M_{hh}/4$ , were chosen as central values.

The NLO  $q\bar{q}A$  cross-section as well as the LO ggF were implemented in a private FORTRAN code utilising the VEGAS integration algorithm, and NNPDF30 parton distribution functions (PDF’s)[394] implemented via the LHAPDF-6 package [395]. For the one-loop integrals appearing in the form factors of the box and triangle diagrams, we have used the COLLIER library [396] to ensure numerical stability of the loop integral calculation

for massless quarks inside the loops<sup>2</sup>. The resulting NLO  $K$ -factor was found to be

$$K_{NLO} = \frac{\sigma_{NLO}}{\sigma_{LO}} = 1.28 \pm 0.02, \quad (9.20)$$

with the error denoting the theoretical uncertainty. The  $K$ -factor does not depend on the scaling of the couplings, nor the flavour of the initial  $q\bar{q}$  since the LO cross section factors out (with exception of the different integration in the real contributions).

The  $q\bar{q}A$  channel will enhance the overall Higgs pair production cross-section, but if one considers the ggF as a SM background for the Yukawa enhancement “signal”  $q\bar{q}A$  channel, it would be interesting to estimate qualitatively when this signal becomes dominant. This estimates how sensitive is Higgs pair to enhanced light Yukawa couplings as ?? demonstrates. The dominant term for  $q\bar{q}A$  comes from the  $hhq\bar{q}$  vertex diagram, such that the  $q\bar{q}A$  cross-section behaves for large values of  $\kappa$  as (assuming that  $\sigma_{SM}^{qqA} \sim 0$ )

$$(\sigma^{qqA} - \sigma_{SM}^{qqA}) \sim g_{hhq\bar{q}}^2 \sim v^{-4} m_q^2 \kappa_q^2. \quad (9.21)$$

The ggF cross-section instead gets contributions from light quark loops interfering with top quark loops in the triangle SM diagram, leading to a scaling of

$$(\sigma^{ggF} - \sigma_{SM}^{ggF}) \sim \kappa_q \frac{m_q^2}{v^2 M_{hh}^2} \ln^2 \left( \frac{M_{hh}}{m_q} \right). \quad (9.22)$$

Taking the ratio we get

$$\frac{(\sigma^{qqA} - \sigma_{SM}^{qqA})}{(\sigma^{ggF} - \sigma_{SM}^{ggF})} \sim \frac{\kappa_q}{v^2 \left( \frac{\ln^2 \left( \frac{M_{hh}}{m_q} \right)}{M_{hh}^2} \right)}. \quad (9.23)$$

This ratio approaches one (neglecting effects from different PDFs) when

$$\kappa_q^{qqA=ggF} \sim \frac{v^2 \ln^2 \left( \frac{M_{hh}}{m_q} \right)}{M_{hh}^2}. \quad (9.24)$$

Using this order of magnitude estimate, we see that the two cross sections are roughly equal if  $\kappa_c^{qqA=ggF} \sim 1$ ,  $\kappa_s^{qqA=ggF} \sim 10$  and  $\kappa_u^{qqA=ggF} \sim \kappa_d^{qqA=ggF} \sim 10^3$ . The actual values of  $\kappa_q^{qqA=ggF}$  for the first generation quarks can be read from fig. ?? . It is interesting to point out to the pact that these  $\kappa_q$  values are not yet excluded.

### 9.2.2 Higgs decays

The same way  $hh$  production squires additional channels due to enhanced Yukawa couplings, also Higgs decays to light quarks will become significant compared to the SM

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<sup>2</sup>I have expanded code to include other SMEFT operators, and it can be found in the GitHub repository [https://github.com/alasfar-lina/HH\\_XS\\_in\\_SMEFT](https://github.com/alasfar-lina/HH_XS_in_SMEFT)

case with Higgs decays to first generation BR’S being  $< \mathcal{O}(10^{-9})$  [346]. In addition to the contribution of light quarks in the loop-level decays  $h \rightarrow \gamma\gamma/Z\gamma$  and  $h \rightarrow gg$ , though this effect is small. Since the  $h \rightarrow q\bar{q}$  decay are near impossible to detect with the current technologies, the effect of opening these decay channels is reduction in the branching ratios of the Higgs final states that are typically sought after, like  $h \rightarrow b\bar{b}$  and  $h \rightarrow \gamma\gamma$ .

In order to compute the Higgs partial widths and branching ratios (BR) at higher orders in QCD, I have modified the FORTRAN programme **HDECAY** [397, 398] to include the light fermion decay channels and loops in the above-mentioned decays<sup>3</sup>. The overall change of the Higgs total width is given by

$$\Gamma_H \approx \Gamma_{\text{SM}} + \sum_{q=c,s,u,d} \frac{g_{h\bar{q}_iq_i}^2}{(g_{h\bar{q}_iq_i}^{\text{SM}})^2} \Gamma_q, \quad (9.25)$$

where  $\Gamma_q$  can be obtained at NLO QCD from the modified **HDECAY** code. Detailed results for the Branching ratios for the final states of interest have been published in [375].

In order to have a preliminary estimate about the sensitivity of Higgs pair production to light Yukawa enhancements, it is important to consider both production and decay effects in terms of signal strength

$$\mu_i := \frac{\sigma \text{BR}_i}{\sigma^{\text{SM}} \text{BR}_i^{\text{SM}}}. \quad (9.26)$$

Comparing the production of single Higgs vs. Higgs pair signal strengths, for any final state of interest, we could see in Figure 9.5 that for first generation  $C_{q\phi} \lesssim 0.8$  Higgs pair production has a higher signal strength than single-Higgs production despite having double the reduction in the signal strength from the decays of two Higgs bosons as opposed to a single one. In fact, and as we shall see in section 9.7, values of  $C_{q\phi} > 0.4$  have been already excluded by multiple searches.

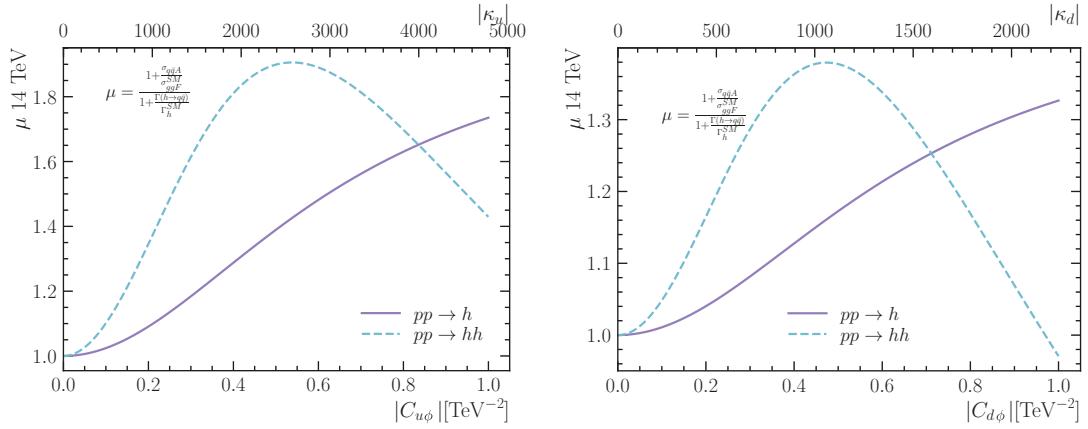
### 9.3 Event generation for the final state $hh \rightarrow b\bar{b}\gamma\gamma$

For this study, the final state  $b\bar{b}\gamma\gamma$  is considered, as this channel has the most potential for Higgs pair searches [220]. It has the “clean”  $h \rightarrow \gamma\gamma$  decay, but also the other Higgs decay to  $b$ -quark pair is a channel with large branching ratio  $\sim 58\%$  and b-tagging capabilities for ATLAS and CMS are continuously improving.

For the cut-based analysis, the FORTRAN codes used to compute the  $hh$  cross-section and decay have been interfaced with **Pythia** 6.4 [399], where the  $q\bar{q}A$  process was generated at NLO and the  $ggF$  at NLO, then multiplied with the NLO  $k$ -factor. The generated events were written to a ROOT file via **RootTuple** tool [400] for further

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<sup>3</sup>The modified **HDECAY** code can be found in the GitHub repository [https://github.com/Alasfar-lina/hdecay\\_lightflavour](https://github.com/Alasfar-lina/hdecay_lightflavour)



**Figure 9.5.** Signal strength at 14 TeV LHC, of the single Higgs (purple solid line) vs. Higgs pair (blue dashed line) as functions of  $C_{u\phi}$  (left) and  $C_{d\phi}$  (right). Both plots show that for  $C_{q\phi} \lesssim 0.8$  the signal strength of Higgs pair production is higher than the single Higgs one. This implies that Higgs pair production is more sensitive to enhancements of light quark Yukawa in SMEFT. This is independent of the final state (except for  $h \rightarrow q\bar{q}$ ).

analysis.

The backgrounds were not simulated for this analysis, rather, the results from [366] were used, because we have used the same cuts as this reference.

For the improved analysis which is based on interpretable BDT, the backgrounds and signal events needed to be generated. The backgrounds described in Table 8.3 were generated using `MadGraph_aMC@NLO` [253], then showered via `Pythia 8.3` [401] and a detector simulation is done using `Delphes 3` [402], the QED/QCD background  $b\bar{b}\gamma\gamma$ ,  $Zh$  and  $b\bar{b}h$  events were taken from the analysis data of Ref. [20], while  $t\bar{t}h$  events were generated specifically for this analysis. In order to obtain the NLO cross-section for these process, the events were multiplied by their respective  $K$ -factors that have been obtained from  $t\bar{t}h$  [403],  $b\bar{b}\gamma\gamma$  [404],  $Zh$  [405] and the remaining part of the  $b\bar{b}h$  processes from [406].

The Higgs pair signals were generated in a slightly different pipeline, the ggF channel events were simulated first using `POWHEG` [188, 339, 340], which has been modified to separate the individual contributions from the box, triangle and their interference individually. This is done in order to easily scale by  $\kappa_\lambda$  (or  $C_\phi$ ), as the box does not depend on it, while the triangle and the interference have quadratic and linear dependence on the trilinear coupling, respectively. The  $q\bar{q}A$  channel events were generated via `MadGraph_aMC@NLO` using a UFO model created with `FeynRules` [292]. Samples for both up- and down-quark initiated  $q\bar{q}A$  processes have been generated. Parton showering and fast detector simulation for both Higgs pair processes were run thorough the same pipeline as the backgrounds, this also goes for the scaling by the NLO of  $q\bar{q}A$  and NNLO for ggF  $K$ -factors after the event generation. The Higgs bosons were decayed with the assumption of narrow width approximation, and the BR values were computed

Channel	LO $\sigma$ [fb]	$K$ -fact.	Order	$6 \text{ ab}^{-1}$ [\#evt @ order]
$hh^{\text{ggF}}_{\text{tri}}$	$7.288 \cdot 10^{-3}$	2.28		96
$hh^{\text{ggF}}_{\text{box}}$	0.054	1.98	NNLO	680
$hh^{\text{ggF}}_{\text{int}}$	-0.036	2.15		-460
$u\bar{u}\text{A}$ ( $C_{d\phi} = 0.1$ )	2.753	1.29	NLO	28
$d\bar{d}\text{A}$ ( $C_{u\phi} = 0.1$ )	4.270	1.30		43

**Table 9.1.** The LO cross-section for Higgs pair production processes (including the decay  $hh \rightarrow b\bar{b}\gamma\gamma$ ) for  $6 \text{ ab}^{-1}$  14 TeV HL-LHC.

in the modified `HDECAY` code.

To be inclusive and to explore the capabilities and importance of the full detector coverage, no generator-level cuts were applied on these processes except for the  $b\bar{b}\gamma\gamma$  processes to avoid divergences. These minimal generator-level cuts for  $b\bar{b}\gamma\gamma$  are

$$\begin{aligned} X p_T^b &> 20 \text{ GeV}, \\ \text{generator level cuts: } \eta_\gamma &< 4.2, \Delta R_{b\gamma} > 0.2, \\ &100 < m_{\gamma\gamma} (\text{GeV}) < 150. \end{aligned} \quad (9.27)$$

Here  $X p_T$  implies a minimum  $p_T$  cut for at least one  $b$ -jet. After the showering and detector simulation, further basic selection cuts were applied to select events with

$$\begin{aligned} \text{basic cuts: } n_{\text{eff}}^{b\text{jet}} &\geq 1, n_{\text{eff}}^{\gamma\text{jet}} \geq 2, \\ p_T^{b\text{jet}} &> 30 \text{ GeV}, p_T^{\gamma\text{jet}} > 5 \text{ GeV}, \\ \eta_{b\text{jet},\gamma\text{jet}} &< 4, 110 \text{ GeV} < m_{\gamma_1\gamma_2} < 140 \text{ GeV}, \end{aligned} \quad (9.28)$$

and  $n_{\text{eff}}^{b/\gamma\text{jet}}$  representing the number of  $b/\gamma$ -jets that pass the basic selection. The cross-section,  $K$ -factors, number of events with  $6 \text{ ab}^{-1}$  luminosity at 14 TeV are given in Table 8.3 for the background and in Table 9.1 for the Higgs pair signals. Both analysis methods included sensitivity analysis for the HL-LHC, i.e. 14 TeV and  $6 \text{ ab}^{-1}$ <sup>4</sup> luminosity and projections for a future hadron circular collider (FCC-hh), with 100 TeV and the luminosity of  $30 \text{ ab}^{-1}$  has been done for the ML based analysis, the results for the FCC can be found in the Appendix C

## 9.4 Cut-based analysis

A cut and count analysis has been performed mainly as a “proof of concept”, in order to demonstrate the sensitivity of Higgs pair production for probing light quark Yukawa couplings. The analysis used the same cuts and  $m_{hh}$  binning as ref. [366] such that their

<sup>4</sup>In the published cut-based analysis [375]  $3 \text{ ab}^{-1}$  luminosity for the HL-LHC was used. However, here I used  $6 \text{ ab}^{-1}$  when reporting fit results

background events counts can be used.

#### 9.4.1 Analysis strategy

In order to derive sensitivity bounds, the number of expected background  $N_b$  and signal  $N_s$  events needs to be estimated from simulated events. Since  $N_b$  is taken from [366], the task is to estimate  $N_s$  for the  $q\bar{q}A$  process as a function of  $C_{q\phi}$ , and to reproduce  $N_s$  of the ggF SM process published in the reference as a cross-check.

Since the cross-section, branching fraction and the integrated luminosity, it is only needed to estimate the selection efficiency  $\epsilon_{SEL}$  from the applied cuts appearing in eq (3.1) to obtain the number of signal events.

The basic cuts of trigger-level selection are jets and photons with minimal  $p_T$  and maximal  $\eta$ .

$$p_T(\gamma/j) > 25 \text{ GeV}, \quad |\eta(\gamma/j)| < 2.5. \quad (9.29)$$

Additionally, a veto on the events with hard leptons is applied

$$p_T(\ell) > 20 \text{ GeV}, \quad |\eta(\ell)| < 2.5, \quad (9.30)$$

Jets were clustered using `fastjet` [407] with the anti-kt algorithm with a radius parameter of  $R = 0.5$ .

The  $b$ -tagging efficiency of  $\epsilon_b = 0.7$ , as well as the photon identification efficiency  $\epsilon_\gamma = 0.8$  have been simulated, in accordance with the ATLAS and CMS performance [408–410, 410, 411]. The selection cuts we used are the same ones as in [366], starting with the cuts of the transverse momentum  $p_T$  of the photons and  $b$ -tagged jets. The two hardest photons/ $b$ -tagged jets, with transverse momentum  $p_{T>}$ , and the softer ones with  $p_{T<}$  are selected to satisfy

$$p_{T>}^>(b/\gamma) > 50 \text{ GeV}, \quad \text{and} \quad p_{T<}^>(b/\gamma) > 30 \text{ GeV}. \quad (9.31)$$

In order to ensure well-separation of the photons and  $b$ -jets, we require the following cuts on the jet radius,

$$\Delta R(b, b) < 2, \quad \Delta R(\gamma, \gamma) < 2, \quad \Delta R(b, \gamma) > 1.5. \quad (9.32)$$

The mass windows used are about three times the photon resolution of ATLAS and CMS [410, 411], such wide windows were used in order to avoid significant signal loss.

$$105 \text{ GeV} < m_{b\bar{b}} < 145 \text{ GeV}, \quad 123 \text{ GeV} < m_{\gamma\gamma} < 130 \text{ GeV}. \quad (9.33)$$

The selection cuts are summarised in table Table 9.2 with their corresponding efficiency. The total selection efficiency for the ggF channel was found to be  $\epsilon_{ggF} = 0.044$ , consistent with the results of [366], while the  $q\bar{q}A$  channel efficiency is slightly higher  $\epsilon_{qq} = 0.05 \pm 0.001$  for the up and down quark initiated  $q\bar{q}A$ , results for second generation quarks can be found in [375].

cut	$\epsilon_{\text{cut}}$	$\delta\epsilon_{\text{cut}}$
Trigger-level in eq. (9.29) and (9.30)	0.71	0.04
$p_T$ cuts in eq. (9.31)	0.35	0.07
$\Delta R$ cuts in eq. (9.32)	0.69	0.21
total	0.11	0.06

**Table 9.2.** The cuts used in the analysis with their efficiency  $\epsilon_{\text{cut}}$  and uncertainties on these efficiencies  $\delta\epsilon_{\text{cut}} = \sqrt{\epsilon(1 - \epsilon) N}$ , where  $N$  is the total number of events. The analysis was performed on 100K SM simulated events. This table is published in [375].

#### 9.4.2 Statistical analysis

The likelihood ratio test statistic  $q_\mu$  was used in order to estimate the HL-LHC sensitivity, and set projected limits on the SMEFT Wilson coefficients  $C_{q\phi}$ , with and without the modifier of the trilinear coupling  $C_\phi$ . Additionally to the HEFT parameters  $c_q$  and  $c_{qq}$ . The likelihood function was constructed from the signal and background events in each bin of the  $m_{hh}$  distribution described in [366]

$$-\ln \mathcal{L}(\mu) = \sum_{i \in \text{bins}} (N_{bi} + \mu N_{si}) - n_i \ln(N_{bi} + \mu N_{si}), \quad (9.34)$$

with  $N_{bi}$  and  $N_{si}$  being the number of background and signal events in the  $i$ th  $m_{hh}$  distribution, respectively. In order to include the theoretical uncertainties on the expected number of signal events, the above likelihood was extended by a Gaussian distribution for  $N_{si}$  in which the mean equals to the central value of the bin values and standard deviation  $\sigma$  equals to its theoretical uncertainty. The signal strength  $\mu$  was then estimated by minimising  $-\ln \mathcal{L}(\mu)$  to obtain the estimator for  $\hat{\mu}$  by injecting SM signal + background events  $n_i$ . The test statistic is then given by

$$q_\mu = 2(\ln \mathcal{L}(\mu) - \ln \mathcal{L}(\hat{\mu})), \quad (9.35)$$

following the procedure described in [412], and using the Python package pyhf [413, 414]. The expected  $6 \text{ ab}^{-1}$  HL-LHC sensitivity for the signal strength at 95% (68 %) CL is found to be  $\mu = 1.5(1.1)$ .

## 9.5 Optimised search for Higgs pair via Interpretable machine learning

When dealing with a multi-variate problem, such as the separation of the Higgs pair signal from its backgrounds, the use of “simple” cuts is not the most efficient method for accomplishing this task. This is mainly due to the fact that in multivariate analysis, the various features used in the classification correlate with each other. This is not captured with the cut and count method. With boosted decision tree (BDT) classifier,

it is possible to capture these correlations and introduce highly non-trivial cuts .

### 9.5.1 Constructing features

The simulated events of the signal and background described in the event selection section are required to contain at least two reconstructed photons and at a  $b$ -tagged jet. From these events, the following high-level features were constructed

- $p_T^{b_1}, p_T^{b_2}, p_T^{\gamma_1}, p_T^{\gamma\gamma},$
- $\eta_{b_{j1}}, \eta_{b_{j2}}, \eta_{\gamma_1}, \eta_{\gamma\gamma},$
- $n_{bjet}, n_{jet}, \Delta R_{\min}^{b\gamma}, \Delta\varphi_{\min}^{bb},$
- $m_{\gamma\gamma}, m_{bb}, m_{b_1 h}, m_{b\bar{b}h}, H_T.$

$p_T^{b/\gamma_{1,2}}$  and  $\eta^{b/\gamma_{1,2}}$  are the  $p_T$  and pseudorapidity for the tagged leading and sub-leading  $b/\gamma$ -jets (in our definition the subleading  $b$ -jet could be a null four-vector since we require one  $b$ -jet inclusive),  $n_{bj}$  is the number of tagged and passed  $b$ -jets.  $\Delta R_{\min}^{b\gamma}$  and  $\Delta\varphi_{\min}^{bb}$  are the minimum  $R$ -distance and  $\varphi$ -angle between a tagged  $b$ -jet and a photon jet. The remaining variables are the invariant masses and  $H_T$  is the scalar sum of the transverse mass of the system.

These features are the same as the ones studied in ref. [20] for  $b\bar{b}h$ . However, they are, by no means, unique. It is possible to run the analysis with another set of features and obtain the same results, as long as these features are independent and highly correlated. Figure 9.6 shows the distributions four most important features from this list, the  $m_{\gamma\gamma}$  is very important in distinguishing the large  $b\bar{b}\gamma\gamma$  background from the signal and  $t\bar{t}h$  ( or other background that contain  $h \rightarrow \gamma\gamma$ ). While the rest, particularly  $H_T$  distinguishes the different  $hh$  channels and also  $hh$  from other Higgs channels backgrounds.

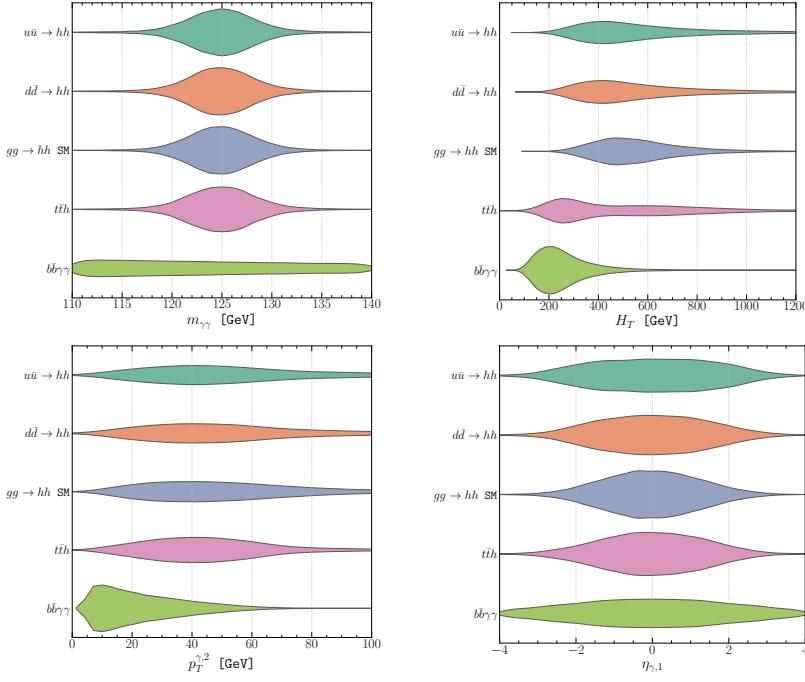
### 9.5.2 Exploratory network analysis

The aim of this analysis is to explore how the kinematic variables constructed in the previous section are related to each other. Moreover, we are interested in examining their variation from channel to channel. This can be achieved by calculating the intra-feature correlations stratified according to the signal channels ( $ggF, u\bar{u}A, d\bar{d}A$ ) and background. Then draw them as network diagrams that can be seen in (a) of Figure 9.7. The Pearson's correlation networks show some differences amongst the different signal strata.<sup>5</sup>. These differences can be further investigated by a post-hoc hypothesis test, based on a linear mixed effects model for each pair of the features  $X_i, X_j$  stratified according to the processes ( $ggF, u\bar{u}A, d\bar{d}A$  and background )  $S_k$ , given as follows

$$X_i = \beta_{ij} X_j + \beta_k S_k + \beta_0, \quad (9.36)$$

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<sup>5</sup>for network plots of the backgrounds see [20]



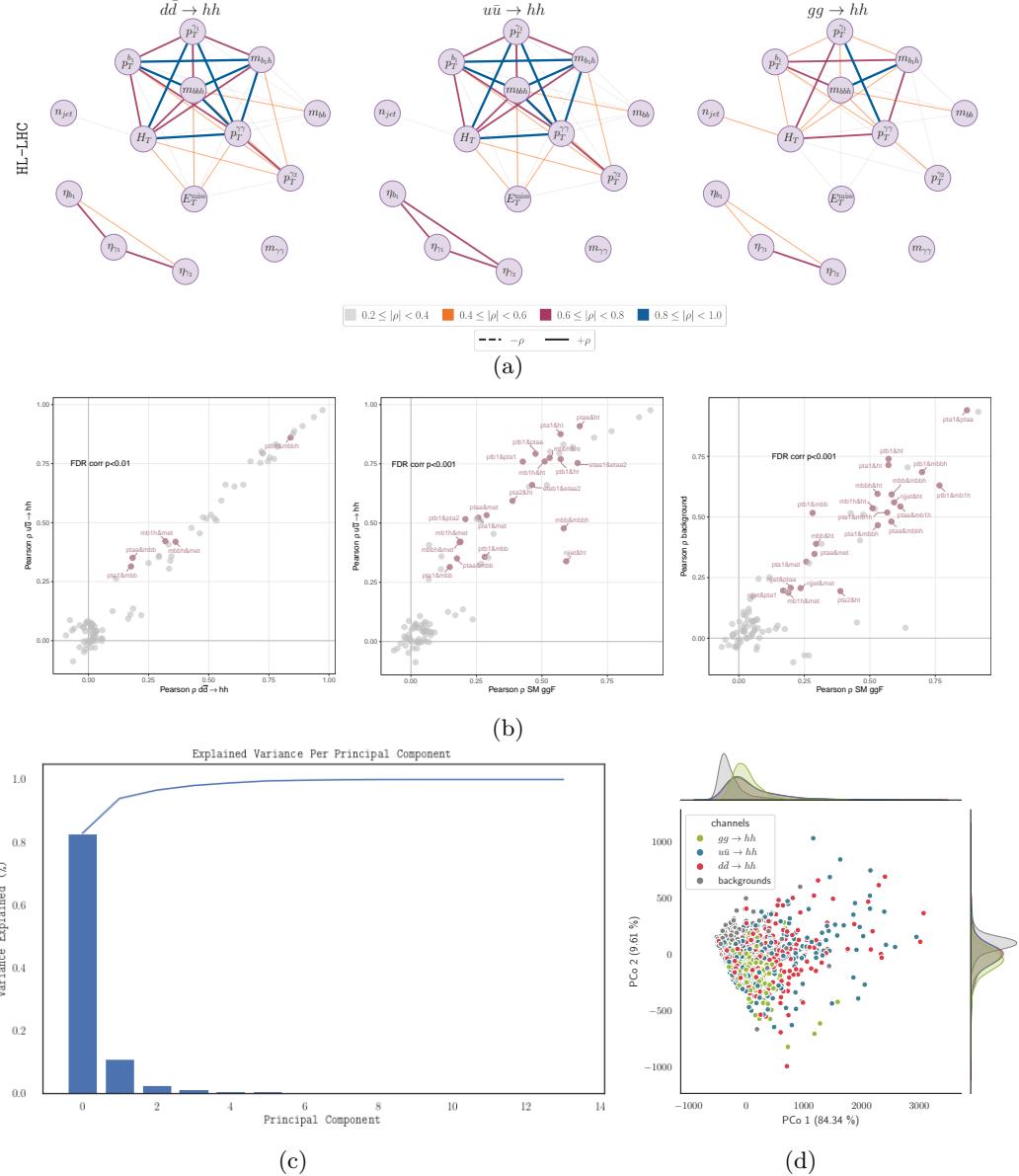
**Figure 9.6.** violin plots of the most significant features used by the BDT classifier for the signal channels, and the two most significant backgrounds  $b\bar{b}\gamma\gamma$ .

where  $\beta_{ij}$ ,  $\beta_k$  and  $\beta_0$  are the constants for the fit . The hypothesis test is therefore preformed by taking the ratio of log likelihood for the linear model of eq. (9.36), defined as

$$t = \frac{\mathcal{L}(\beta_{ij}, \beta_k, \beta_0)}{\mathcal{L}(\beta_{ij}, \beta_k = 0, \beta_0)}. \quad (9.37)$$

This analysis of variation (ANOVA) yields a  $p$ -value for each feature pair, these  $p$ -values are false discovery rate (FDR)-corrected, and the correlation difference amongst the strata is considered significant if the FDR-corrected  $p$ -values pass the threshold  $p < 0.001$  or  $p > 0.01$  when comparing  $u\bar{u}A$  against  $d\bar{d}A$ <sup>6</sup>. The result of these comparisons can be seen in sub-figures (b). We can see that many of the features do not have significant variation across the strata. This indicates that these features are not important in the separation of the signal from the background. The most significant variation is between the ggF (equivalently  $q\bar{q}A$ ) and the background. While for the  $q\bar{q}A$  channels, the correlation patters are almost identical except for the correlation between the observables related to the PDF's, which is expected since the only kinematic difference between the up- and down- initiated  $q\bar{q}A$  emerges from the PDF's of the up and down quarks.

<sup>6</sup>The threshold for this comparison is related due to the high degree of similarity between the two channels.



**Figure 9.7.** (a) Network diagrams of the signal channels of their Pearson correlation  $\rho$  between the features, showing slightly different patterns of correlation amongst these channels. (b) The same Pearson correlations of figure (a) plotted against each other for the different signals, with the colouring indicating whether the difference between the correlation passes the hypothesis testing (ANOVA) passes the threshold FDR-corrected  $p$ -value indicated at each figure. (c) Scree plot of the Principal-component clustering (PCo) of the the signal channels and the backgrounds, almost full variance coverage is obtained by the first four PCo's.(d) The clustering in the first two PCo's, one can see that even with unsupervised clustering the di-Higgs signals have a significantly different distribution than the background. However, it is hard to see an marked clustering for the different signal channels.

This network analysis allows for better understanding of the feature set at hand. When considering that many intra-feature correlations do not vary much across the channels as seen in (b) of [Figure 9.7](#) and the features themselves cluster into four groups according to their correlations, it is tempting to further reduce the dimensionality of the feature space by performing an unsupervised clustering via Principle Component analysis (PCoA). Panel (c) in [Figure 9.7](#) contains a scree plot showing that the variance explained by the first few PCo's is very high, thus reducing the dimensionality of our feature space significantly. When the first two PCo's are plotted (d), the clustering of signal and the background channels can be seen. The distinction between the signals vs. backgrounds is visible, but less marked between the signal channel themselves, in particular  $u\bar{u}A$  against  $d\bar{d}A$ . The first PCo contains, from highest weight to lowest,  $m_{\gamma\gamma}$ ,  $H_T$ ,  $n_{jet}$ ,  $m_{bb}$  and  $p_T^{\gamma\gamma}$ . The rest of the features have a negligible weight.

It is not surprising to see these features contribute the most in the clustering of events given how they are distributed as we have seen in [Figure 9.6](#). In the next step of the analysis we will see them appear once more.

### 9.5.3 Classification analysis

The unsupervised clustering and network analysis merely offers a method to explore how the Higgs pair signal is different from the backgrounds. It is useful to reduce the dimensionality of the feature space and offer “hints” on which subset of features has the highest discriminant power. However, for analysis of the sensitivity and full resolution of the signal against backgrounds, the gold standard is rule-based machine learning. The use of BDT's and random forests in particle physics analysis has been explored since early LHC era, nowadays it became widespread, their popularity becomes evident when one examines the literature-review of this thesis for instance. Many of the recent Higgs experimental analyses were performed using some rule-based ML algorithm.<sup>7</sup>

In this analysis, the EXtreme gradient BDT (XGBoost), with its Python implementation [415], has been used as the classifier algorithm. The standard procedure for training and testing the classifier was followed, starting with the complete list of features listed in [subsection 9.5.1](#) and then the most important features were shortlisted to improve the efficiency and performance of the classifier. This was possible due to the introduction of interpretability to the ML analysis, which provided variable importance measures, by which features with low importance index can be removed.

Interpretability is achieved by incorporating a mathematically robust measure from game theory known as **Shapley values** [416]. This measure formulate an axiomatic prescription for fairly distributing the payoff of a game amongst the players in a  $n$ -player co-operative game. When applied to ML, Shapley values estimate the significance of the features used in the classification. The process naturally and mathematically lends itself to examining the correlations amongst the features used in the classification, since all possible combinations of variables can be taken out of the game to check the outcome.

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<sup>7</sup>Rule-based ML algorithms outperform deep neural networks (DNN) in terms of simplicity of implementation and computational requirements. In addition, rule-based algorithms, such as decision trees, are more transparent as far as the signal vs. background separation is concerned

Predicted no. of events at HL-LHC							
Actual no. of events	Channel	$hh_{\text{tri}}^{\text{ggF}}$	$hh_{\text{tri}}^{\text{ggF}}$	$hh_{\text{box}}^{\text{ggF}}$	$Q\bar{Q}h$	$b\bar{b}\gamma\gamma$	total
$hh_{\text{tri}}^{\text{ggF}}$		28	14	18	38	10	108
$hh_{\text{int}}^{\text{ggF}}$		89	80	129	178	41	517
$hh_{\text{box}}^{\text{ggF}}$		77	105	266	265	50	763
$Q\bar{Q}h$		177	98	191	5,457	1,835	7,758
$b\bar{b}\gamma\gamma$		1,743	845	1,074	30,849	287,280	321,791

**Table 9.3.** The confusion matrix output of the trained BDT five-channel classifier. The separation between the ggF topologies allows for setting constraints on  $C_\phi$ . The number of events shown are for the HL-LHC at 14 TeV and integrated luminosity of  $6 \text{ ab}^{-1}$ , assuming the SM signal.

Further information regarding the application of Shapley values in particle physics analysis can be found in refs. [20, 417, 418]. For Higgs pair production study presented here, the same procedure described in [20] was followed. The importance of a variable in determining the outcome of a classification will be quantified by the mean of the absolute Shapley value,  $\overline{|S_v|}$ , larger values signifying higher importance. The SHAP (SHapley Additive exPlanations) [419] package implemented in Python was used. This package computes the feature importance using Shapley values calculated exactly from tree-explainers [420, 421]. This analysis is to be published soon [310]

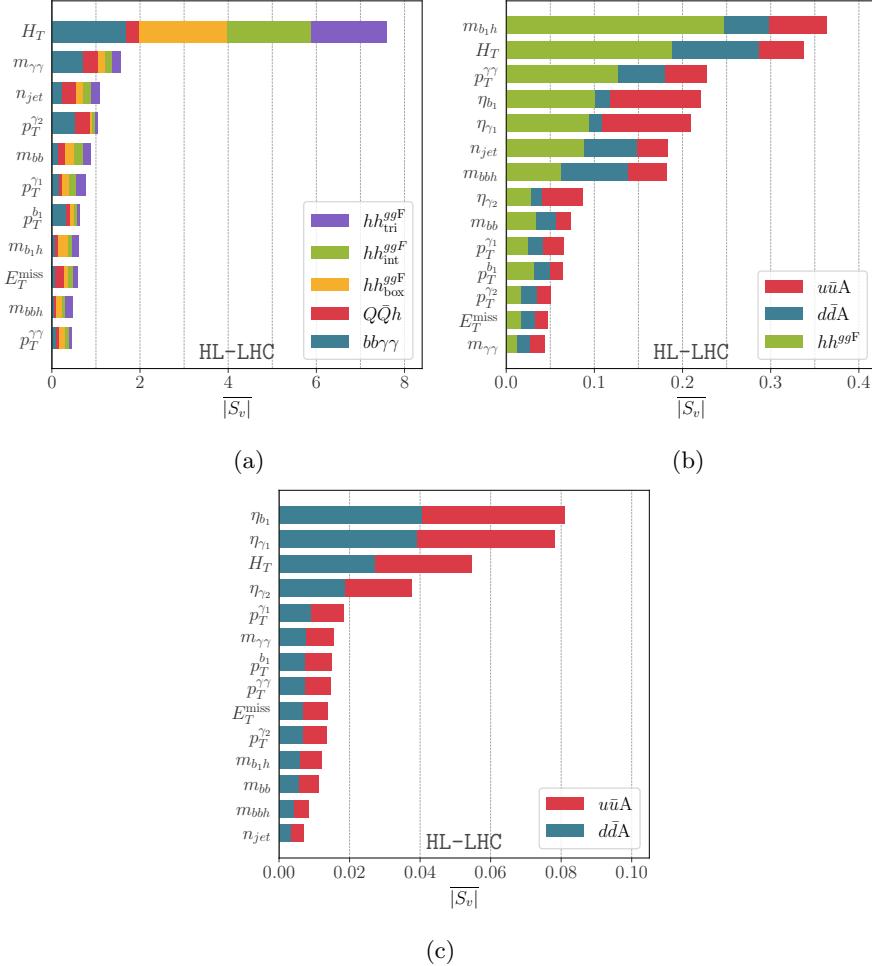
### Classifier output

The trained BDT's outputs are extracted in terms of confusion matrices, with number of events as entries. The diagonal elements of these matrices represent the true positive (TP) identification of the signal and true negative (TN) rejection of the background, while the upper triangular part represents the signal loss, or false negative counts (FN). Finally the lower triangular part shows the remaining background contamination of the signal, or the false positive counts (FP). Using these counts it is possible to estimate the accuracy score ACC of the classifiers

$$ACC = \frac{TP + TN}{TP + TN + FP + FN} \approx 0.7, \quad (9.38)$$

And the sensitivity  $TP/P \approx 0.2$ , which corresponds to the  $\epsilon_{SEL}$  of the cut-based analysis. Here we see that the ML- based analysis yielded a four- to five-fold increase in  $\epsilon_{SEL}$  compared to the cut and count method. Table 9.3 shows one of these matrices from the classification of the ggF SM signal separated into the topologies according to their dependence on  $C_\phi$ . For up- and down-quark  $q\bar{q}A$ , the same matrices were constructed, and since the number of events for these processes scale with  $C_{q\phi}^2$  it is only required to produce one matrices for each classification procedure, like the case of the ggF channel. For the fitting procedure, a Bayesian framework based on an MCMC method was used, analogous to the procedure described in section 7.2

The full analysis code, including the BDT training and fits as well as the confusion ma-



**Figure 9.8.** The feature importance output in terms of  $|S_v|$ . The higher the value of  $|S_v|$ , the more important the kinematic variable is in separating the different channels : (a) The hierarchy of variables important for the separation of  $hh_{\text{tri}}^{ggF}$  from  $hh_{\text{int}}^{ggF}$  events from  $hh_{\text{box}}^{ggF}$ ,  $Q\bar{Q}h$  and  $b\bar{b}\gamma\gamma$  QCD-QED background (b) The hierarchy of variables important for the separation of  $hh^{ggF}$ ,  $u\bar{u}A$  and  $d\bar{d}A$  events. (c) The hierarchy of variables important for the separation of  $u\bar{u}A$  from  $d\bar{d}A$  events.

trices for the classification procedures preformed can be found in the **Github** repository: <https://github.com/talismanbrandi/IML-diHiggs.git>.

### Feature importance and Shapley values

Another output of the interpretable BDT is the SHAP scores for the features used in the classification. The  $|S_v|$  values are used to order the features used for the classification. The most important features in different classifiers used in this analysis is seen in Figure 9.8. Panel (a) shows the hierarchy of the features used for the separation of the SM ggF signal from the backgrounds, the same features that showed the most

significant change in the network analysis and unsupervised clustering appear in the top of the list. However, the BDT was able to distinguish between the different signals, a task the unsupervised clustering was unable to fulfil. Figure (b) shows the list of feature importance for the ggF vs  $q\bar{q}A$  classification, while (c) demonstrates the full strength of the BDT in distinguishing  $u\bar{u}A$  from  $d\bar{d}A$  despite having very little variation of their kinematic distributions. As expected,  $u\bar{u}A$  vs  $d\bar{d}A$  classification, the features appeared on top of the list, are related to the different PDF's but their ranking was unintuitive because this classification is a truly a multivariate problem, where the intra-variable correlations and differences have been fully extorted.

## 9.6 Fit results

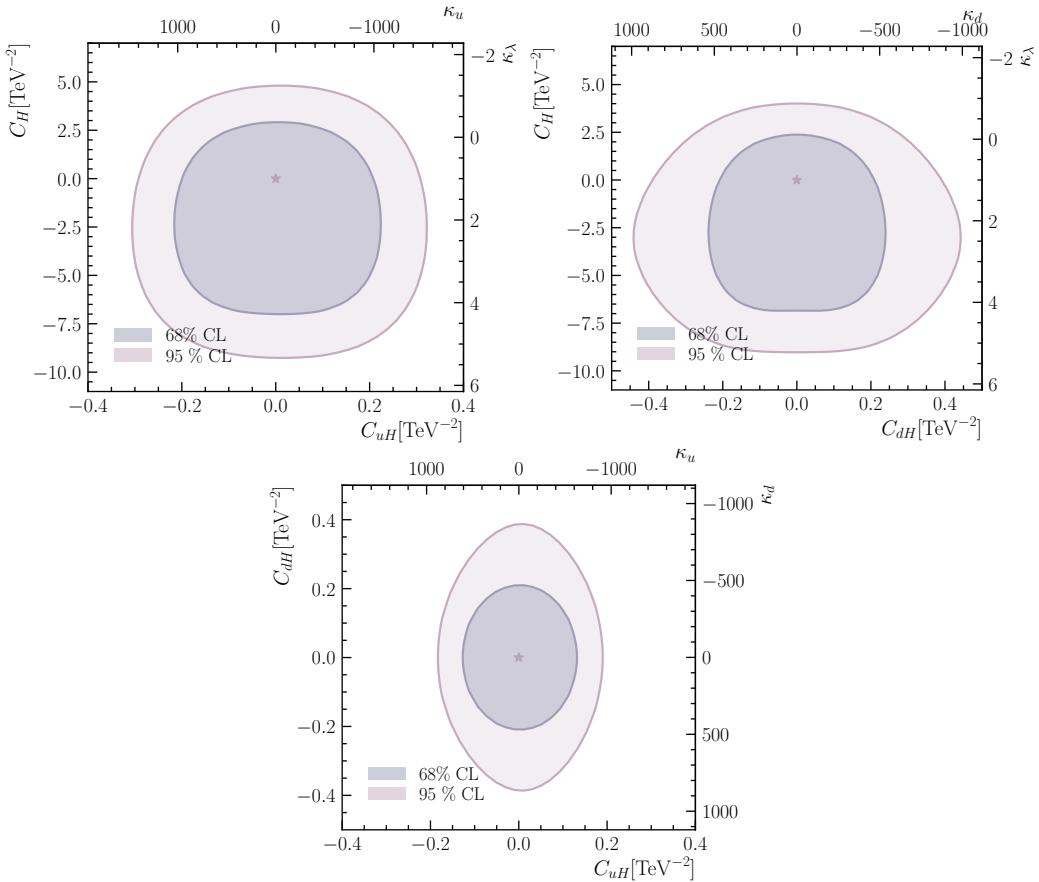
The fit from the cut-bases analysis was originally made for  $3 \text{ ab}^{-1}$  and published in [375], but for a better comparison with the optimised multi-variate analysis the fit for this thesis was carried out again for  $6 \text{ ab}^{-1}$ , and with SMEFT Wilson coefficient parametrisation. Thus harmonising it with the results of the rest of the thesis. The fits were done in the  $C_\phi - C_{q\phi}$  plane shown the top plots of Figure 9.9. As well as the  $C_{u\phi} - C_{d\phi}$  one in the low panel of the same figure. We see that even with the traditional technique, two-parameter fits were possible. However, the bounds obtained on the trilinear self-coupling modifier are weaker than the projected bounds for the HL-LHC, made by ATLAS and CMS [128, 316, 422], which is expected due to the dilution of these bounds by adding Light Yukawa coupling modifiers and the loss of some signal due to the analysis technique. For the  $C_{u\phi} - C_{d\phi}$  combined fit, no correlation between the two parameters is seen.

To demonstrate the power of multi-variate (MV) analysis, we compare the fit results from single parameter fits of this analysis to the cut-and count technique (CC) for both up and down quark coupling modifiers at 68% CL/CI

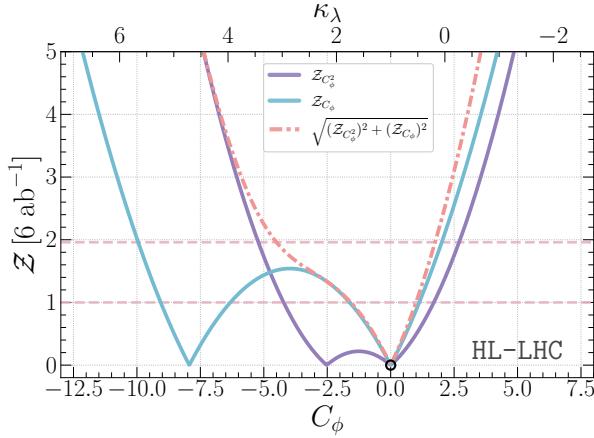
$$\begin{aligned} C_{u\phi}^{MV}(\kappa_u^{MV}) &= [-0.09, 0.10] \quad ([ -466, 454]), & C_{u\phi}^{CC}(\kappa_u^{CC}) &= [-0.18, 0.17] \quad ([ -841, 820]), \\ C_{d\phi}^{MV}(\kappa_d^{MV}) &= [-0.16, 0.16] \quad ([ -360, 360]), & C_{d\phi}^{CC}(\kappa_d^{CC}) &= [-0.18, 0.18] \quad ([ -405, 405]). \end{aligned} \tag{9.39}$$

A significant improvement of the bounds from using MV analysis over CC one of two-fold for  $C_{u\phi}$ , but a mild one for  $C_{d\phi}$  with  $\mathcal{O}(10\%)$  improvement.

In order to compare the ML multi-variate analysis used to other sensitivity projections, the projections on the trilinear coupling modifier  $C_\phi$  are shown in Figure 9.10. These bounds are obtained by using the we pe BDT classification showcased in Table 9.3. These bounds are similar or slightly better than the results quoted by the experimental sensitivity analysis quoted before. This was achieved by optimising the BDT with separating the signal and background channels, as well as the exclusion of less-important features. The projected  $1\sigma$  bound on  $C_\phi$  is  $[-1.57, 1.00]$  at HL-LHC. Another advantage

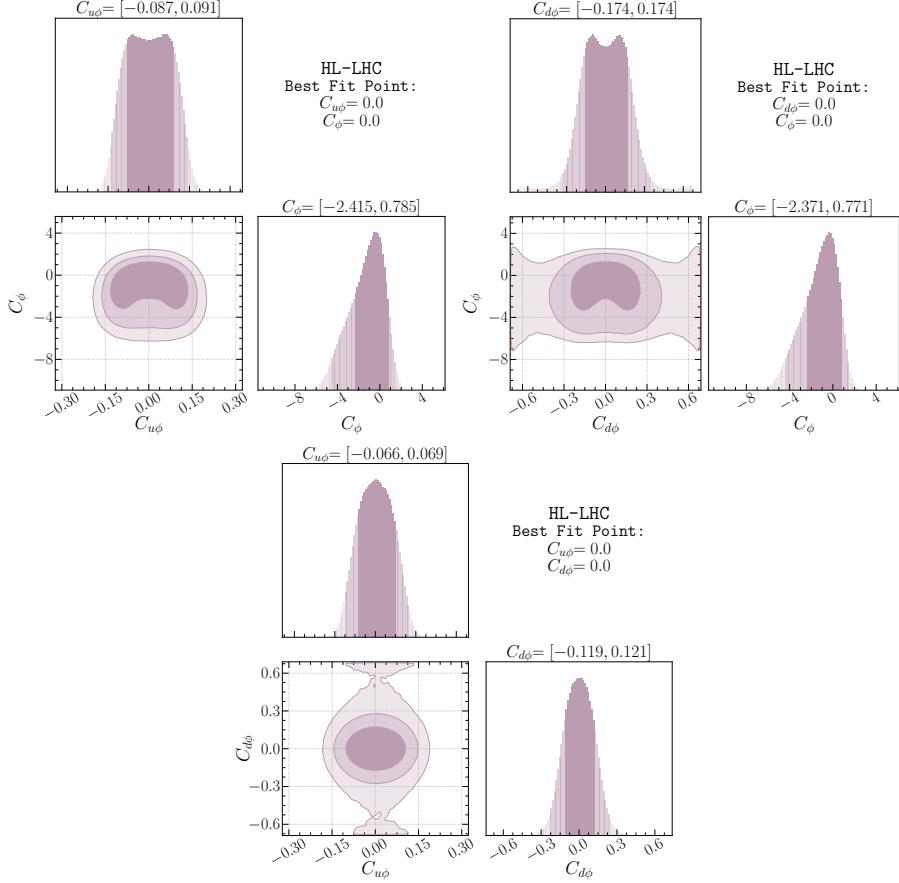


**Figure 9.9.** The 68% and 95% CL contours of the constraints on up and down Yukawa coupling modifiers as well as  $C_\phi$  from two-parameter fits using the results of the cut-based analysis for the HL-LHC at 14 TeV and 6 ab $^{-1}$  integrated luminosity.



**Figure 9.10.** Bounds on  $C_\phi$  (or  $\kappa_\lambda$ ) at the HL-LHC from single parameter fit. The solid blue lines are the constraints coming from the  $hh_{\text{int}}^{\text{ggF}}$  contribution which scales linearly with the modified coupling and the solid purple line is that from the  $hh_{\text{tri}}^{\text{ggF}}$  contribution that scales quadratically with the modified coupling. The red dashed line is the combination of the quadratic and linear channel. The horizontal light red dashed lines marks the 68% and 95%CI's.

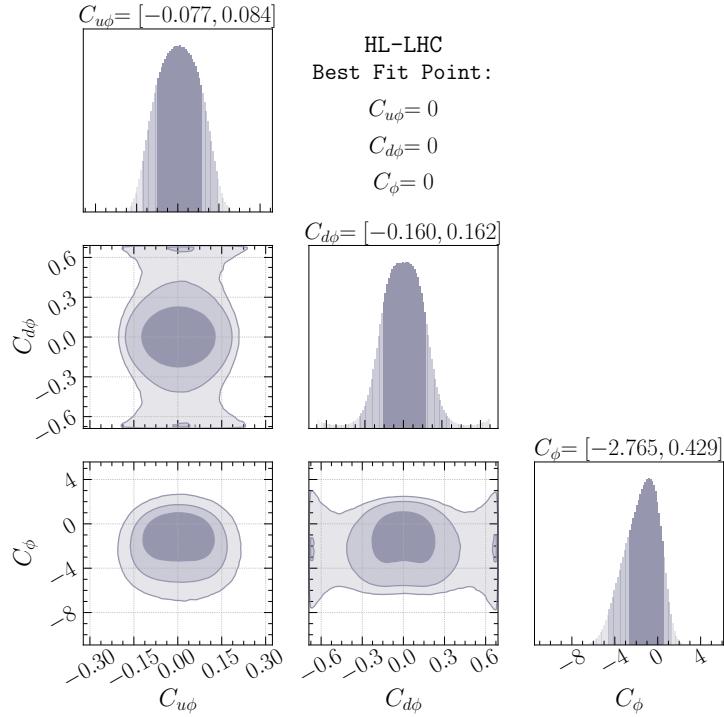
of the optimised multi-variate analysis is the ability to perform two-parameter fits in the same planes described above, shown in Figure 9.11 while maintaining the improvement over the cut-based one. Since the BDT training was able to achieve sufficient accuracy for seven-channel classifier, including up and down  $q\bar{q}A$ , the three ggF topologies and the backgrounds. It was possible to resolve all of the signal channels strata and their parametric dependence on the three Wilson coefficients  $C_{u\phi}$ ,  $C_{d\phi}$  and  $C_\phi$ . A three-parameter fit is possible without degeneracies, as seen in Figure 9.12. However, the posterior distribution of the three-parameter fit show no marked correlations amongst the Wilson coefficients. In both two- and three-parameter fits degeneracy in the  $C_{d\phi}$  direction is observed at 99.7% CI. This due to the reduction of the Higgs pair signal when the  $h \rightarrow d\bar{d}$  decay channel is opened, particularly for high values of this Wilson coefficient as highlighted by Figure 9.5. In fact, when this analysis is applied for the strange quark, the overall effect of enhanced strange quark is a reduction in the  $b\bar{b}\gamma\gamma$  signal, making this Higgs pair final state insensitive to strange Yukawa enhancements, more details on this were discussed in [375]. Comparing with the constraints on  $C_\phi$  from a single parameter fit in Figure 9.10, it can be seen from the two- and three-parameter fits in Figure 9.11 and Figure 9.12, respectively, that, the constraints on  $C_\phi$  become diluted when the light-quark Yukawa coupling modifiers  $C_{q\phi}$  are taken into account. This effect is somewhat more prominent for  $C_{d\phi}$  than for  $C_{u\phi}$  and stems from the fact that away from  $C_{u\phi,d\phi} = 0$  larger negative values of  $C_\phi$  are allowed by the crescent shaped curves of the HDP contours. The bounds on  $C_{u\phi}$  and  $C_{d\phi}$  from the fit with two-parameters including  $C_\phi$  remain the same as the bounds on these Wilson coefficient from the single parameter  $C_{u\phi,d\phi}$  fits. The fit results are summarised in Table 9.4.



**Figure 9.11.** The 68%, 95% and 99.7% HDP contours, for Bayesian fits preformed on pairs of Wilson coefficients for  $C_\phi$ ,  $C_{u\phi}$  and  $C_{d\phi}$  form the multi-variate analysis output.

Operators	$C_{u\phi}$	$C_{d\phi}$	$C_\phi$		$\kappa_u$	$\kappa_d$	$\kappa_\lambda$
HL-LHC 14 TeV $6 \text{ ab}^{-1}$ @ 68% CI							
$\mathcal{O}_\phi$	—	—	[-1.57, 1.00]		—	—	[0.53, 1.73]
$\mathcal{O}_{u\phi}$	[-0.09, 0.10]	—	—		[-477, 431]	—	—
$\mathcal{O}_{d\phi}$	—	[-0.16, 0.16]	—		—	[-360, 360]	—
$\mathcal{O}_{u\phi} \& \mathcal{O}_\phi$	[-0.087, 0.091]	—	[-2.42, 0.79]		[-434, 417]	—	[0.63, 2.13]
$\mathcal{O}_{d\phi} \& \mathcal{O}_\phi$	—	[-0.17, 0.17]	[-2.73, 0.77]		—	[-381, 379]	[0.63, 2.27]
$\mathcal{O}_{u\phi} \& \mathcal{O}_{d\phi}$	[-0.065, 0.069]	[-0.12, 0.12]	—		[-331, 312]	[-268, 272]	—
All	[-0.077, 0.084]	[-0.160, 0.162]	[-2.77, 0.43]		[-400, 369]	[-362, 359]	[0.79, 2.30]

**Table 9.4.** Summary of the 68% projected bounds on  $C_{u\phi}$ ,  $C_{d\phi}$  and  $C_\phi$  from single-, two- and three-parameter fits for HL-LHC with  $6 \text{ ab}^{-1}$  of data and FCC-hh with  $30 \text{ ab}^{-1}$  of data. The corresponding bounds on the rescaling of the effective couplings,  $\kappa_u$ ,  $\kappa_d$  and  $\kappa_\lambda$  are presented on the right side of the table.

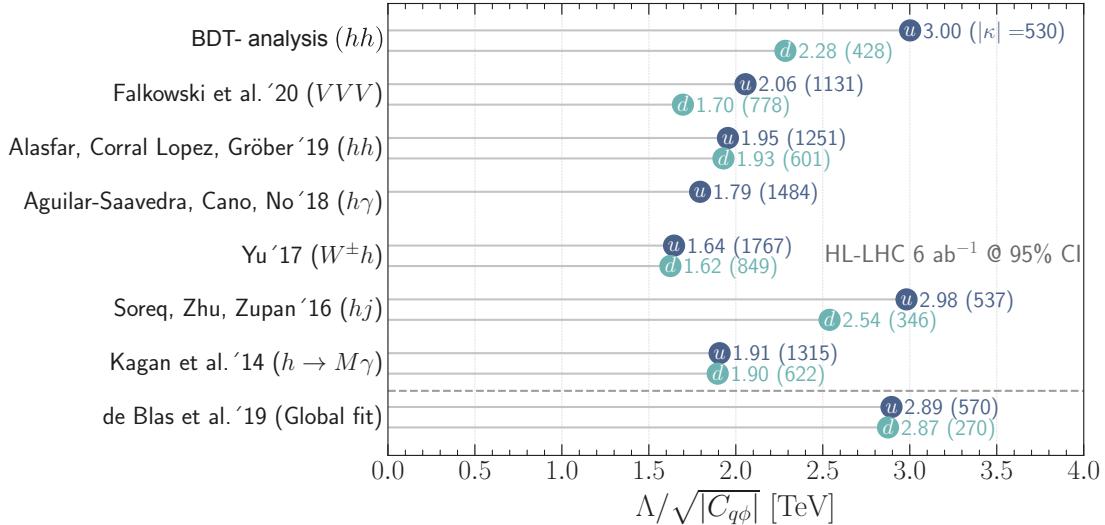


**Figure 9.12.** Three parameter Bayesian fits with  $C_{u\phi}$ ,  $C_{d\phi}$  and  $C_\phi$ , the HDP contours are the same as Figure 9.11 .

## 9.7 Overview of Light Yukawa searches

There are additional measurements of the light-quark Yukawa couplings that might become relevant at HL-LHC or FCC-hh, a careful study of which is beyond the scope of the current work. Yet we attempt to include a discussion here, so as to provide a comparison with our study and to put it into proper context, or to serve as proposal for further studies. The channel  $pp \rightarrow h + j$  has been suggested as a probe for charm Yukawa coupling [423] with charm-tagged jet having a potential bound of  $\kappa_c \sim 1$  for the HL-LHC, depending on the charm-tagging scheme. This process could be used for the first and second generations Yukawa couplings by looking at the shapes of kinematic distributions, the most important one being the  $p_T$  distribution [424–426]. The expected HL-LHC 95% CL bounds are  $\kappa_c \in [-0.6, 3.0]$ ,  $|\kappa_u| \lesssim 170$  and  $|\kappa_d| \lesssim 990$ . The use of  $h + j$  process along with other single Higgs processes have also been suggested as indirect probes for Higgs self coupling [148–152, 154], due to the contribution of the trilinear coupling to NLO electroweak corrections to these processes. In addition, experimental fits have been carried out for the trilinear coupling from single Higgs observables [178, 427].

It seems that for the HL-LHC, an optimal bound for the trilinear coupling can be obtained by combining both the data from single-Higgs process as well as Higgs pair production [177], with 68% CL bound on  $\kappa_\lambda \in [0.1, 2.3]$ , compared to the expected



**Figure 9.13.** Summary of the 95% CI/CL sensitivity bounds on the SMEFT Wilson coefficients  $C_{u\phi}$  (blue), and  $C_{d\phi}$  (green). The bounds are interpreted in terms of the NP scale  $\Lambda$  that can be reached through the measurements of the Wilson coefficient at the HL-LHC at  $6 \text{ ab}^{-1}$ , the corresponding  $\kappa_q$ 's are shown inside the parentheses. Single parameter fit 95% CI bounds are used from this analysis for comparison with previous studies.

bound of  $\kappa_\lambda \in [0.0, 2.5] \cup [4.9, 7.4]$  coming from using di-Higgs measurements alone. Moreover, single Higgs processes, namely  $Zh$  and  $W^\pm h$  production, could also be useful in probing charm-Yukawa coupling using a mixture of  $b$ - and  $c$ -tagging schemes leveraging the mistagging probability of  $c$ -jets as  $b$ -jets in  $b$ -tagging working points, and vice-versa, in order to break the degeneracy in the signal strength [428]. The use of this technique could probe  $\kappa_c \sim 1$  in the FCC-hh. Of course, for the charm-Yukawa coupling, the constraints are set to improve significantly, as there has been recent direct observation of  $h \rightarrow c\bar{c}$  [141]. Therefore, from here on, we will mainly concentrate on the process with more potential for constraining Yukawa couplings of the first generation quarks.

Rare Higgs decays to mesons,  $h \rightarrow M + V$ ,  $M = \Upsilon, J/\Psi, \phi, \dots$ , were also suggested as a probe for light-quark Yukawa couplings [429–431], and there have been experimental searches for these decays [141, 432] with bounds on the branching ratios,  $\mathcal{B}(h \rightarrow X, \gamma, X = \Upsilon, J/\Psi, \dots) \sim 10^{-4} - 10^{-6}$  at 95% CL. It was shown in Ref. [433], that the charge asymmetry of the process  $pp \rightarrow hW^+$  vs  $pp \rightarrow hW^-$  can be used as a probe for light-quark Yukawa couplings as well as to break the degeneracy amongst quark flavours. Moreover, the rare process  $pp \rightarrow h\gamma$  is also a possible way to distinguish between enhancements of the up- and down-Yukawa couplings [434] where the authors have estimated the bounds on the up-Yukawa coupling of  $\kappa_u \sim 2000$  at the HL-LHC. Despite some processes appearing more sensitive than others, one should think of these processes as complementary to each other.

One of the main features of the effective couplings  $hhq\bar{q}$  and  $hhhq\bar{q}$  emerging from SMEFT operator  $\mathcal{O}_{q\phi}$ , or the Chiral Lagrangian for that matter, is that these couplings

are either free from propagator suppression for  $hhq\bar{q}$  or scale with energy for  $hhhq\bar{q}$  while being safe from strong unitarity constraints. This feature gives processes with multiple Higgs and/or vector bosons  $V = W^\pm, Z$  an advantage in constraining  $\mathcal{O}_{q\phi}$ . The latter constrains come from the longitudinal degrees of freedom of the gauge bosons which can be understood from the Goldstone boson equivalence theorem. The use of the final state  $VV$  as a probe for  $\mathcal{O}_{q\phi}$  is difficult due to the large SM background. However, the three-boson final state  $VVV$  was shown to give strong projected bounds for light-quark Yukawa couplings for HL-LHC with 95% CL bounds on  $\kappa_u \sim 1600$ , and  $\kappa_d \sim 1100$ . A ten fold improvement is expected at FCC-hh [435] with bounds of order  $\kappa_d \sim 30$ . Higgs pair production has a smaller SM background compared to  $VV$  production, but it has a significantly smaller cross section too, even when compared to  $VVV$ , as the latter process has already been observed at the LHC [436, 437].

On the contrary, Higgs pair production is inaccessible with the runs I-III of the LHC, but it is potentially accessible at the HL-LHC [438] having a  $\sigma \cdot BR \sim 1\text{fb}^{-1}$ . However, Higgs pair production, particularly the channel  $h \rightarrow b\bar{b}\gamma\gamma$ , is of significant interest as it has unique features. The first being the ability to constrain the trilinear and light-quark Yukawa couplings simultaneously, as we have already seen in the previous sections. Secondly, Higgs pair production could probe non-linear relations between Yukawa interaction and  $hhq\bar{q}$  couplings [439]. Lastly, Higgs pair production is expected to be significant enhanced in certain models involving modification of light-quark Yukawa couplings (cf. [440–442]) A numerical comparison of the strongest bounds from HL-LHC on the first-generation Yukawa couplings from the studies discussed above in Figure 9.13. In comparison to the global fit bounds that have been obtained with no invisible or untagged Higgs decays allowed [443]. For  $C_{d\phi}$ , the most stringent bound comes from the global fit, and the  $h + j$  channel, as a model-independent bound, while this analysis provides the second most stringent model-independent bound. For  $C_{u\phi}$  this analysis provides the most stringent constraint while the bound from  $h + j$  and the global analysis are comparable. The figure is interpreted in terms of the reach of NP scale  $\Lambda$  that can be achieved by the measurement of these Wilson coefficients. For future colliders, like the FCC-hh at 100 TeV, in addition to Higgs pair production, triple Higgs production might be an interesting channel for constraining the operators with Wilson coefficient  $C_{u\phi}$  and  $C_{d\phi}$  due to the energy increase of a Feynman diagram coupling the quarks to three Higgs bosons.

Finally, it should be noted that there are also non collider signatures for enhanced light-quark Yukawa couplings, manifesting in frequency shifts in atomic clocks from Higgs forces at the atomic level [444].

## 9.8 Discussion and conclusion

The chapter walked through the potential of Higgs pair production to glean information about the elusive Yukawa couplings of the first generation quarks, from the final state  $b\bar{b}\gamma\gamma$ . This has been done in two different approaches: The first is the traditional cut and count method. Afterwards, significant improvement of the analysis has been

achieved using interpretable machine learning. In order maintain harmony with other chapters of this thesis, the enhancements of light Yukawa couplings were parametrised within the SMEFT framework

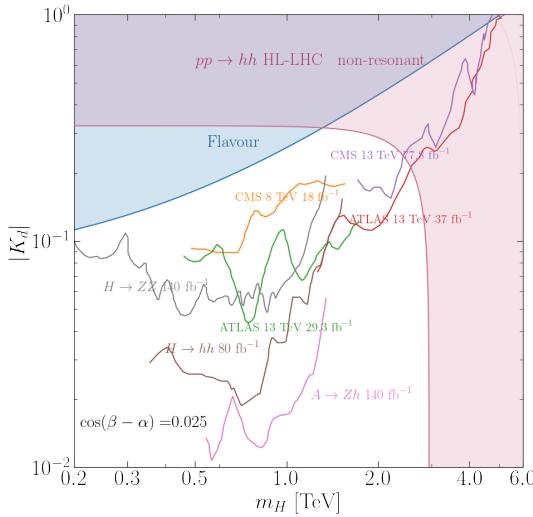
Despite the limitations of the cut-based analysis for Higgs pair, it was still possible to estimate notable sensitivity for both up- and down-type Yukawa coupling to the Higgs boson, comparable with other channels and the model-dependent global fit. Superior estimated bounds, in particular for the up quark, emanated from the full exploitation of the kinematical shapes and their correlations in a multi-variate analysis. This was achieved by using a High-level kinematical distributions as features in a BDT classification, then interfacing it with an explainer based on Shapley values.

The precedence of using an interpretable ML framework over DNN's stems from the ability to optimise the training procedure by means of physics-motivated dimensionality reduction by excluding less important features. Interpretable ML not only outperforms black-box models, but also provides physics understanding of the processes at hand, pointing to kinematic variables like  $H_T$  and  $m_{\gamma\gamma}$  as being important variables that instrument this separation. Lastly, but most importantly, interpretable models provide higher confidence in the results of their classification or regression.

The use of a BDT classifier was not only beneficial for increasing the  $hh$  signal selection efficiency, but also to classify the signal channels strata, such that it is possible to parametrise it in terms of  $C_\phi$ ,  $C_{u\phi}$  and  $C_{d\phi}$ . By decomposing the ggF channel into its sub-topologies depending on their  $C_\phi$  parametrisation cross-section into the box topologies which do not depend on the trilinear coupling, the triangle scaling quadratically with  $C_\phi$ . Lastly the interference between the two, which has a linear dependence on  $C_\phi$ . The  $q\bar{q}A$  is considered exclusively a NP channel. It scales quadratically with  $C_{q\phi}$ . The outcome of this technique is the ability to preform, two and three-parameter fits including all of the Wilson coefficients in question.

With the HL-LHC Higgs pair searches, it is expected to constrain the Higgs trilinear coupling to  $\mathcal{O}(1\%)$ . A result match by other sensitivity analysis based on ML analysis done by experimentalists at the CMS and ATLAS [128, 316, 422]. This clearly indicates the desideratum of Higgs pair production observation for understanding the Higgs potential. In spite of light Yukawa modifiers like  $C_{q\phi}$  being typically overlooked when studying Higgs pair production, this study showed that they can dilute the bounds on the trilinear Higgs coupling, and thus these coefficients need to be considered in any phenomenological studies of Higgs pair. Particularly that these Wilson coefficients are weakly bounded from other measurements, unlike other coefficients that can be constrained from single-Higgs, EWPO or top data.

There exist a handful of potential UV-complete models in which both light Yukawa as well as the Higgs trilinear couplings are enhanced. For example, a model proposed in ref [440] based on vector-like quarks (VLQ) with AFV assumptions. The original assumption of this model is excluded, as the authors assumed an enhancement of all light quark-Higgs couplings to be equal to the beauty Yukawa. One could still get significant enhancement to light Yukawa fro VLQ masses of  $\sim 2$  TeV, which is well above the current direct searches excluding VLQ of masses  $M_{VLQ} < 1.6$  TeV [445, 446] for the hadronic



**Figure 9.14.** Example of constraints on the 2HDM with SFV presented in [442, 450] from flavour observables, LHC dijet,  $Zh$ ,  $ZZ$  and resonant  $hh$  searches. The region shaded in Red is the bounds projected for the HL-LHC from the analysis presented in this chapter. This plot was inspired by the plots found in ref. [442].

final state, and  $M_{VLQ} < 1.2$  TeV for the leptonic one [447]. Due to the AFV manifested in this model, the VLQ could be made not to couple to the third generation quarks, and evade the tree-level EWPO bounds [18]. In addition, the trilinear Higgs coupling could be modified by the inclusion of an additional scalar singlet cf. [90, 448, 449]. Another example of models with enhanced light Yukawa is a two-Higgs-doublet model (2HDM) model proposed in refs. [442, 450]. This model has a special kind of AFV, known as spontaneous flavour violation (SFV). Enhancements to light Yukawa couplings come from the second Higgs Yukawa couplings, which make diagonal in the flavour space  $K_q$  ( $q = u, d$ ). SFV has the constraint that either the up-type or the down-type couplings can be enhanced, while the couplings of the other type maintain the SM hierarchy. The Higgs potential is modified by the addition of the second doublet, and consequently the Higgs self-coupling will be modified as well. Like any other 2HDM, the parameter space is rather large. The bounds on this model will depend on the region of its parameter space we are interested in. Figure 9.14 shows the bounds on this model for a point near the alignment limit. For small mass of the “heavy” Higgs  $H$  and large Yukawa coupling  $K_d$  flavour bounds dominate, while for larger  $m_H$  the dijet searches [451–453] would dominate due to the decay  $H \rightarrow d\bar{d}$ . On the contrary, the decay  $H \rightarrow hh$  would become dominant from smaller values of  $K_d$  and larger  $H$  mass, but still  $m_H < 2$  TeV. In this regime, resonant Higgs pair searches give strong constraints for light Yukawa enhancement [454, 455]. Similar light Yukawa bounds in this region of the parameter space could also be derived from  $Zh$  [456] and  $ZZ$  [457, 458] searches. Lastly, for  $m_H > 2$  TeV, the non-resonant Higgs pair production will become the dominant bound on light Yukawa enhancement, coming from the analysis of this chapter.

## Part IV

# Flavour physics



## 10 Data-inspired models for $b \rightarrow s\ell\ell$ anomalies

Recent results from  $B$ -factories including Belle and Babar as well as the LHCb-experiment involving the semileptonic decays of the beauty mesons  $B^0, B^\pm, B_s \dots$  point to marked deviation of  $\sim 2.5\sigma$  from the SM prediction, particularly in the branching fractions ratios

$$R_{K^{(*)}} \equiv \frac{Br(B \rightarrow K^{(*)}\mu^+\mu^-)}{Br(B \rightarrow K^{(*)}e^+e^-)}, \quad (10.1)$$

in the low dilepton mass bins [21–25]<sup>1</sup>. In addition to the results of angular analysis of the decay  $B \rightarrow K^*\mu^+\mu^-$  [459, 460], with the observable  $P'_5$  showing similar deviation from the SM, the most recent measurement was published by LHCb [461] if the light cone sum rules for modelling the hadronic effects are considered. Other observables derived from the branching fractions of semileptonic and full leptonic final states of  $B$  mesons decays, e.g.  $B_s \rightarrow e^+e^-$  also showed deviations from the SM with the  $2\sigma - 3\sigma$  range [26–29]. These observables have in common the FCNC transition  $b \rightarrow s\ell\ell \ell = e, \mu$ , and are in conflict with the SM lepton universality of EW couplings. This tension could be translated into a strong case for the evidence of BSM physics with lepton flavour universality violation (LUV) [462–464].

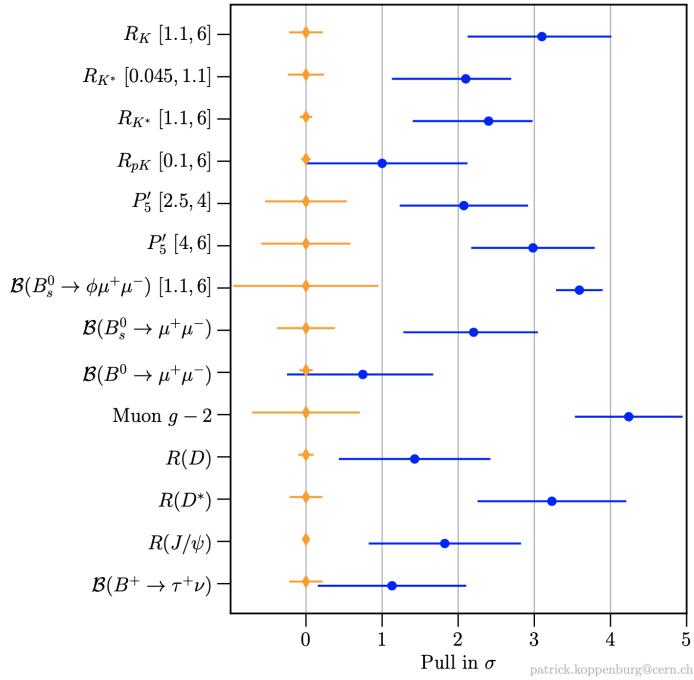
When these aberrant results are added to the recent muon anomalous magnetic moment  $g - 2$  measurement by Fermilab [465] or measurements of differential dilepton branching fractions of  $B$ -mesons, grounds for the muons being the source of LUV are established, i.e. the NP degrees of freedom contain muon-flavoured couplings. However, the hadronic contributions in the decay amplitudes and  $g - 2$  corrections [466–470], that require non-perturbative QCD [471–474], make such a conclusion debatable, see, e.g. [475, 476] and the most recent analysis, with the updated lepton flavour universality tests [477].

Another class of  $B$  decays involving the tree-level  $b \rightarrow c\tau\nu_\tau$  transitions has shown similar tension with the SM [478–481]. Amongst other, the observable  $R_{D^{(*)}} \equiv Br(B \rightarrow D^{(*)}\tau\nu)/Br(B \rightarrow D^{(*)}\ell\nu)$ , originally found at Babar [482] and subsequently measured at Belle [483] and LHCb [484] has shown a  $\sim 20\%$  deviation from the SM. All of the anomalous flavour observables as summarised in Figure 10.1 with their pull in  $\sigma$ 's shown in blue, compared the standard model predictions with their uncertainties in orange.

The simultaneous resolution for the anomalies emerging from  $b \rightarrow s\ell\ell$  and the semileptonic  $b \rightarrow c$  transitions, requires models with complicated flavour structure [30–39], as such models need to accommodate for similar deviations from the SM for both classes

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<sup>1</sup>The data from the most recent measurement of the  $R_{K^*}$  [25] has not been used in this work, as the fits shown in this chapter predates these results.



**Figure 10.1.** Forest plot summarising the flavour observables in tension with the SM predictions, the experimental pull in terms of standard deviations  $\sigma$  is shown in blue, while the SM prediction with the theoretical uncertainties is highlighted in orange. This figure is made by P. Koppenburg [485].

albeit these two transitions occur at different orders in the SM. Such models are often being at the edge of flavour physics constraints [312, 486] and collider bounds [487, 488]. On the other hand, most up-to-date measurements of  $R_{D^{(*)}}$  from the Belle collaboration [489, 490] turns out to be in good agreement with the SM [491–494], thereby casting some doubt on the potential for NP lurking within  $b \rightarrow c$  transitions. Furthermore, the ratios of branching fractions of decays involving the FCNC  $b \rightarrow s\ell\ell$  transitions have a much lower dependence on the non-perturbative QCD effects, than  $g - 2$ , and differential distributions of semileptonic  $B$ -decays [495–498]. Therefore, the LUV information extracted from such “clean” observables have the highest potential for extracting LUV insights, see [499] for more details.

The  $b \rightarrow s\ell\ell$  anomalies have been studied in a model-independent manner, in particular SMEFT framework in refs. [500–504] and more recently revisited in refs. [505–511]. Additionally, many UV-complete models were investigated, particularly leptoquarks (LQ), like in refs. [512–516]. Another class of models of special interest are  $Z'$  models, in which the  $B$  anomalies can be realised at loop-level. The simplest of these models has been proposed in ref. [517], extending the SM with a single new  $U(1)$  gauge group, together with the presence of top- and muon-partners, resulting in a top-philic  $Z'$  boson capable of evading present collider constraints [518] and responsible for the required LUV signatures. This model has the advantage of not introducing extra flavour spurions to the

SM, i.e. similar to the MFV ansatz [377, 519, 520]. More general set of models with the same features can be found in ref. [521], and subsequently elaborated upon in greater detail in the phenomenological study of ref. [522].

While evading flavour constraints, models with topophilic  $Z'$  are in strong tension with the  $Z$ -pole measurements [522, 523]. In fact, it has been shown in [505], that in spite of large hadronic uncertainties for the amplitude of the  $B \rightarrow K^* \mu^+ \mu^-$  decay, a tension of at the  $3\sigma$  level at least would persist between  $B$  data and EWPO for muonic LUV effects, and an even stronger tension would be found in the case of LUV scenarios involving electron couplings. This elucidates the interplay between  $B$ -physics and EWPO [505, 506, 514–516, 521, 524, 525].

This chapter aims to review a global fit including both EWPO and flavour observables related to the  $B$ -anomalies. Then present UV models that are based on ones present in the literature [517, 518, 521], that accommodate the resulting fit constraints are investigated. This work is an extension of several studies done by some of my collaborators [472, 475, 503, 505, 526–528], and published in [168]. This chapter is organized as follows: in section 10.1, the SMEFT analysis of the flavour anomalies is presented; in ?? I discuss a viable  $Z'$  model in relation to our EFT results. After that, I present a possible alternative leptoquark scenarios in in ?? . Lastly, the conclusions are summarised in section 10.4.

## 10.1 Flavour anomalies in SMEFT

### 10.1.1 Theoretical preamble

Global fits from  $b \rightarrow s\ell\ell$  anomalies show that if NP degrees of freedom enter at tree-level, they would have an energy scale  $\Lambda \sim 10$  TeV [500–504]. Highlighting that for LHC phenomenology, the use of SMEFT is justified. The operators of interest for the explanation of these  $B$  anomalies are [505, 506, 521]:

$$\mathcal{O}_{LQ^{(1)}}^{\ell\ell 23}, (\mathcal{O}_{LQ}^{(1,3)})^{\ell\ell 23}, \mathcal{O}_{Qe}^{23\ell\ell}, \mathcal{O}_{Ld}^{\ell\ell 23}, \mathcal{O}_{ed}^{\ell\ell 23}. \quad (10.2)$$

Following the same convention for the SM fields in Table 2.1 and operator definitions in the Warsaw basis presented in Table 4.1. Current data, taking non-perturbative QCD effects into an account using light-cone sum rules, both left- and right-handed operators are permitted [505, 507–509]. Nevertheless, the statistical significance for the right-handed  $b \rightarrow s$  interaction remains small, coming only from  $R_{K^*}/R_K \neq 1$  [504, 505]. Hence, one can effectively consider the left-handed operators  $(\mathcal{O}_{LQ}^{(1,3)})^{2223}$  and  $\mathcal{O}_{Qe}^{2322}$  for addressing the flavour anomalies. Additionally, when conservative hadronic uncertainties are considered [471–473], the preference of NP coupling to the muons exclusively becomes mitigated and the inclusion of electron interactions becomes viable as well [503]. From these considerations, we conclude that the operator  $(\mathcal{O}_{LQ}^{(1,3)})^{\ell\ell 23}$  with either or both  $\ell = e, \mu$  offers the minimal resolution of these anomalies within the SMEFT framework [505].

Introducing these operators at tree-level will lead to flavour violation beyond the SM, as these operators are flavour spurions unrelated to the SM ones. This can be avoided if they get generated at loop level from the RGE of operators involving the leptons and the Higgs [521]

$$(\mathcal{O}_{\phi L}^{(1,3)})^{\ell\ell}, \quad \mathcal{O}_{\phi e}^{\ell\ell}, \quad (10.3)$$

or alternatively, from the semileptonic four-fermion (SL-4F) operators with right-handed top-quark currents:

$$\mathcal{O}_{Lu}^{\ell\ell 33}, \quad \mathcal{O}_{eu}^{\ell\ell 33} \quad (10.4)$$

The leading log solution of the RGE for these operators [295, 296], with the matching conditions for the left-handed quark-current operators in eq. (10.2) at the EW scale  $\mu_{\text{EW}} \sim v$  are:<sup>2</sup>

$$\begin{aligned} C_{LQ}^{(1)} \ell\ell 23 &= V_{ts}^* V_{tb} \left( \frac{y_t}{4\pi} \right)^2 \log \left( \frac{\Lambda}{\mu_{\text{EW}}} \right) \left( C_{Lu}^{\ell\ell 33} - C_{\phi L}^{(1)} \ell\ell \right), \\ C_{LQ}^{(3)} \ell\ell 23 &= V_{ts}^* V_{tb} \left( \frac{y_t}{4\pi} \right)^2 \log \left( \frac{\Lambda}{\mu_{\text{EW}}} \right) C_{\phi L}^{(3)} \ell\ell, \\ C_{Qe}^{23\ell\ell} &= V_{ts}^* V_{tb} \left( \frac{y_t}{4\pi} \right)^2 \log \left( \frac{\Lambda}{\mu_{\text{EW}}} \right) \left( C_{eu}^{\ell\ell 33} - C_{\phi e}^{\ell\ell} \right). \end{aligned} \quad (10.5)$$

In heavy quark physics,  $B$  decays are typically studied within the low energy weak effective theory [531–533], in which we have the vector and axial currents defined as

$$\begin{aligned} \mathcal{O}_{9V,\ell} &= \frac{\alpha_e}{8\pi} (\bar{s}\gamma_\mu(1-\gamma_5)b)(\bar{\ell}\gamma^\mu\ell), \\ \mathcal{O}_{10A,\ell} &= \frac{\alpha_e}{8\pi} (\bar{s}\gamma_\mu(1-\gamma_5)b)(\bar{\ell}\gamma^\mu\gamma_5\ell), \end{aligned} \quad (10.6)$$

and matched at the EW scale  $\mu_{\text{EW}}$  with the SMEFT operators in eq. (10.3) - (10.4) follow:

$$\begin{aligned} C_{9,\ell}^{\text{NP}} &= \frac{\pi v^2}{\alpha \Lambda^2} \left( \frac{y_t}{4\pi} \right)^2 \log \left( \frac{\Lambda}{\mu_{\text{EW}}} \right) \left( C_{\phi L}^{(3)} \ell\ell - C_{\phi L}^{(1)} \ell\ell - C_{\phi e}^{\ell\ell} + C_{Lu}^{\ell\ell 33} + C_{eu}^{\ell\ell 33} \right), \\ C_{10,\ell}^{\text{NP}} &= \frac{\pi v^2}{\alpha \Lambda^2} \left( \frac{y_t}{4\pi} \right)^2 \log \left( \frac{\Lambda}{\mu_{\text{EW}}} \right) \left( C_{\phi L}^{(1)} \ell\ell - C_{\phi L}^{(1)} \ell\ell - C_{\phi e}^{\ell\ell} - sC_{Lu}^{\ell\ell 33} + C_{eu}^{\ell\ell 33} \right). \end{aligned} \quad (10.7)$$

The overall normalization in the weak Hamiltonian follows the standard conventions adopted in refs. [472, 503, 505]. As anticipated, the set of operators of interest for the study of  $R_{K(*)}$  in eq. (10.5) is also sensitive to EWPO. The operators involving the Higgs field and lepton bilinears in the SMEFT induce tree-level modifications to EW-boson

<sup>2</sup>similar to the previous chapters, for one-loop effects, the NP scale is set to be  $\Lambda = 1$  TeV. The renormalisation scale is set to  $\mu_{\text{EW}} = m_t \simeq v/\sqrt{2}$  to minimise the matching-scale dependence with the inclusion of the NLO corrections [529, 530].

couplings, while modifications of the  $Z$  couplings to the leptons can be also induced via top-loop contribution [64]. In the leading-log approximation and at the leading order in the top Yukawa coupling, LUV effects can be generated by:

$$\begin{aligned}\Delta g_{Z,L}^{\ell\ell}|_{\text{LUV}} &= -\frac{1}{2} \left( C_{\phi L}^{(1)\ell\ell} + C_{\phi L}^{(3)\ell\ell} \right) \frac{v^2}{\Lambda^2} - 3 \left( \frac{y_t v}{4\pi\Lambda} \right)^2 \log \left( \frac{\Lambda}{\mu_{\text{EW}}} \right) C_{Lu}^{\ell\ell 33}, \\ \Delta g_{Z,R}^{\ell\ell}|_{\text{LUV}} &= -\frac{1}{2} C_{\phi e}^{\ell\ell} \frac{v^2}{\Lambda^2} - 3 \left( \frac{y_t v}{4\pi\Lambda} \right)^2 \log \left( \frac{\Lambda}{\mu_{\text{EW}}} \right) C_{eu}^{\ell\ell 33},\end{aligned}\quad (10.8)$$

where  $\Delta g_{Z,L(R)}^{\ell\ell} \equiv g_{Z,L(R)}^{\ell\ell} - g_{Z,L(R)}^{\ell\ell,\text{SM}}$  is the deviation with respect to the left-handed (right-handed) leptonic couplings to the  $Z$  boson in the SM theory. Since EW couplings to leptons have been precisely measured at LEP/SLC, they provide an important test bed for lepton universality [65, 523].

These observations motivates a global SMEFT fit of the operators explaining the  $B$ -anomalies, as well as their interplay with EWPO. Assuming that the LUV affects are generated by NP via radiative effects, matching what is seen in eq. (10.7). Consequently, the NP will contribute to EWPO at tree-level, whilst other SMEFT operators appearing from the REG mixing are assumed to be small or constrained from other processes. In order for these assumptions to be fulfilled within SMEFT, the operators modifying the EW coupling of the quarks need to be included as well

$$\mathcal{O}_{\phi Q}^{(1)qq}, \mathcal{O}_{\phi Q}^{(3)qq}, \mathcal{O}_{\phi u}^{qq}, \mathcal{O}_{\phi d}^{qq}, \quad (10.9)$$

where  $q = 1, 2, 3$  identifies quark generations. These operators are considered to be flavour aligned, in a similar fashion to  $C_{q\phi}$  of the previous chapter, in particular they are assumed to be aligned with the down-quark basis. This is needed to avoid pathological tree-level FCNC [312]. The same holds for the leptonic operators, being aligned with the charged lepton mass bases.

The EWPO have a degeneracy between the first and second generation quarks, particularly in the down-type quarks sector. Therefore, it is natural to impose a  $U(2)^3$  symmetry between first and second generator quark operators, thus imposing  $C_{\phi Q}^{(1,3)11} = C_{\phi Q}^{(1,3)22}$ ,  $C_{\phi u}^{11} = C_{\phi u}^{22}$ . This also helps to suppress large FCNC contributions from these operators. Additionally, the RGE boundary condition  $C_{\phi u}^{33} = 0$  is assumed. This is motivated by the fact that this Wilson coefficient cannot be constrained by EWPO, as modifications to  $Z$ -coupling to right-handed top quarks cannot be probed by  $Z$ -pole measurements. Finally, for completeness, the four-lepton operator is also included in the fit:

$$O_{LL}^{1221} = (\bar{L}_1 \gamma^\mu L_2)(\bar{L}_2 \gamma_\mu L_1), \quad (10.10)$$

which contributes to the muon decay amplitude, and therefore alters the extraction of the value of the Fermi constant,  $G_F$ , which is one of the inputs of the SM EW sector.

The operators in eqs. (10.3), (10.9) and (10.10), with the assumptions mentioned before, saturate all the 17 degrees of freedom, i.e. combinations of operators, that can

be constrained in a fit to EWPO in the dimension-six SMEFT framework while keeping flavour changing neutral currents in the light quark sector under control. Together with the 4 four-fermion operators from eq. (10.4), this completes a total of 21 operators, which is included in the fit setup described in the next section.

### 10.1.2 SMEFT fit

The global fit combining both flavour observables related to the  $b \rightarrow s\ell\ell$  anomalies, in addition to EWPO is carried out in a Bayesian statistical framework. The experimental observables are modelled via state-of-the-art theoretical information already implemented and described in ref. [505] for flavour physics, and for EW and Higgs physics in ref. [62] and, more recently, in ref. [65]. EWPO are extended by flavour non-universal SMEFT contributions described in ref. [523, 534]. The statistical and physics frameworks are both available within the publicly available `HEPfit` [306] package, a MCMC framework built using the Bayesian Analysis Toolkit [535]<sup>3</sup>

The experimental input used for the global fit is summarised in the following, which are also implemented in `HEPfitcode`:

- The set of EWPO including the  $Z$ -pole and  $W$  properties measurements from LEP and SLD, in addition to Tevatron and LHC measurements of EW bosons properties and rates [47, 49, 536–540]. The following EWPO used in the fit are

$$\begin{aligned} & M_H, m_t, \alpha_S(M_Z), \Delta\alpha_{\text{had}}^{(5)}(M_Z), \\ & M_Z, \Gamma_Z, R_{e,\mu,\tau}, \sigma_{\text{had}}, A_{FB}^{e,\mu,\tau}, A_{e,\mu,\tau}, A_{e,\tau}(P_\tau), R_{c,b}, A_{FB}^{c,b}, A_{s,c,b}, R_{u+c}, \\ & M_W, \Gamma_W, \text{BR}_{W \rightarrow e\nu, \mu\nu, \tau\nu}, \Gamma_{W \rightarrow cs}/\Gamma_{W \rightarrow ud+cs}, |V_{tb}|; \end{aligned}$$

- The angular distribution of the decay  $B \rightarrow K^{(*)}\ell^+\ell^-$  including both the  $\mu$  and  $e$  final states in the large  $m_{\ell\ell}$  bins<sup>4</sup>. The data from ATLAS [541], Belle [497], CMS [542, 543] and LHCb [544, 545], in addition to the branching fractions from LHCb [546], the charged  $B^+$  meson measured by LHCb [547], and the HF-LAV average [548] for the branching fraction of the decay  $B \rightarrow K^*\gamma$ ;
- The angular distribution of  $B_s \rightarrow \phi\mu^+\mu^-$  [549] and the branching ratio of the decay  $B_s \rightarrow \phi\gamma$  [550], measured by LHCb;
- The LUV ratios  $R_K$  [23] and  $R_{K^*}$  [22] from LHCb and Belle [24];
- Branching ratio of  $B_{(s)} \rightarrow \mu^+\mu^-$  measured by LHCb [27], CMS [26], and ATLAS [28]; in addition to the upper bound on the decay  $B_s \rightarrow e^+e^-$  reported by LHCb [29].

Modelling the decays of hadrons involves factorisable (in terms of decay constant) and non-factorisable non-perturbative QCD effects. The non-factorisable effects emerge from

<sup>3</sup>`HEPfit` is developed by some of my collaborators, who have co-authored this work

<sup>4</sup>The measurements of  $B \rightarrow K^{(*)}\ell^+\ell^-$  decays in the low di-lepton invariant mass region are plagued by large uncertainties for the charmonium resonance, and thus not included in the fit.

long-distance hadronic contributions to [466, 467, 471] QCD loops appearing in radiative corrections to these decays. In this analysis, the  $B \rightarrow K^* \ell^+ \ell^-$  has two different scenarios to describe these hadronic effects, discussed also in other previous works of my collaborators [475, 503, 505, 526–528]. The first is a conservative approach (Phenomenological Data Driven or PDD) as originally proposed in [472], and refined in ref. [475], whilst the second is a more optimistic one based on the results in [466] (Phenomenological Model Driven or PMD). The PDD scenario is based on a generic model of the hadronic effects, which is simultaneously fitted to  $b \rightarrow s \ell \ell$  data alongside the NP effects. Contrary to PDD approach, the PMD scenario the dispersion relations specified in [466] is used constrain the hadronic contributions in the entire large-recoil region considered in the analysis, ergo PMD has smaller hadronic effects in the  $B \rightarrow K^* \ell^+ \ell^-$  amplitudes [526]. The choice of the hadronic uncertainties model significantly affects the outcome of the fits to the  $B$ -decays observables [505].

In order to be as general as possible, the SMEFT global fit is done for four different scenarios, described as follows:

- **EW:** Using EWPO data only with the assumptions discussed in section 10.1. This fit includes the operators in eqs. (10.3), (10.9), and (10.10), summing up to a total of 17 Wilson coefficients.
- **EW (SL-4F Only):** This refers to a fit done with the Wilson coefficients of the SL-4F operators involving the right-handed top current, reported in eq. (10.4). This scenario incorporates the assumption that BSM enters the modifications of the  $Z$  couplings to muons and electrons through top-quark loops only.
- **EW & Flavour:** Wilson coefficients of all the 21 operators given in eq. (10.3), (10.9), and eq. (10.10), together with eq. (10.4) are varied.  
All of the EW data as well as the flavour observables listed above are used. This scenario comes in two varieties, PDD and PMD, as explained above.
- **Flavour:** These fits exclusively include the Wilson coefficients of the *4 operators* (both electrons and muons) appearing in eq. (10.4), and are done including only flavour data, i.e. excluding EW measurements. Results are again distinguished for the PDD and PMD cases. This fit is typically done when flavour anomalies are discussed in the literature. Hence, it was included here to emphasize the importance of including EWPO .

### 10.1.3 Fit results

The fit was preformed for each of the aforementioned scenarios and the extracted average values of the Wilson coefficients and the corresponding 68% CI are summarised in Figure 10.2.

The EW only fits, involving 17 out of the total 21 Wilson coefficients are shown in orange. The EWPO fit shows good agreements with the SM within  $2\sigma$  level. Furthermore, the Wilson coefficients involved in the fit seems to be highly correlated from the EWPO data as indicated by the correlation matrix in Figure 10.3.

The impact of these operators on the  $b \rightarrow s\ell\ell$  observables is shown in Figure 10.4, where it collects the mean and standard deviation on the shift in the  $Z$  coupling to light leptons w.r.t the SM, it should be noted that these deviations of the  $Z$  couplings are related to the LUV ratios  $R_{K^{(*)}}$  in the dilepton-mass range [1.0, 6.0] GeV<sup>2</sup> by:

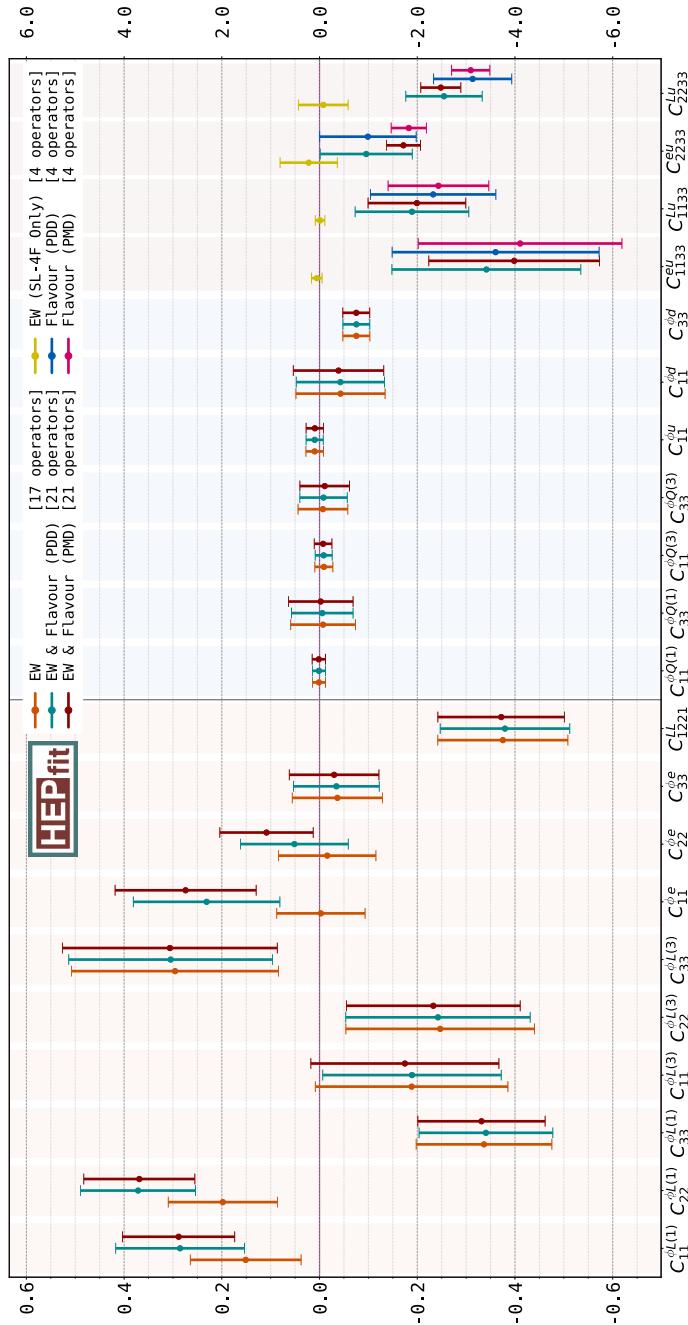
$$\delta g_{Z,L(R)}^{ee(\mu\mu)} \equiv g_{Z,L(R)}^{ee(\mu\mu)} / g_{Z,L(R)}^{ee(\mu\mu),\text{SM}} - 1 , \quad \delta R_{K^{(*)}} \equiv R_{K^{(*)}} - R_{K^{(*)}}^{\text{SM}} , \quad (10.11)$$

which is tightly constraint by the EWPO data to per-mille level.

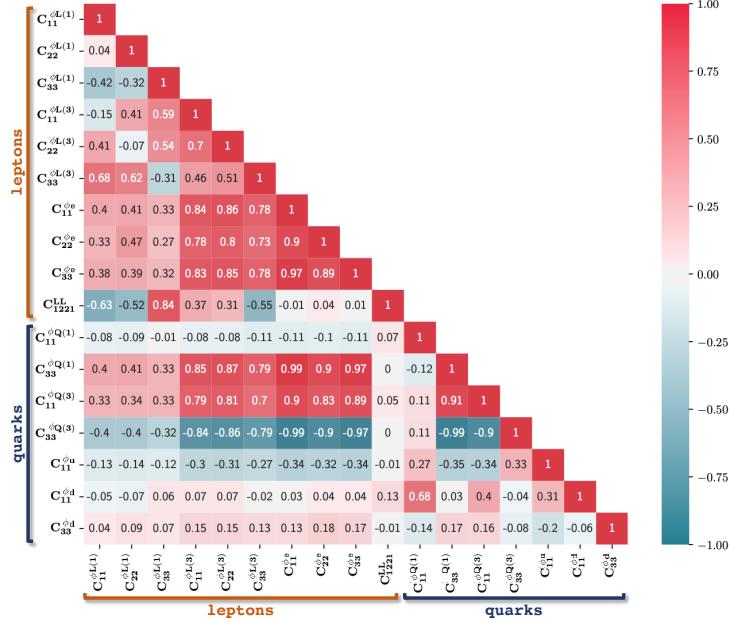
The other fit scenario only involves the **SL-4F** coefficients constraint from EWPO data, shown in yellow in Figure 10.2. Although EW data allows for more relaxed constraints on these operators, for example  $\mathcal{O}_{Lu}^{\ell\ell 33}$  compared to the ones modifying  $Z$  couplings at tree-level e.g.  $\mathcal{O}_{\phi L}$ , the bounds remain compatible with the null (SM) Hypothesis and in about  $3\sigma$  conflict with the experimental measurements on  $R_{K^{(*)}}$  (indicated by the shaded red boxes in the right side of Figure 10.4).

We now move to the flavour data fits, with both ansätze for the hadronic contributions PDD highlighted in blue and PMD in pink. For this fit, deviations of the muonic  $C_{Lu}^{2233}$  shows deviation from the SM hypothesis of  $3\sigma$  for PDD and up to  $6\sigma$  for the optimistic PMD scenario. The difference of the significance between the two cases stems from the interpretation of the angular analysis –namely the  $P'_5$  observable– of the  $B \rightarrow K^*\mu\mu$  decay. The PDD approach favours the fully left-handed NP coupling, i.e.  $C_{9,\ell} = -C_{10,\ell}$ , and allows for NP coupling to electrons, while the PMD exclusively predicts the muonic resolution [503, 505].

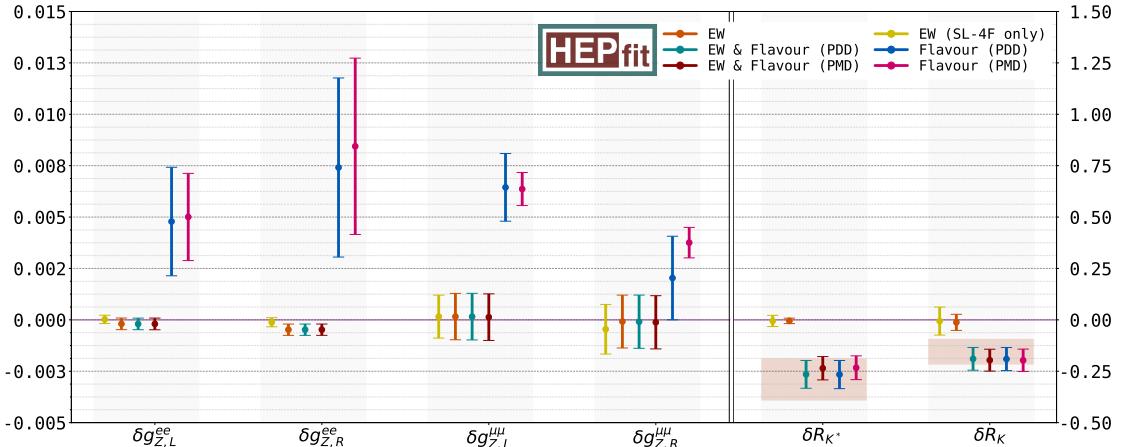
Flavour data seem to predict deviations in the  $Z$  coupling modifiers, which implies tension between the flavour fits and EWPO exacerbated by the PMD modelling of the long distance QCD effects. This tension between  $B$ - anomalies and EW data reaches  $3(6)\sigma$  level for PDD(PMD). Of course, introducing a tree-level resolution of the  $b \rightarrow s\ell\ell$  anomalies would decouple EW sensitive SMEFT operators from the four-fermion operators required for these anomalies, but it will not be compatible with the MFV ansatz. In fact, the size of flavour violation introduced by the tree-level resolution of the  $B$  anomalies brings any model with such structure to the brick of exclusion by other flavour observables [312, 486–488].



**Figure 10.2.** The marginalised fit results of the Wilson coefficients considered in the scenarios detailed in subsection 10.1.2. The central points denotes the mean of the marginalised posterior distribution, while the error bars are the 68% CI constraint of the Wilson coefficients. (Note the different scaling in the axes quantifying the size of the bounds presented in each half of the figure.) This figure is published in [168].



**Figure 10.3.** The correlation matrix resulting from the Bayesian fit of the Wilson coefficient of the operators listed in eqs. (10.3), (10.9), (10.10) in the EW scenario introduced in subsection 10.1.2. The two distinct groups of Wilson coefficients associated to leptonic and quark interactions are remarked as “leptons” and “quarks”, respectively. This figure is published in [168].



**Figure 10.4.** Fit results following the same convention as Figure 10.2 for the  $Z$  boson coupling modifiers for the muons and electrons, as well as the lepton universality violating ratios, see eq. (10.11), with the red boxes indicating the region selected by the experimental measurements of  $R_{K,(K^*)}$ . This figure is published in [168].

A global fit with the 21 coefficients, combining both flavour and EW data is the way to reach consensus between what is required by  $b \rightarrow s\ell\ell$  observations resolution and EW precision tests. Similarly to the flavour scenario, the fits are performed for PDD in teal

and PMD in red in Figure 10.2 and Figure 10.4. In these scenarios, the tension between EWPO and flavour data is lifted as the deviation from the SM  $Z$ -couplings remain within the EW data predictions, also the LUV generated by the Wilson coefficients matches the experimental observations. The resolution comes from deviation of  $C_{\phi L(e)}^{\ell\ell}$  and the 4F operators  $\mathcal{O}_{(L)eu}^{\ell\ell 33}$  from the SM hypothesis.

Another interesting observation from the global fit can be seen in the network graphs in [?], where in the EW only fits the SL-4F Wilson coefficients are degenerate with the Higgs-lepton bilinear currents  $C_{\phi L(e)}^{\ell\ell}$ , having Pearson's correlation of  $\rho \sim -1.0$ . This degeneracy is broken once both EW and flavour data are taken into an account, as seen in the lowe panels of this figure. The breaking of the degeneracy is the reason for the shifts in the posterior distributions of  $C_{\phi L(e)}^{\ell\ell}$  from the SM hypothesis.

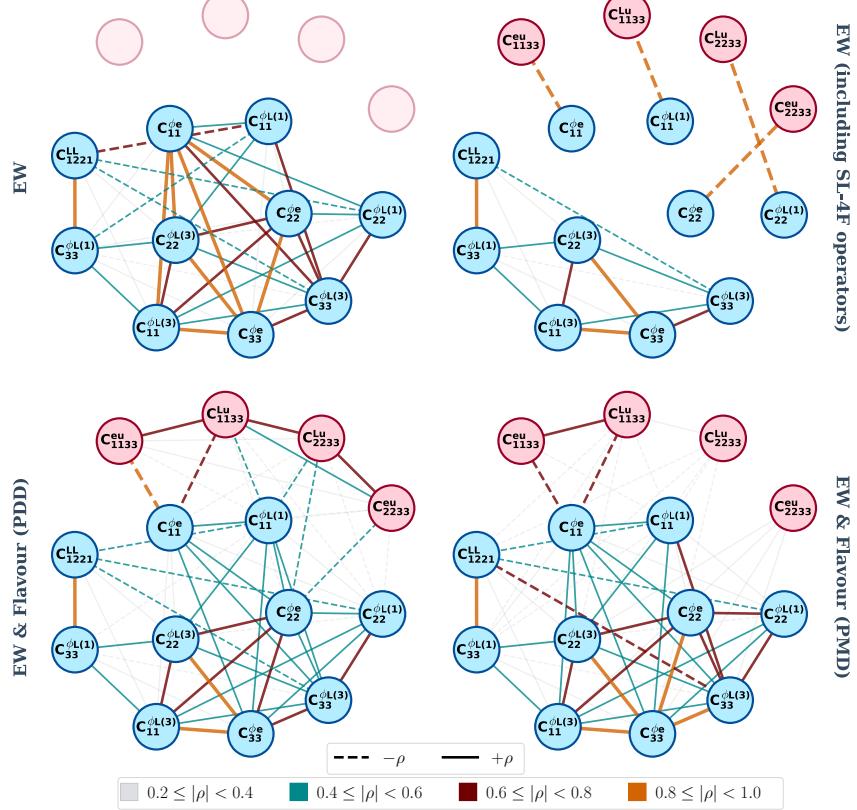
It is not necessary to invoke all of the 21 SMEFT operators considered in the EW & Flavour scenario in order to have a resolution for the flavour anomalies and EWPO. A simpler picture using two or four operator suffices to fulfil the experimental need to explain LUV and respect EW measurements. This picture contain the fully left-handed operator,  $\mathcal{O}_{LQ}^{\ell\ell 23}$  and  $\mathcal{O}_{\phi L}^{(1)\ell\ell}$ . The former operator would be generated at a loop-level by  $\mathcal{O}_{Lu}^{\ell\ell 33}$ , while the latter at tree-level. This minimalist SMEFT approach would then include only  $\mathcal{O}_{\phi L}^{(1)\ell\ell}$  and  $\mathcal{O}_{Lu}^{\ell\ell 33}$ , and  $\ell = \mu, e$ . So the model could involves either muons, electrons or both of them.

In Figure 10.6, the EWPO (grey), flavour with PDD (orange) and combined (magenta) fits for this minimal SMEFT model. For the muonic (left) and electronic (right) solutions. We observe that the tension between EWPO and  $b \rightarrow s\ell\ell$  data if individual fits were preformed, whilst this is resolved in the combined fit. However, this induces correlation between the four-fermion operator  $\mathcal{O}_{Lu}^{\ell\ell 33}$  and the one involving the Higgs-doublet and lepton bilinears. This model also obeys MFV assumptions, which protects it from other flavour observables. However, as mentioned earlier, the  $B$  anomalies has to be explained at one-loop level. Finally, note that the role played here by  $\mathcal{O}_{Lu}^{\ell\ell 33}$  could be shared, in part, with  $\mathcal{O}_{eu}^{\ell\ell 33}$ , depending on how much departure is actually required from the fully left-handed solution to  $B$  anomalies. As already noted, this fact critically depends on the information stemming from the angular analysis of  $B \rightarrow K^* \mu\mu$  [505]. On general grounds, to relieve the bounds from EWPO, the presence of  $\mathcal{O}_{eu}^{\ell\ell 33}$  would also necessitate sizeable NP effects from  $\mathcal{O}_{\phi e}^{\ell\ell}$ , thus leaving us with a maximum of four needed operators to explain the flavour anomalies without being excluded by EWPO or including complex flavour structures.

## 10.2 $Z'$ with vector-like partners

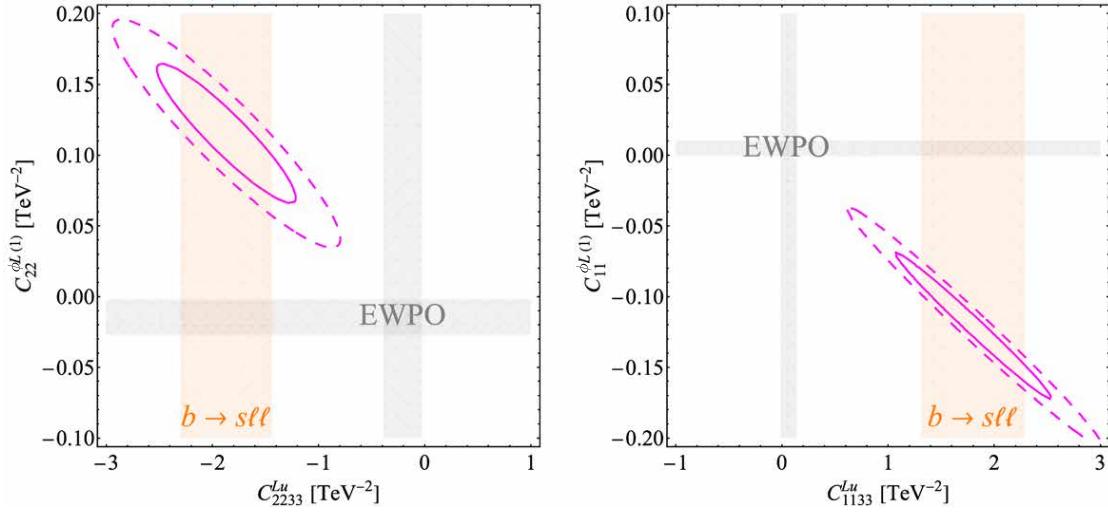
Exhilarated by the SMEFT fit and the consequent simplified model discussed in the previous section, we discuss some UV-complete models that explain the  $B$ -anomalies at loop-level; without adding extra flavour violation; and respects the EWPO bounds.

The first model that satisfies these requirements is based on a  $Z'$  model published in ref. [517]. This model is a simple extension of the SM gauge group by an additional



**Figure 10.5.** Network plot of the correlation between the Wilson coefficients considered in this study. The upper left panel shows the correlations from the **EW** fit, the upper right panel for the same fit but with the SL-4F Wilson coefficients included in the fit. The lower panel includes the flavour anomalies data in the **EW+Flavour** scenario, in which the degeneracy is broken. The lower left panel is for the PDD hadronic effects, while the lower right one is for the PMD case. This figure is published in [168].

Abelian group  $U(1)_X$  with a corresponding gauge boson  $X_\mu$  identified as the  $Z'$ . All of the SM fields have no  $X$  charge. This gauge symmetry is spontaneously broken by a vev of an additional scalar singlet  $S$ , which gives a mass to the  $Z'$  boson  $m_{Z'} = g_X \langle S \rangle$ . A top-quark  $T$ , (VLQ) and a muonic partners  $M$  are added as well. These two fields mix with the top  $u_3$  and muon  $L_2$  via a Yukawa interaction terms with the scalar field  $S$ . Kinematic mixing between the  $Z'$  and the SM  $Z$  boson, as well as the between the Higgs and the new scalar are assumed to be negligible. The new fields and their representation under the SM plus the new gauge group are summarised in Table 10.1.



**Figure 10.6.** A minimal solution for the flavour anomalies within SMEFT, while respecting EWPO. The left panel shows the four-fermion operators involving the muon and on the right the electronic solution is shown. EWPO fits are the grey regions, while the  $b \rightarrow s\ell\ell$  measurements fits with PDD ansatz are highlighted by the orange bands. The combined fit's  $1$  and  $2\sigma$  contours are magenta coloured. This plot has been published in [168].

This model is completely characterized by eight new parameters: the gauge coupling  $g_X$ , the mass  $\mu_S$  and quartic  $\lambda_S$  of the renormalizable potential of  $S$ , the new Yukawa couplings  $Y_{T,M}$ , here taken to be real, and the vector-like mass-term parameters  $M_{T,M}$ . In particular, the Lagrangian of the model contains the following terms:

$$\begin{aligned} \mathcal{L} = & M_T \bar{T}_R T_L + M_M \bar{M}_R M_L + Y_t \bar{u}_3 \tilde{\phi}^\dagger Q_3 \\ & + Y_T \bar{u}_3 T_L S + Y_\mu \bar{e}_2 \phi^\dagger L_2 + Y_M \bar{M}_R L_2 S + \text{h.c.} . \end{aligned} \quad (10.12)$$

From this Lagrangian, we can read off the mixing terms between the SM fields and

Particle/Field	$G_{\text{SM}} \otimes U(1)_X$	multiplicity
<b>VL fermions</b>		
T	$(\mathbf{3}, \mathbf{2})_{Y=\frac{1}{6}, X=-1}$	
M	$(\mathbf{1}, \mathbf{2})_{\frac{1}{2}, -1}$	
<b>Gauge boson</b>		
$X_\mu$	$(\mathbf{1}, \mathbf{1})_{0,0}$	
<b>Scalar</b>		
S	$(\mathbf{1}, \mathbf{1})_{0,1}$	

**Table 10.1.** The added fields of this model and their representation under the SM gauge group and the new  $U(1)_X$ . Note that the new charge assignment here is not unique, and the model would produce the same phenomenology with different but consistent assignment.

vector-like (VL) partners.<sup>5</sup> The spontaneous symmetry breaking of  $U(1)_X$  is achieved in the way discussed at the beginning of this thesis in section 2.1 by a non-vanishing vev  $\langle S \rangle^2 = -\mu_S^2/(2\lambda_S) \equiv \eta^2 \neq 0$ , that implies the following fermionic mixing patterns:

$$\begin{aligned} \text{top sector: } & \left( \begin{array}{cc} \bar{u}_3 & \bar{T}_R \end{array} \right) \left( \begin{array}{cc} \frac{Y_t v}{\sqrt{2}} & \frac{Y_T \eta}{\sqrt{2}} \\ 0 & M_T \end{array} \right) \left( \begin{array}{c} U_3 \\ T_L \end{array} \right) + \text{h.c.}, \\ \text{muon sector: } & \left( \begin{array}{cc} \bar{e}_2 & \bar{M}_R \end{array} \right) \left( \begin{array}{cc} \frac{Y_\mu v}{\sqrt{2}} & 0 \\ \frac{Y_M \eta}{\sqrt{2}} & M_M \end{array} \right) \left( \begin{array}{c} E_2 \\ M_L \end{array} \right) + \text{h.c.}, \end{aligned} \quad (10.13)$$

where  $U_i$  ( $E_i$ ) indicates the  $Q_i$ -component ( $L_i$ -component) with weak isospin  $1/2$  ( $-1/2$ ). Using the determinant and trace of the squared mass matrices, one can easily show that

<sup>5</sup>Note that upon an opposite  $U(1)_X$  charge assignment for the vector-like fermionic partners than the one implicitly assumed, one should replace in eq. (10.12)  $S$  with  $S^\dagger$ .

the eigenvalues  $m_{t,\mathsf{T}}$  and  $m_{\mu,\mathsf{M}}$  must satisfy [517]:

$$\begin{aligned} m_{t,\mu} m_{\mathsf{T},\mathsf{M}} &= \frac{1}{\sqrt{2}} Y_{t,\mu} v M_{\mathsf{T},\mathsf{M}}, \\ m_{t,\mu}^2 + m_{\mathsf{T},\mathsf{M}}^2 &= M_{\mathsf{T},\mathsf{M}}^2 + \frac{1}{2} (Y_{t,\mu} v)^2 + \frac{1}{2} (Y_{\mathsf{T},\mathsf{M}} \eta)^2, \end{aligned} \quad (10.14)$$

that in the decoupling limit clearly yield:  $m_{t,\mu} \simeq Y_{t,\mu} v / \sqrt{2}$ ,  $m_{\mathsf{T},\mathsf{M}} \simeq M_{\mathsf{T},\mathsf{M}}$ . Defining for the top sector the rotation matrix from the interaction to the mass basis following the convention:

$$\begin{pmatrix} t_{R(L)} \\ \mathsf{T}'_{R(L)} \end{pmatrix} = \begin{pmatrix} \cos \theta_{R(L)}^t & -\sin \theta_{R(L)}^t \\ \sin \theta_{R(L)}^t & \cos \theta_{R(L)}^t \end{pmatrix} \begin{pmatrix} u_3(U_3) \\ \mathsf{T}_{R(L)} \end{pmatrix}, \quad (10.15)$$

and doing similarly for the muonic sector, the mixing angles between SM fields,  $t$  and  $\mu$ , and their partner mass eigenstates,  $\mathsf{T}'$  and  $\mathsf{M}'$ , can be conveniently expressed in terms of the dimensionless ratios  $\xi_{\mathsf{T},\mathsf{M}}$  and  $\varepsilon_{t,\mu}$ :

$$\begin{aligned} \tan 2\theta_R^t &= \frac{2\xi_{\mathsf{T}}}{\xi_{\mathsf{T}}^2 - \varepsilon_t^2 - 1}, \quad \tan 2\theta_L^t = \frac{2\varepsilon_t}{\xi_{\mathsf{T}}^2 - \varepsilon_t^2 + 1}, \text{ with } \varepsilon_t \equiv \frac{Y_t v}{Y_{\mathsf{T}} \eta}, \quad \xi_{\mathsf{T}} \equiv \frac{\sqrt{2} M_{\mathsf{T}}}{\eta Y_{\mathsf{T}}}; \\ \tan 2\theta_R^\mu &= \frac{2\varepsilon_\mu}{\xi_{\mathsf{M}}^2 - \varepsilon_\mu^2 + 1}, \quad \tan 2\theta_L^\mu = \frac{2\xi_{\mathsf{M}}}{\xi_{\mathsf{M}}^2 - \varepsilon_\mu^2 - 1}, \text{ with } \varepsilon_\mu \equiv \frac{Y_\mu v}{Y_{\mathsf{M}} \eta}, \quad \xi_{\mathsf{M}} \equiv \frac{\sqrt{2} M_{\mathsf{M}}}{\eta Y_{\mathsf{M}}}. \end{aligned} \quad (10.16)$$

In a perturbative expansion in  $\varepsilon_{t,\mu}$ , eq. (10.16) clearly shows that the mixing in the top sector proceeds mainly through  $\tan \theta_R^t \simeq 1/\xi_{\mathsf{T}}$ , while in the muonic sector one has  $\tan \theta_L^\mu \simeq 1/\xi_{\mathsf{M}}$  and negligible  $\tan \theta_R^\mu$ .

Hence, for  $\varepsilon_{t,\mu}/\xi_{\mathsf{T},\mathsf{M}} = Y_{t,\mu} v / \sqrt{2} M_{\mathsf{T},\mathsf{M}} < 1$ , the leading couplings of the  $Z'$  boson to the SM fields correspond to right-handed top-quarks and to left-handed muons as well as neutrinos, these couplings are given by

$$g_{Z't_R} = g_X \sin^2 \theta_R^t = \frac{g_X}{1 + \xi_{\mathsf{T}}^2} + \mathcal{O}\left(\varepsilon_t^2/\xi_{\mathsf{T}}^2\right), \quad (10.17)$$

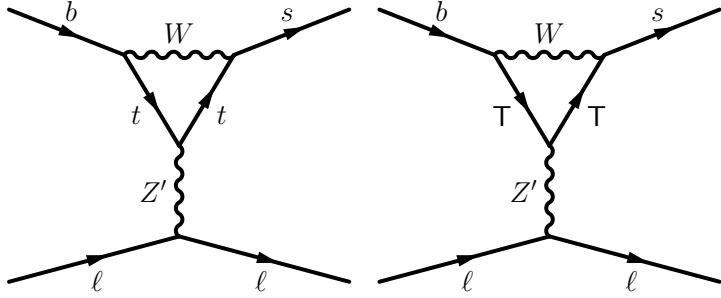
$$g_{Z'\mu_L(\nu)} = g_X \sin^2 \theta_L^\mu = \frac{g_X}{1 + \xi_{\mathsf{M}}^2} + \mathcal{O}\left(\varepsilon_\mu^2/\xi_{\mathsf{M}}^2\right), \quad (10.18)$$

with  $g_{Z't_L(\mu_R)}$  contributing only at order  $\varepsilon_{t(\mu)}^2/\xi_{\mathsf{T}(\mathsf{M})}^2$ . The  $b \rightarrow sll$  anomalies can be explained in this model via penguin (also box) diagrams with LUV as shown in Figure 10.7. Since the  $Z'$  couples to the muons and not the electrons, LUV is generated at loop-level.

### 10.2.1 SMEFT matching and constraints

Integrating out the  $Z'$  will generate the operator  $\mathcal{O}_{Lu}^{2233}$  with the matching condition:

$$C_{Lu}^{2233} = -\frac{g_{Z't_R} g_{Z'\mu_L}}{m_{Z'}^2} \simeq -\frac{1}{(1 + \xi_{\mathsf{T}}^2)(1 + \xi_{\mathsf{M}}^2)\eta^2}, \quad (10.19)$$



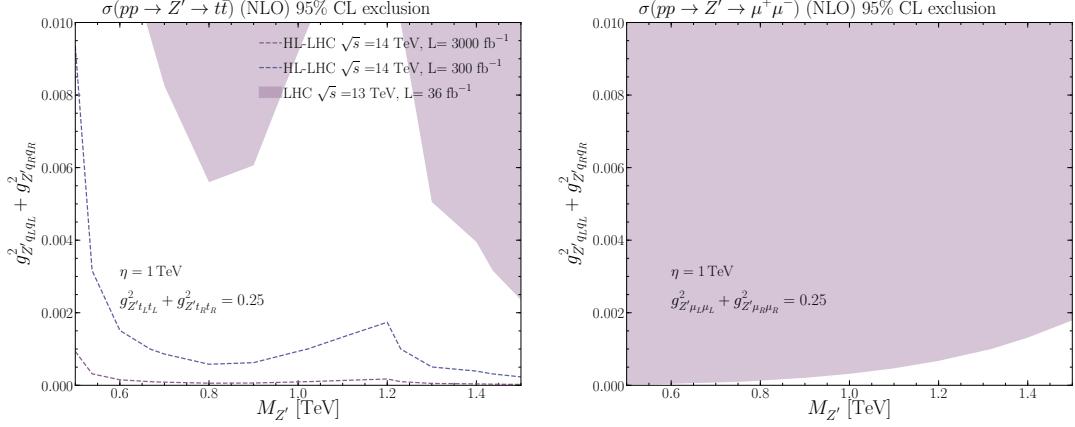
**Figure 10.7.** FCNC penguins with LUV emerging from the  $Z'$  model, explaining the  $b \rightarrow s\ell\ell$  anomalies at loop-level. The penguin diagrams with the top-partners in the loop are the dominant ones.

together with four-fermion operators built of  $t_R$  or  $\mu_L, \nu$  fields that can be potentially probed at collider and by experimental signatures like  $\nu$ -trident production. From eq. (10.19) it is clear that in order to have  $|C_{Lu}^{2233}| \sim 2 \text{ TeV}^{-2}$  as required by the fit in Figure 10.6, the SSB of the new gauge group needs to happen at a scale close to the EW, namely  $\eta \lesssim \text{TeV}$ ;<sup>6</sup> for  $m_{Z'} \sim \text{TeV}$  this leads to a natural coupling  $g_X \gtrsim 1$ .

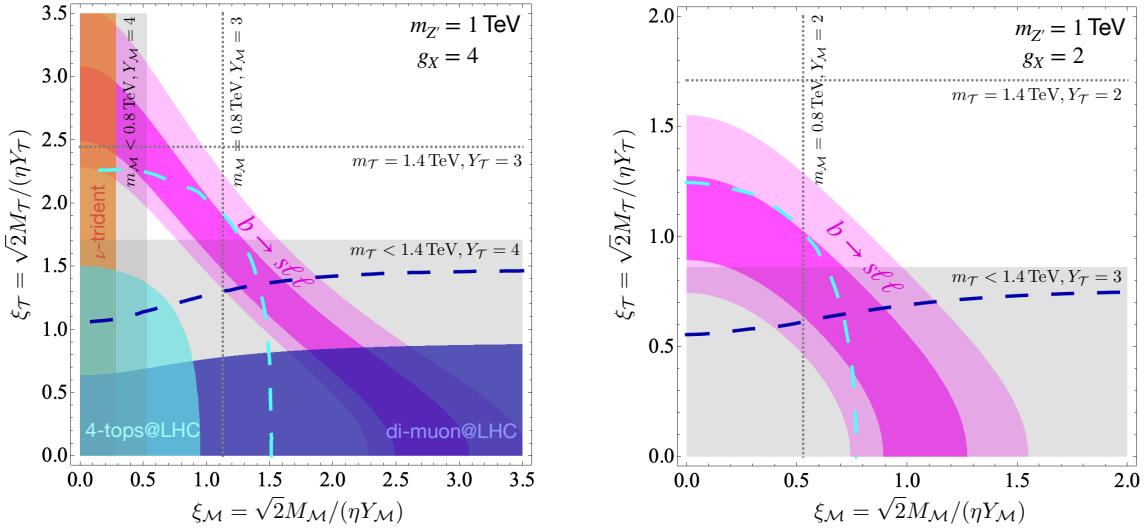
The main collider constraints come from the production of top pair  $pp \rightarrow Z' \rightarrow t\bar{t}$ , with the most stringent bounds at the time of conducting this work came from ATLAS [551]. In addition to the resonant di-muon searches [552]. In Figure 10.8 these searches are projected onto our model, with the choice of  $\eta = 1 \text{ TeV}$  and other parameters chosen to be preferred by the  $B$  anomalies observables, we see that the constraints on this model are dominated by the resonant di-muon searches. The theoretical prediction of the resonant top pair and di-muon production via gluon fusion  $gg \rightarrow Z' \rightarrow t\bar{t}/\mu\mu$  in this model has been calculated at NLO using the two-loop triangle calculations presented in chapter 6. Further emphasizing the importance of the calculation of Higher order corrections for the gluon fusion  $gg \rightarrow Z^*$  for NP processes on top of Higgs measurements.

In Figure 10.9 collects the constraints on this model, starting with the  $1\sigma$  region corresponding to the explanation of  $B$  anomalies via eq. (10.19) in the parameter space  $\xi_{T,M}$ , fixing the gauge coupling  $g_X = m_{Z'}/\eta$  for a tentative  $Z'$  gauge boson at the TeV scale and the VEV of the new scalar field  $S$  set to  $\eta = 250 \text{ GeV}$  and  $\eta = 500 \text{ GeV}$  in the left and right panel, respectively. In the same plot, the collider searches are also presented, re-interpreted from the results presented in ref. [522]. The bounds from neutrino-trident production performed in [553] constrain small  $\xi_M$ , where the 95 %CL bounds are shown in the orange band of the plot. The top-philic  $Z'$  is predominantly produced at tree-level in association with top-pair, in the blue region we show the 95% high- $p_T$  constraint stemming from the recasting of the  $pp \rightarrow \mu^-\mu^+t\bar{t}$  search at ATLAS [552, 554]. The cyan contours are constraints coming from four-tops production analysis from CMS [555], see

<sup>6</sup>Note that even for masses as low as  $\mu_S \sim \mathcal{O}(v)$ , for  $\eta \simeq v$  and  $\lambda_S \sim \mathcal{O}(1)$ , the interactions of  $S$  do not alter the phenomenology discussed here since the largest  $S$ -generated effects are still suppressed as  $\mathcal{O}(\varepsilon_t^2/\xi_T^2)$ .



**Figure 10.8.** Direct searches for  $Z'$  using top pair production [551] and di-muon searches [551], the gluon fusion cross-sections for this model were calculated at NLO using the results of the two-loop triangle calculations of the process  $gg \rightarrow Z^*$  preformed in chapter 6.



**Figure 10.9.** Collective of the constraints on the  $Z'$  model. the magenta regions show the 68% and 95% CL constraints from  $b \rightarrow s\ell\ell$  anomalies, while the rest are the collider searches re-interpreted from the ones in ref. [522]. The projections for the early HL-LHC (at  $300 \text{ fb}^{-1}$ ) constraints are shown as dashed lines. Gray regions underlie the parameter space where the mass of the vector-like partner lies below current collider limits for a fixed Yukawa coupling as explicitly reported, while dashed lines show the corresponding shift of the limit due to a smaller value of the same type of Yukawa coupling. The left panel is for  $\eta = m'_Z/4$  and the right panel is for  $\eta = m'_Z/2$ . This figure is published in [168].

ref. [522] for more details. These constraints' prospects for the early runs of the HL-LHC at  $300 \text{ fb}^{-1}$  are also explored and indicated with the dashed lines. One can see that the model benchmark shown in the right panel of Figure 10.9 old promising potential for discovery at the HL-LHC. Finally, in the same figure, fixing the partner Yukawa coupling to  $\mathcal{O}(1)$  values as reported in the two panels, we mark in gray the region corresponding to the bound on the mass of the vector-like partner expected from collider, taken to be  $m_T = 1.4 \text{ TeV}$  from the search at ATLAS in ref. [556], and  $m_M = 0.8 \text{ TeV}$  from the CMS analysis of ref. [557].

### 10.2.2 Expanding the model

For this model to survive EWPO constraints, it needs to induce  $O_{\phi L}^{(1) 22}$  with the same correlation patterns observed in Figure 10.6. In principle, it is possible to achieve that by inducing tree-level  $z - Z'$  mixing by charging  $S$  under  $U_Y(1)$  in addition to  $U_X(1)$ , thereby inducing some misalignment in the weak hypercharge  $Y$ . However, this will create a tree-level (Drell Yan-like) resonant di-muon production enhancement, far beyond what is allowed by current collider searches [552]. Therefore, this mechanism is not possible. In order to accommodate for EW precision constraints, this model needs to be expanded further by including new VL leptonic states, like the ones discussed in refs. [558, 559]. These new degrees of freedom are interesting by their own merit, in particular with resolving the anomaly associated with  $(g - 2)_\mu$  [465, 560], also for neutrino mass source and other interesting collider phenomenology [561, 562].

The simplest resolution can be accomplished by the inclusion of two new vector-like muonic partners: a singlet under  $SU(2)_L$ ,  $S_Y$ , and a triplet of  $SU(2)_L$ ,  $T_Y$ , where in both cases the subscript  $Y$  denotes the hypercharge of the fermion. Since they are VL fermions, they have a mass term, thus adding new parameters  $M_{S_Y, T_Y}$ . Their mixing with the SM leptons comes -like M- from Yukawa term, which for  $Y = 0$  is given by

$$\mathcal{Y}_{S_0} \bar{S}_{0,R} \tilde{\phi}^\dagger L_2 + \mathcal{Y}_{T_0} \bar{T}_{0,R}^A \tau^A \tilde{\phi}^\dagger L_2 + \text{h.c.} , \quad (10.20)$$

Another possibility of interest may be the one of replacing in eq. (10.20)  $\tilde{\phi}$  with the Higgs doublet,  $\phi$ , and involve then the pair of vector-like partners with hypercharge  $Y = 1$ . The matching condition for these new fields produces the needed SMEFT operators, and the values and sign of the corresponding Wilson coefficients is given by the interplay between these fields [559, 560] of the form:

$$\begin{aligned} C_{\phi L}^{(1) 22} &= \frac{\mathcal{Y}_{S_0}^2}{4M_{S_0}^2} - \frac{\mathcal{Y}_{S_1}^2}{4M_{S_1}^2} + \frac{3\mathcal{Y}_{T_0}^2}{4M_{T_0}^2} - \frac{3\mathcal{Y}_{T_1}^2}{4M_{T_1}^2} , \\ C_{\phi L}^{(3) 22} &= -\frac{\mathcal{Y}_{S_0}^2}{4M_{S_0}^2} - \frac{\mathcal{Y}_{S_1}^2}{4M_{S_1}^2} + \frac{\mathcal{Y}_{T_0}^2}{4M_{T_0}^2} + \frac{\mathcal{Y}_{T_1}^2}{4M_{T_1}^2} . \end{aligned} \quad (10.21)$$

In order to obtain the needed value and sign of  $C_{\phi L}^{(1) \ 22} \sim 0.1$  but also vanishing or negligible  $C_{\phi L}^{(3) \ 22}$  some tuning between the singlet and the  $Y = 0$  triplet is needed, although this tuning is stable under the RGE once generated at the NP scale.

Working under the PDD ansatz, it is possible to consider that the model would couple to the electron instead of the muon. Not much would change in terms of the particle content of this model, except for opposite charge assignment to get correct signs for the Wilson coefficients of  $O^{Lu}$  and  $O_{\phi L}^{(1)}$  seen in the right panel off [Figure 10.6](#), but making the electron and top partners having opposite  $X$  charges and then making the proper adjustments to the Lagrangian. A final comment is needed for the electron scenario reported in the right panel of [Figure 10.6](#), that involves opposite signs for the Wilson coefficients of  $O^{Lu}$  and  $O_{\phi L}^{(1)}$  discussed so far. For the former, we note that the sign highlighted in the matching in eq. (10.19) follows from having assumed the same sign for the charge of the vector-like top and muon partners under  $U(1)_X$ . For what concerns the generation of  $C_{\phi L}^{(1) \ 11} < 0$ , according to eq. (10.21) one needs to correlate once again the contribution stemming from  $S_0$ , or from  $S_1$ , with the effect coming from a  $SU(2)_L$  triplet, that now needs to be identified with  $T_1$ , namely the triplet of hypercharge  $Y = 1$ .

## 10.3 Leptoquark scenarios

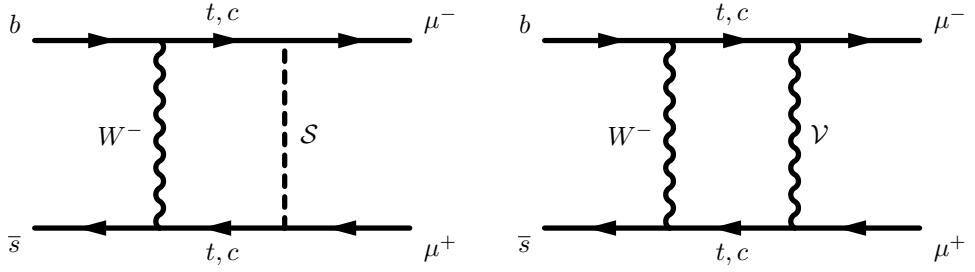
Leptoquark models generically predicted in grand unified theories (GUTs) [563, 564]. They typically generate baryon violating process that lead to proton decay, which are severely constrained. However, in light of the simplified SMEFT model discussed earlier [Figure 10.6](#), it is possible to introduce leptoquarks (LQ) that couple non-universally to quark and lepton generations. These LQs are within reach of colliders, and not pushed to the GUT scale like their flavour-universal counterparts. Actually, they are potential candidates for explaining the flavour anomalies [522, 565]. Such models typically involve highly non-trivial flavour structure. For a comprehensive survey of LQ models, see for instance [314, 513, 567–569].

In this section, we limit ourselves to LQs that generate  $C_{Lu}^{\ell\ell 33}$  and  $C_{eu}^{\ell\ell 33}$ , and introduce LUV in  $b \rightarrow s\ell\ell$  transition at loop-level as shown in [Figure 10.10](#). With that in mind, only a handful of LQs models remain, they are summarised in [Table 10.2](#). From this table, we can recognise the suitable models that explain the  $B$  anomalies at one loop as predicted in [Figure 10.6](#). Unlike the  $Z'$  model, we have distinct cases for NP coupling to the electron or the muon. The case of the vector LQ  $\mathcal{V}^\mu \sim (\bar{\mathbf{3}}, \mathbf{2}, -1/6)$  for LUV effects originating from electron couplings, and the scalar  $\mathcal{S} \sim (\bar{\mathbf{3}}, \mathbf{2}, -7/6)$  for the ones associated to muons. The interaction terms of interest are:

$$\mathcal{L}_{\mathcal{V}\bar{f}f} = \tilde{\lambda}_{te} \bar{L}_1^c \gamma_\mu u_3 i\tau^2 \mathcal{V}^\mu + \text{h.c.} , \quad \mathcal{L}_{\mathcal{S}\bar{f}f} = \lambda_{t\mu} \bar{L}_2 u_3 \mathcal{S} + \text{h.c.}, \quad (10.22)$$

When the LQs are integrated out, we arrive to the matching to SMEFT

$$C_{Lu}^{1133} = +\frac{|\tilde{\lambda}_{te}|^2}{M_{\mathcal{V}}^2} , \quad C_{Lu}^{2233} = -\frac{|\lambda_{t\mu}|^2}{2M_{\mathcal{S}}^2} . \quad (10.23)$$

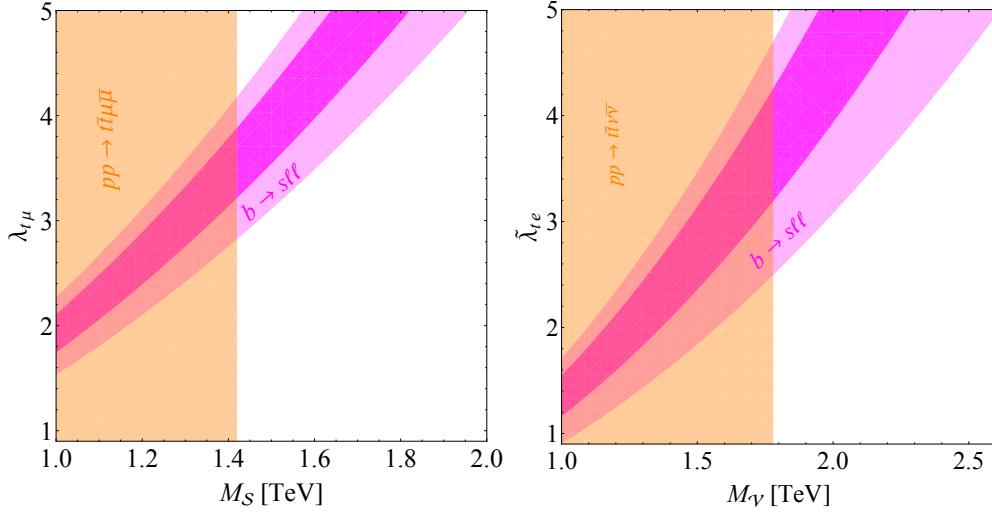


**Figure 10.10.** Box diagrams generated by scalar  $\mathcal{S}$  (left) and vector  $\mathcal{V}$  leptoquarks, of the  $b \rightarrow s\ell\ell$  transition with LUV.

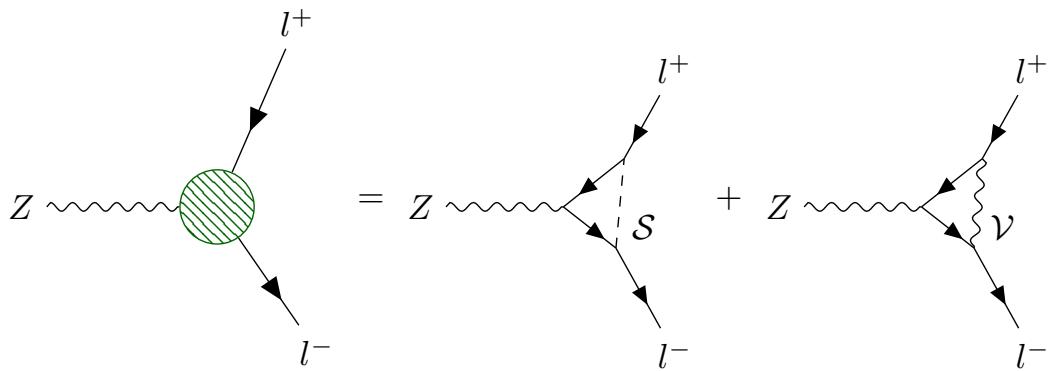
Vector LQ: $\mathcal{V}^\mu$	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	Comments
$\bar{L}_\ell \gamma_\mu (\tau^A) Q_3 \mathcal{V}^{\mu(A)}$	$(\bar{\mathbf{3}}, \mathbf{1} \text{ or } \mathbf{3})_{-\frac{2}{3}}$	not of interest
$(\mathcal{V}^\mu)^\dagger \bar{e}_\ell^c \gamma_\mu Q_3$	$(\bar{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}}$	not of interest
$\bar{L}_\ell^c \gamma_\mu u_3 i\tau^2 \mathcal{V}^\mu$	$(\bar{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	generates $C_{\ell\ell 33}^{Lu} > 0$
$\bar{e}_\ell \gamma_\mu u_3 \mathcal{V}^\mu$	$(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{5}{3}}$	generates $C_{\ell\ell 33}^{eu} < 0$
Scalar LQ: $\mathcal{S}$		
$\bar{L}_\ell (\tau^A) (i\tau^2) Q_3^c \mathcal{S}^{\dagger(A)}$	$(\bar{\mathbf{3}}, \mathbf{1} \text{ or } \mathbf{3}, 1/3)$	not of interest
$\bar{e}_\ell Q_3 i\tau^2 \mathcal{S}$	$(\bar{\mathbf{3}}, \mathbf{2})_{-\frac{7}{6}}$	not of interest
$\bar{L}_\ell u_3 \mathcal{S}$	$(\bar{\mathbf{3}}, \mathbf{2})_{-\frac{7}{6}}$	generates $C_{\ell\ell 33}^{Lu} < 0$
$\bar{e}_\ell^c u_3 \mathcal{S}$	$(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	generates $C_{\ell\ell 33}^{eu} > 0$

**Table 10.2.** Scalar and vector LQ interactions under scrutiny: LQs of interest for this analysis have to generate the dimension-six operators  $O_{Lu,eu}^{\ell\ell 33}$ . This table is published in [168].

The LQ models are simpler in terms of added fields and parameters than the  $Z'$ , this also reflects on their collider constraints. The scalar LQ with muonic coupling, has only one constraint coming from  $pp \rightarrow t\bar{t}\mu\mu$ , while the vector electro-philic LQ has bounds from  $t\bar{t}2\nu$  searches. These bounds are based on the dedicated collider study of ref. [570], and highlighted in orange in Figure 10.11, thy are also confronted with the  $b \rightarrow s\ell\ell$  predictions shown as magenta regions. These leptoquarks can generate o  $C_{\phi L}^{(1)} \ell\ell$  only at loop-level, see Figure 10.12, which is insufficient to fulfil the requirements of the flavour + EWPO fit Figure 10.6. Hence, the addition of the extra singlet and triplet leptonic partners discussed in the previous section is again needed to survive EWPO constraints.



**Figure 10.11.** Constraints on the mass and LQ coupling with the muon for the scalar LQ model  $\mathcal{S}$  on the left panel; while on the right panel the vector eletro-philic LQ model parameters constraints are shown. The orange band shown the collider bounds based on comprehensive analysis found in ref. [570]. The magenta regions show the models phase space at 68% and 95% CL that explains the flavour anomalies at one-loop.



**Figure 10.12.** The leptoquarks considered can only generate  $C_{\phi L}^{(1)} \ell\ell$  via loop matching.

## 10.4 Conclusion

This chapter addressed the  $b \rightarrow s\ell\ell$  anomalies' resolution based on NP models that fall under the assumptions of MFV. Which required that LUV effects being generated at loop-level. Moreover, the interplay between EWPO and these anomalies was portrayed in Figure 10.2 and, supported with Figure 10.4.

The global SMEFT fit preformed hints that a unifying solution for both EWPO and LUV anomalies can be achieved with the inclusion of the right operators. Furthermore, the picture can be simplified by only including a minimum of 2-4 SMEFT operators, portrayed in Figure 10.6. Like any multivariate analysis, the correlation amongst the coefficients played an important role in finding the proper resolution of the EWPO and flavour observables conundrum.

Inspired by the simplified SMEFT model, we have discussed a top-phillic  $Z'$  model with top and muon vector-like partners. Moreover, an alternative, simpler models based on leptoquarks can also produce the  $B$  anomalies at loop-level. Both models can be amended to include muonic or electronic solutions as in the SMEFT simplified scenario. For the  $Z'$  model, the top and lepton partners need to have the same  $X$  charge for the muon case, while they need to carry an opposite one for the electronic NP coupling. The LQ models are different for the muonic and electronic, the former is compatible with scalar and the latter with a vector LQs.

Both of these models required the inclusion of correlated pair of vector-like leptons, a  $SU(2)_L$  singlet and a triplet, in order to realise the minimal EFT scenario depicted on Figure 10.6. We observe that the existence of these particles may be independently motivated by the heavy new dynamics underlying the origin of neutrino masses and/or by a tentative explanation of the  $(g - 2)_\mu$  anomaly [465, 560].

Future measurements of  $B$  decays by the LHCb and Belle-II is expected to reach precision regime in the upcoming years [499, 571]. These measurements, in addition to high-energy ones at linear colliders [443, 534] will reveal more on the nature of these anomalies and their connection with Higgs physics. This is already hinted by the global fits done here with the current data predicting NP Higgs operators like  $C_{\phi L/e}$ . Understanding how observables from different sectors correlate is an essential key to understanding the nature of NP underlying these anomalies amongst others.

# 11 Conclusion

Constraints on Higgs observables are deeply intertwined with top and flavour physics. This has been highlighted throughout the entirety of this thesis, and the literature reviewed within.

The era of Higgs precision measurements is on the horizon, prompting the inclusion of Higher order corrections to Higgs processes, which requires improved techniques for their calculation. An example of these techniques is the  $p_T$  expansion, first employed to obtain an analytic form for the Higgs pair virtual corrections [11]. This techniques was used in chapter 6, for obtaining the QCD two-loop corrections of the gluon fusion component of  $Zh$ , which is the main source of this process's theoretical uncertainty. The true power of this method is seen when combined with Padé approximants to bridge it with other expansions in order to obtain an analytic form for the virtual corrections covering the entire invariant mass spectrum [341].

The use of higher-order calculations in SMEFT opened the potential for probing the Higgs trilinear self-coupling [149–152, 154], and show connections between four-top operators and EWPO [174]. The nexus between the SMEFT four-heavy quark operators and the Higgs self-coupling is explored in chapter 7, via the inclusion of NLO SMEFT effects in single-Higgs rates.

Precision Higgs measurements will not be complete without the observation of Higgs pair production, the aspired jewel process of the HL-LHC, which carries the most potential for measuring the elusive Higgs self-coupling, consequently revealing the shape of the Higgs potential.

In chapter 9, we have demonstrated the potential of this process in constraining other “difficult” Higgs observable; its interaction with the light quarks. Then, we proceeded to treating Higgs pair production as a multivariate problem and employed aspects of Interpretable machine learning [20]. Using a BDT-classifier interfaced with Shapley values as an interpretability layer, we were able to constrain the trilinear coupling along with the up- and down-quarks Yukawa coupling enhancements, within SMEFT. The interpretability allowed for optimised classifier as well as added physics understanding and confidence. The constraints project for HL-LHC on up-quark Yukawa coupling enhancement obtained from this analysis is the most stringent amongst the proposed channels. The harmony of amongst different observables within SMEFT framework extends towards the newly-discovered flavour anomalies involving  $b \rightarrow s\ell\ell$  transitions, which was explored in chapter 10. When these anomalies are confronted with EWPO, a marked tension of up to  $6\sigma$  is observed between the data from  $B$  decays and EWPO, further highlighting the interplay between these anomalies and EWPO [505, 506, 514–516, 521, 524, 525].

This conundrum can be resolved by a global fit involving EW and flavour data. A

minimalist SMEFT model, assuming no new flavour spurions involved would generate LUV at loop-level, and involves operators from the top and Higgs sectors. We have later showcased UV-complete models ascertained from this fit to explain these flavour anomalies. The first model was based on  $Z'$  models with top-partner VLQ [517], while the other is a set of top-phillic leptoquark models.

Although there is not clear direction for BSM physics hinted by current LHC data, the SMEFT offers a versatile anatomy for new physics searches, with minimal assumptions on its degrees of freedom. More importantly, the correlations seen within SMEFT operators from different sectors could be as a starting point from data-driven model building, after all, physics is .. an *experimental science* !

# Appendices



# A Details of $Zh$ calculation

## A.1 Orthogonal Projectors in $gg \rightarrow ZH$

In this appendix I present the explicit expressions of the projectors  $\mathcal{P}_i^{\mu\nu\rho}$  appearing in eq.(6.2). The projectors are all normalized to 1. They are:

$$\mathcal{P}_1^{\mu\nu\rho} = \frac{m_Z}{\sqrt{2}s'p_T^2} \left[ p_1^\nu \epsilon^{\mu\rho p_1 p_2} - p_2^\mu \epsilon^{\nu\rho p_1 p_2} + q_t^\mu \epsilon^{\nu\rho p_2 p_3} \right] \quad (\text{A.1})$$

$$+ q_u^\nu \epsilon^{\mu\rho p_1 p_3} + s' \epsilon^{\mu\nu\rho p_2} - s' \epsilon^{\mu\nu\rho p_1} \Big], \quad (\text{A.2})$$

$$\mathcal{P}_2^{\mu\nu\rho} = \frac{1}{\sqrt{2}s'p_T} \left[ q_u^\nu \epsilon^{\mu\rho p_1 p_3} + q_t^\mu \epsilon^{\nu\rho p_2 p_3} \right], \quad (\text{A.3})$$

$$\begin{aligned} \mathcal{P}_3^{\mu\nu\rho} &= \frac{\sqrt{3}}{2s'p_T} \left[ s' \epsilon^{\mu\nu\rho p_1} + s' \epsilon^{\mu\nu\rho p_2} - p_1^\nu \epsilon^{\mu\rho p_1 p_2} - p_2^\mu \epsilon^{\nu\rho p_1 p_2} \right. \\ &+ (q_u^\nu \epsilon^{\mu\rho p_1 p_3} - q_t^\mu \epsilon^{\nu\rho p_2 p_3}) \left( \frac{1}{3} + \frac{m_Z^2}{p_T^2} \right) \\ &\left. + \frac{m_Z^2}{p_T^2} (q_t^\mu \epsilon^{\nu\rho p_2 p_1} - q_u^\nu \epsilon^{\mu\rho p_1 p_2}) \right], \end{aligned} \quad (\text{A.4})$$

$$\mathcal{P}_4^{\mu\nu\rho} = \frac{m_Z}{\sqrt{2}s'p_T^2} \left[ q_t^\mu (\epsilon^{\nu\rho p_2 p_1} - \epsilon^{\nu\rho p_2 p_3}) - q_u^\nu (\epsilon^{\mu\rho p_1 p_2} - \epsilon^{\mu\rho p_1 p_3}) \right], \quad (\text{A.5})$$

$$\mathcal{P}_5^{\mu\nu\rho} = \frac{1}{\sqrt{6}s'p_T} \left[ q_t^\mu \epsilon^{\nu\rho p_2 p_3} - q_u^\nu \epsilon^{\mu\rho p_1 p_3} \right], \quad (\text{A.6})$$

$$\begin{aligned} \mathcal{P}_6^{\mu\nu\rho} &= \frac{1}{s'p_T} \left[ g^{\mu\nu} \epsilon^{\rho p_1 p_2 p_3} + s' \epsilon^{\mu\nu\rho p_3} + p_1^\nu \epsilon^{\mu\rho p_2 p_3} - p_2^\mu \epsilon^{\nu\rho p_1 p_3} - \frac{s'}{2} \epsilon^{\mu\nu\rho p_2} \right. \\ &+ \frac{1}{2} (p_1^\nu \epsilon^{\mu\rho p_1 p_2} + p_2^\mu \epsilon^{\nu\rho p_1 p_2} + q_u^\nu \epsilon^{\mu\rho p_1 p_3} - q_t^\mu \epsilon^{\nu\rho p_2 p_3} - s' \epsilon^{\mu\nu\rho p_1}) \\ &\left. + \frac{m_Z^2}{2p_T^2} (q_t^\mu \epsilon^{\nu\rho p_2 p_1} - q_u^\nu \epsilon^{\mu\rho p_1 p_2} + q_u^\nu \epsilon^{\mu\rho p_1 p_3} - q_t^\mu \epsilon^{\nu\rho p_2 p_3}) \right], \end{aligned} \quad (\text{A.7})$$

where we defined  $q_t^\mu = (p_3^\mu - \frac{t'}{s'}p_2^\mu)$  and  $q_u^\nu = (p_3^\nu - \frac{u'}{s'}p_1^\nu)$  and we used the shorthand notation  $\epsilon^{\mu\nu\rho p_2} \equiv \epsilon^{\mu\nu\rho\sigma} p_2^\sigma$ .

Using these projectors we obtained the relations between the form factors  $\mathcal{A}_i$  defined

in eq.(6.2) and those defined in section 2 of ref.[230]:

$$\mathcal{A}_1 = \frac{p_T^2}{2\sqrt{2}m_Z(p_T^2 + m_Z^2)} \left[ (t' + u')F_{12}^+ - (t' - u')F_{12}^- \right], \quad (\text{A.8})$$

$$\begin{aligned} \mathcal{A}_2 &= -\frac{p_T}{2\sqrt{2}(p_T^2 + m_Z^2)} \left[ (t' + u')F_{12}^+ - (t' - u')F_{12}^- \right. \\ &\quad \left. - \frac{p_T^2 + m_Z^2}{2s'} ((t' + u')F_3^+ - (t' - u')F_3^-) \right], \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} \mathcal{A}_3 &= \frac{p_T}{2\sqrt{3}(p_T^2 + m_Z^2)} \left[ (t' + u')F_{12}^- - (t' - u')F_{12}^+ \right. \\ &\quad \left. + (p_T^2 + m_Z^2)(F_2^- + F_4) \right], \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} \mathcal{A}_4 &= -\frac{m_Z}{2\sqrt{2}(p_T^2 + m_Z^2)} \left[ (t' + u')F_{12}^- - (t' - u')F_{12}^+ \right. \\ &\quad \left. + (p_T^2 + m_Z^2) \left( (1 - \frac{p_T^2}{m_Z^2})F_2^- + 2F_4 \right) \right], \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned} \mathcal{A}_5 &= \frac{p_T}{2\sqrt{6}(p_T^2 + m_Z^2)} \left[ (t' + u')F_{12}^- - (t' - u')F_{12}^+ \right. \\ &\quad \left. + (p_T^2 + m_Z^2) \left( 4(F_2^- + F_4) + \frac{3}{2s'} \left( (t' + u')F_3^- - (t' - u')F_3^+ \right) \right) \right], \end{aligned} \quad (\text{A.12})$$

$$\mathcal{A}_6 = \frac{p_T}{2} F_4. \quad (\text{A.13})$$

## A.2 Two-loop Results

The NLO amplitude can be written in terms of three contributions, namely the two-loop 1PI triangle, the two-loop 1PI box and the reducible double-triangle diagrams,

$$\mathcal{A}_i^{(1)} = \mathcal{A}_i^{(1,\Delta)} + \mathcal{A}_i^{(1,\square)} + \mathcal{A}_i^{(1,\bowtie)}. \quad (\text{A.14})$$

In this section, the exact analytic results for the triangle and double triangle topologies are presented.

The two-loop triangle results are

$$\mathcal{A}_1^{(1,\Delta)} = \frac{p_T^2 (\hat{s} - \Delta_m)}{4\sqrt{2}m_Z} \frac{\mathcal{K}_t^{(2l)}}{(p_T^2 + m_Z^2)}, \quad (\text{A.15})$$

$$\mathcal{A}_2^{(1,\Delta)} = -\frac{p_T (\hat{s} - \Delta_m)}{4\sqrt{2}} \frac{\mathcal{K}_t^{(2l)}}{(p_T^2 + m_Z^2)}, \quad (\text{A.16})$$

$$\mathcal{A}_3^{(1,\Delta)} = \frac{p_T (\hat{t} - \hat{u})}{4\sqrt{3}} \frac{\mathcal{K}_t^{(2l)}}{(p_T^2 + m_Z^2)}, \quad (\text{A.17})$$

$$\mathcal{A}_4^{(1,\Delta)} = -\frac{m_Z (\hat{t} - \hat{u})}{4\sqrt{2}} \frac{\mathcal{K}_t^{(2l)}}{(p_T^2 + m_Z^2)}, \quad (\text{A.18})$$

$$\mathcal{A}_5^{(1,\Delta)} = -\frac{p_T (\hat{t} - \hat{u})}{4\sqrt{6}} \frac{\mathcal{K}_t^{(2l)}}{(p_T^2 + m_Z^2)}, \quad (\text{A.19})$$

$$\mathcal{A}_6^{(1,\Delta)} = 0, \quad (\text{A.20})$$

where the  $\mathcal{K}_t^{(2l)}$  function is defined in eq.(4.11) of ref.[264]. While the double-triangle for-factors are found to be.

$$\mathcal{A}_1^{(1,\bowtie)} = -\frac{m_t^2 p_T^2}{4\sqrt{2} m_Z (m_Z^2 + p_T^2)^2} \left[ F_t(\hat{t}) (G_t(\hat{t}, \hat{u}) - G_b(\hat{t}, \hat{u})) + (\hat{t} \leftrightarrow \hat{u}) \right], \quad (\text{A.21})$$

$$\mathcal{A}_2^{(1,\bowtie)} = \frac{m_t^2 p_T}{4\sqrt{2} (m_Z^2 + p_T^2)^2} \left[ F_t(\hat{t}) (G_t(\hat{t}, \hat{u}) - G_b(\hat{t}, \hat{u})) + (\hat{t} \leftrightarrow \hat{u}) \right], \quad (\text{A.22})$$

$$\mathcal{A}_3^{(1,\bowtie)} = \frac{m_t^2 p_T}{4\sqrt{3} \hat{s} (m_Z^2 + p_T^2)^2} \left[ (m_h^2 - \hat{t}) F_t(\hat{t}) (G_t(\hat{t}, \hat{u}) - G_b(\hat{t}, \hat{u})) - (\hat{t} \leftrightarrow \hat{u}) \right], \quad (\text{A.23})$$

$$\begin{aligned} \mathcal{A}_4^{(1,\bowtie)} = & -\frac{m_t^2}{4\sqrt{2} m_Z \hat{s}^2 (m_Z^2 + p_T^2)^2} \left[ (m_Z^2 (m_h^2 - \hat{t})^2 \right. \\ & \left. - \hat{t} (m_Z^2 - \hat{u})^2) F_t(\hat{t}) (G_t(\hat{t}, \hat{u}) - G_b(\hat{t}, \hat{u})) - (\hat{t} \leftrightarrow \hat{u}) \right], \end{aligned} \quad (\text{A.24})$$

$$\begin{aligned} \mathcal{A}_5^{(1,\bowtie)} = & -\frac{m_t^2 p_T}{4\sqrt{6} \hat{s} (m_Z^2 + p_T^2)^2} \left[ (4m_Z^2 - \hat{s} - 4\hat{u}) F_t(\hat{t}) (G_t(\hat{t}, \hat{u}) - G_b(\hat{t}, \hat{u})) \right. \\ & \left. - (\hat{t} \leftrightarrow \hat{u}) \right], \end{aligned} \quad (\text{A.25})$$

$$\mathcal{A}_6^{(1,\bowtie)} = 0, \quad (\text{A.26})$$

where

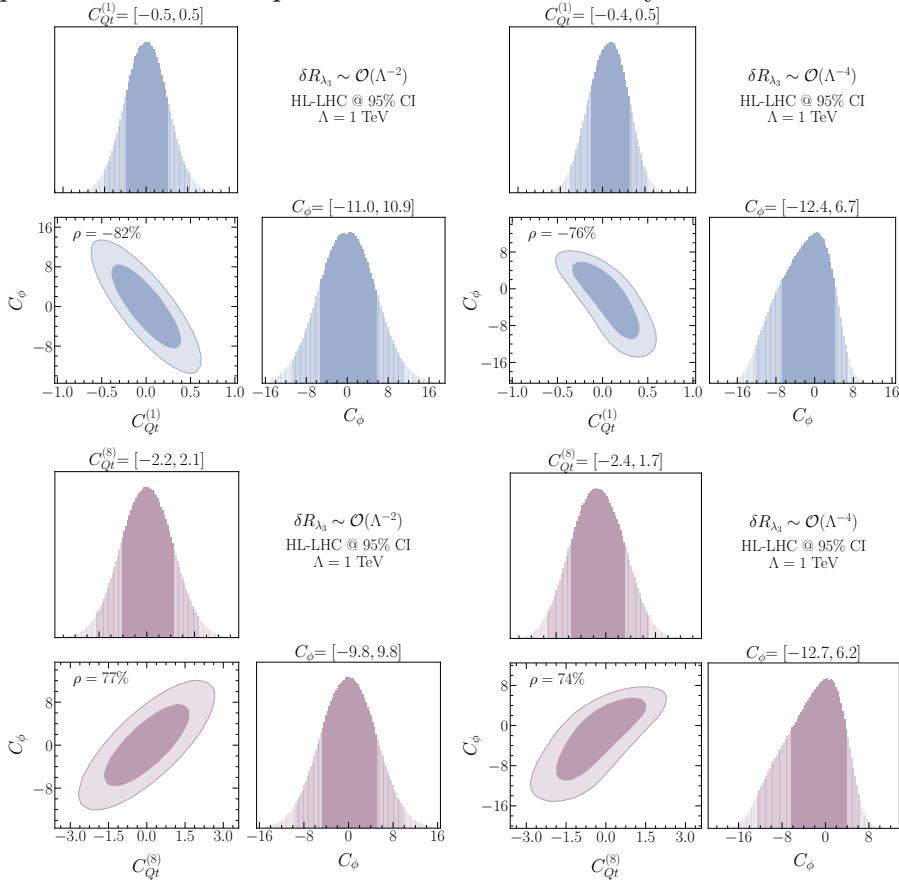
$$\begin{aligned} F_t(\hat{t}) &= \frac{1}{(m_h^2 - \hat{t})^2} \left[ 2\hat{t} \left( B_0(\hat{t}, m_t^2, m_t^2) - B_0(m_h^2, m_t^2, m_t^2) \right) \right. \\ &\quad \left. + (m_h^2 - \hat{t}) \left( (m_h^2 - 4m_t^2 - \hat{t}) C_0(0, m_h^2, \hat{t}, m_t^2, m_t^2, m_t^2) - 2 \right) \right], \end{aligned} \quad (\text{A.27})$$

$$\begin{aligned} G_x(\hat{t}, \hat{u}) &= (m_z^2 - \hat{u}) \left[ m_z^2 \left( B_0(\hat{t}, m_x^2, m_x^2) - B_0(m_z^2, m_x^2, m_x^2) \right) \right. \\ &\quad \left. + (\hat{t} - m_z^2) \left( 2m_x^2 C_0(0, \hat{t}, m_z^2, m_x^2, m_x^2, m_x^2) + 1 \right) \right]. \end{aligned} \quad (\text{A.28})$$

## B Two-parameter fits of four-fermion operators and $C_\phi$ for HL-LHC

I present here in [Figure B.1](#) and [Figure B.2](#) the fit results for the SMEFT four heavy quark operators with the Higgs trilinear self-coupling modifier  $C_\phi$  for the HL-LHC projections by CMS [128, 303] as an extension of the results presented in [chapter 7](#).

The expected constraints improve from the Run-II ones by a factor of

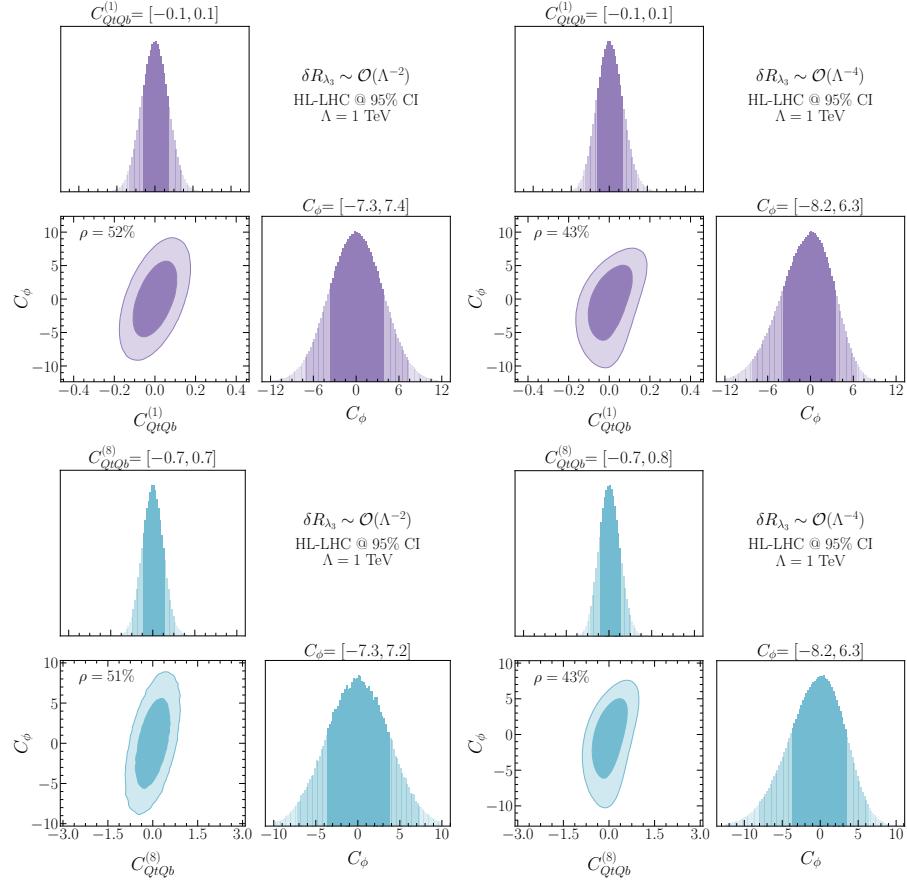


**Figure B.1.** The posterior distributions of the HL-LHC projections fits for  $C_\phi$  with  $C_{Qt}^{(1)}$  (up) and  $C_\phi$  with  $C_{Qt}^{(8)}$  (down). With 68% and 95% highest density posterior contours indicated. The limits shown on top of the plots indicate the 95% CI's. Plots on the left are made for the fully linearised  $\delta R_{\lambda_3}$ , while the ones on the right include the quadratic effects.

$$\sim \sqrt{\frac{\mathcal{L}_{\text{HL-LHC}}}{\mathcal{L}_{\text{Run-II}}}}, \quad (\text{B.1})$$

as expected, from statistical analysis. This comes from the adaptation of the  $S_2$  uncertainties scheme.

The linear fits show similar correlation patterns to the ones from the Run-II in [Figure 7.5](#) and [Figure 7.6](#). However, the quadratic  $R_{\lambda_3}$  scheme shows strong correlation between  $C_\phi$  and the four-heavy quark Wilson coefficients, while this is not seen in the Run-II fits. The implication of these correlations is worsened projected constraints on negative  $C_\phi$  values in the two-parameter fits.



**Figure B.2.** The posterior distributions of the HL-LHC projections fits for  $C_\phi$  with  $C_{QtQb}^{(1)}$  (up) and  $C_\phi$  with  $C_{QtQb}^{(8)}$  (down). With the same annotations as in [Figure B.1](#).

## C Prospects for Higgs pair production at the FCC

The analysis done in section 9.5 for Higgs pair at the HL-LHC can be repeated for the future hadron circular collider (FCC-hh), with centre-of-mass energy of 100 TeV and integrated luminosity of  $30 \text{ ab}^{-1}$ . The Higgs pair events and the backgrounds were generated in the same manner for the FCC-hh as for the HL-LHC. Moreover, the ML analysis and the consequent statistical framework were also identical to the ones done for the HL-LHC. With the caveat of using the 14 TeV  $K$ -factors for the 100 TeV cross-section scaling, as the 100 TeV  $K$ -factors were not available for all processes. I should note that we have explicitly checked that at least within the SM, for Higgs pair production via gluon fusion the difference is of  $\mathcal{O}(1\%)$  [206] and hence small. An example output of the BDT-classifier for the FCC-hh is shown for the SM signal as a confusion matrix in Table C.1

Preforming a single-parameter fit on the light Yukawa modifiers, we see the projected bounds on these operators at FCC-hh are given by

$$\begin{aligned} C_{u\phi}^{MV}(\kappa_u^{MV}) &= [-0.012, 0.011] \quad ([ -57.8, 54.7 ]), \\ C_{d\phi}^{MV}(\kappa_d^{MV}) &= [-0.012, 0.012] \quad ([ -26.3, 28.4 ]). \end{aligned} \quad (\text{C.1})$$

These projected bounds for FCC-hh are an order of magnitude better than those for HL-LHC. In addition, the bounds on  $C_{u\phi}$  and  $C_{d\phi}$  are numerically the same displaying a much greater improvement in the bounds on  $C_{d\phi}$  than on  $C_{u\phi}$  at the higher energy collider. The results of the FCC-hh analysis are summarised in Table C.2

From this table, we observe that the constraints on the trilinear self-coupling reach the precision-level of  $\sim 4\%$  at 68% CI. As for light Yukawa, the up-type will reach  $\mathcal{O}(50)$

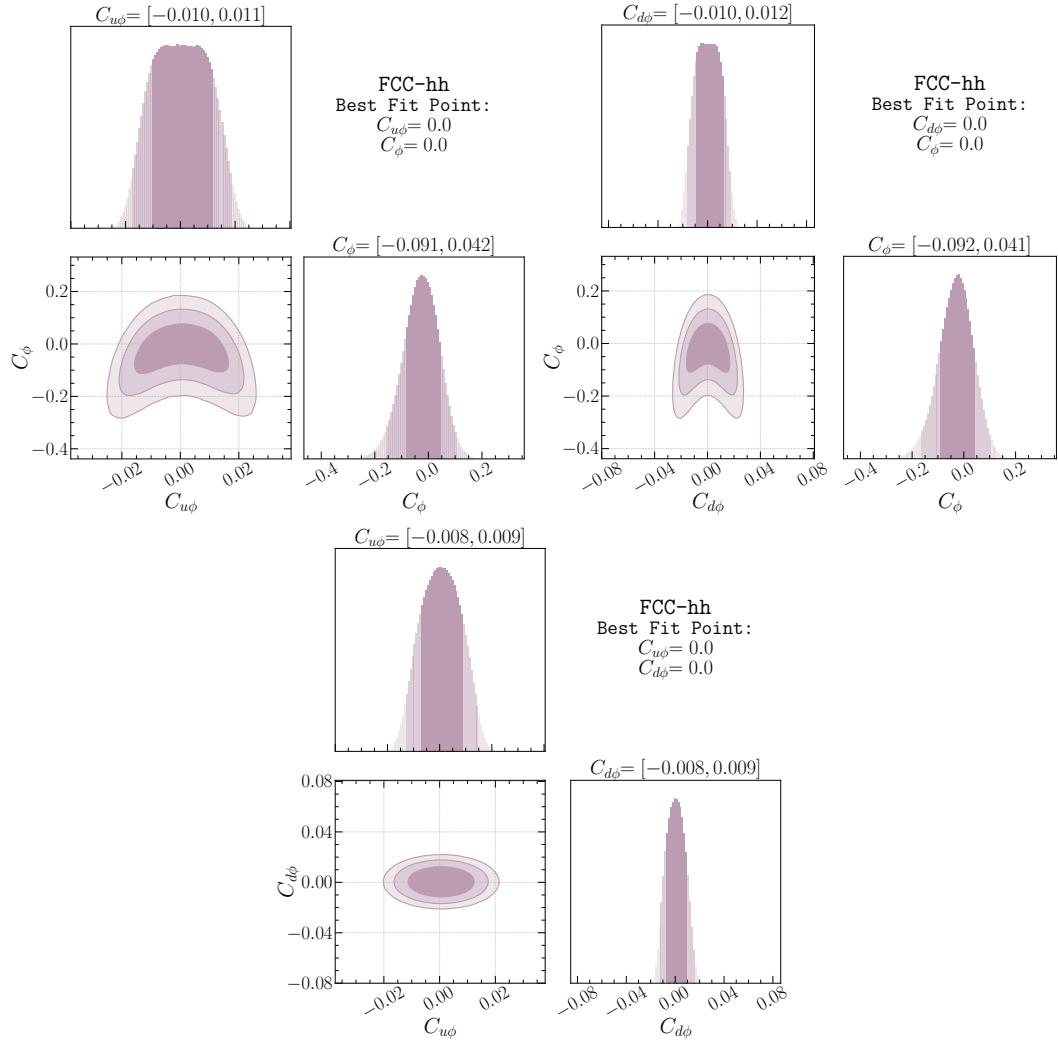
Predicted no. of events at FCC-hh							
Actual no. of events	Channel	$hh_{\text{tri}}^{\text{ggF}}$	$hh_{\text{tri}}^{\text{ggF}}$	$hh_{\text{box}}^{\text{ggF}}$	$Q\bar{Q}h$	$b\bar{b}\gamma\gamma$	total
	$hh_{\text{tri}}^{\text{ggF}}$	3,579	1,303	2,372	4,697	337	12,288
	$hh_{\text{int}}^{\text{ggF}}$	13,602	7,300	17,075	24,716	1523	64,216
	$hh_{\text{box}}^{\text{ggF}}$	14,534	11,416	35,988	415,26	1,996	105,460
	$Q\bar{Q}h$	29,611	12,355	23,279	1,238,266	214,564	1,518,075
	$b\bar{b}\gamma\gamma$	45,574	22,290	26,213	150,935	227,142	24,317,657
	$Z_j$	10.95	31.22	111.1	737.7	4,743	

**Table C.1.** The confusion matrix output of the trained BDT five-channel classifier for the FCC-hh analysis. This table is antireligious to for the HL-LHC ??

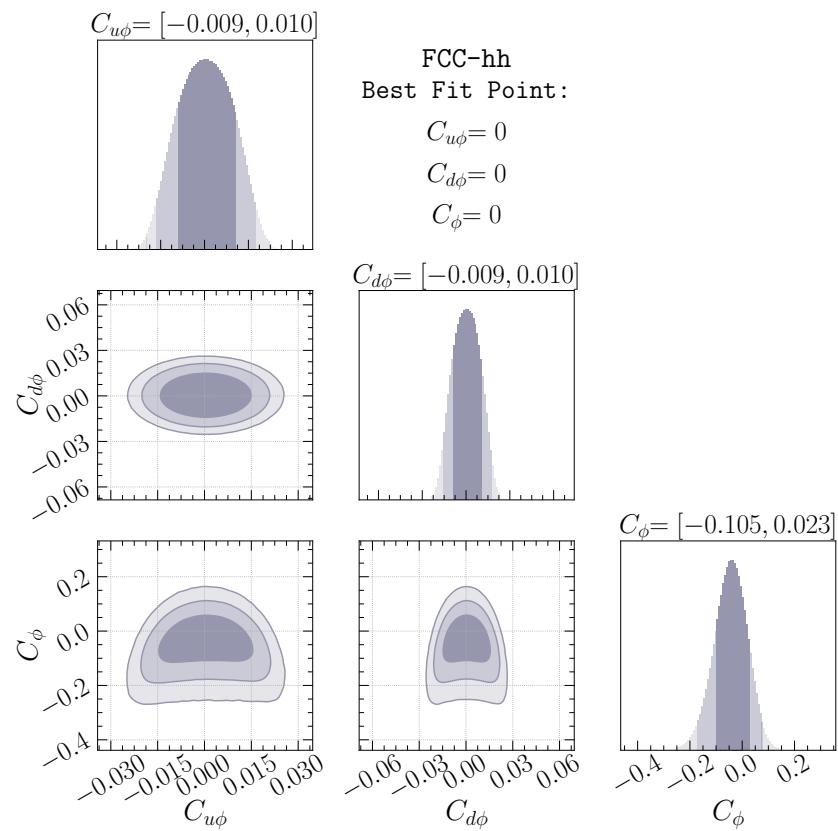
Operators	$C_{u\phi}$	$C_{d\phi}$	$C_\phi$		$\kappa_u$	$\kappa_d$	$\kappa_\lambda$
$\mathcal{O}_\phi$	—	—	[-0.066, 0.064]		—	—	[0.97, 1.03]
$\mathcal{O}_{u\phi}$	[-0.012, 0.011]	—	—		[-57.8, 54.7]	—	—
$\mathcal{O}_{d\phi}$	—	[-0.012, 0.011]	—		—	[-26.3, 28.4]	—
$\mathcal{O}_{u\phi} \& \mathcal{O}_\phi$	[-0.010, 0.011]	—	[-0.091, 0.042]		[-52, 49]	—	[0.98, 1.04]
$\mathcal{O}_{d\phi} \& \mathcal{O}_\phi$	—	[-0.010, 0.012]	[-0.092, 0.041]		—	[-24, 26]	[0.98, 1.04]
$\mathcal{O}_{u\phi} \& \mathcal{O}_{d\phi}$	[-0.008, 0.009]	[-0.008, 0.009]	—		[-42, 39]	[-19, 19]	—
All	[-0.009, 0.010]	[-0.009, 0.010]	[-0.105, 0.023]		[-47, 44]	[-21, 21]	[0.99, 1.05]

**Table C.2.** The  $1\sigma$  bounds on  $C_{u\phi}$ ,  $C_{d\phi}$  and  $C_\phi$  from one-, two- and three-parameter fits for FCC-hh with  $30\text{ ab}^{-1}$  integrated luminosity.

times the SM value showing significant improvement over the HL-LHC, and  $\mathcal{O}(20 - 30)$  for the down Yukawa. The posterior distributions for the two-parameter fits are shown in Figure C.1, while the three-parameter analysis in Figure C.2. These plots show more significant correlation patterns between  $C : \phi$  and the light Yukawa modifiers compared to the HL-LHC fits in Figure 9.11 and Figure 9.12



**Figure C.1.** Constraints on pairs of Wilson coefficients for  $C_\phi$ ,  $C_{u\phi}$  and  $C_{d\phi}$  for FCC-hh with 30  $\text{ab}^{-1}$  integrated luminosity.



**Figure C.2.** Three parameter fits with  $C_{u\phi}$ ,  $C_{d\phi}$  and  $C_\phi$ , using projection for the FCC-hh.

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