First a quiz for an hour or so...

1 - Cauchy Schwartz Inequality

Remember that $|\mathbf{v} \cdot \mathbf{w}| \leq ||\mathbf{v}||||\mathbf{w}||$ with $\mathbf{v}, \mathbf{w}$ being vectors.

So we can reformulate our question in this form, letting $\mathbf{w} = (1, 1, 1, 1, ..., 1) \in \mathbb{R}$ Then $||\mathbf{w}|| = \sqrt{n}$ for this n-dimensional vector.

Taking $\mathbf{v} = (a_1, a_2, ..., a_n)$, we can rewrite the inequality given as:

$$\frac{|a_1 + a_2 + ... + a_n|}{\sqrt{n}} \leq \sqrt{a_1^2 + a_2^2 + ... + a_n^2}$$
$$\implies |a_1 + a_2 + ... + a_n| \leq \sqrt{n} \cdot \sqrt{a_1^2 + a_2^2 + ... + a_n^2}$$
$$\implies |\mathbf{v} \cdot \mathbf{w}| \leq ||\mathbf{w}|| \cdot ||\mathbf{v}||$$

So by Cauchy-Schwartz, the above holds.

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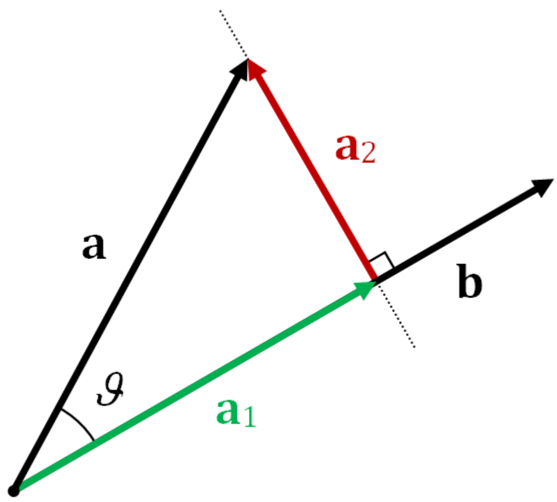

Know that the shortest distance from a point to a line is the length of the vector perpendicular to $\mathbf{b}$ that goes through point p. To calculate this, remember the definition of the projection of vector $\mathbf{b}$ onto vector $\mathbf{a}$ (In the picture above it's $\mathbf{a_1}$).

We know that $\mathbf{a} + (-\mathbf{a_2}) = \mathbf{a_1} = proj_{\mathbf{b}}\mathbf{a}$ in the above picture. We can derive $\mathbf{a_2}$ in this picture and then calculate the magnitude by first finding what $\mathbf{a}$ is, and then finding the projection of it onto $\mathbf{b}$.

So $\mathbf{a} = (0, 2, -3) - (1, -1, -2) = (-1, 3, -1)$ where the second vector is a point contained in the vector between $(1, -1, 2)$ and $(2, -2, -2)$.

and $a_2 = proj_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{b}||^2}\mathbf{b} = \frac{(-1,3,-1)\cdot(1,-1,0)}{2}(1, -1, 0) = -2(1, -1, 0)$ with $b = (1, -1, 0)$ (calculate this by subtracting point $(2, -2, -2)$ and $(1, -1, -2)$)

Now calculate $\mathbf{a_2} = \mathbf{a} - proj_{\mathbf{b}}(\mathbf{a}) = (-1, 3, -1) + 2(1, -1, 0) = (1, 1, -1)$
Now the magnitude of $\mathbf{a_2}$ is $||\mathbf{a_2}|| = \sqrt{3}$, which is the shortest distance between our point and line.

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Want a linear equation such that (1,3) is a solution of this system and this system expresses a line which is parallel to vector [2,-1].

Remember we can express (in vector form) our equation of the line as $\mathbf{r} = \mathbf{p} + \lambda\mathbf{v}$, where $\mathbf{v}$ is parallel to $\mathbf{r}$ and λ is a scalar.

So let $\mathbf{r} = [r_x, r_y] = [1,3] - \lambda[2,-1]$. We can check that (1,3) is contained in this line by letting $\lambda = 0$. You can check whether [2,-1] is parallel the classic way (via dot product if you want).

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Find the equation of a plane going through points $(1,3,-2)$, $(0,6,2)$, $(3,4,-3)$.

Consider two vectors $\mathbf{v} = (1-0, 3-6, -2-2) = (1,-3,-4)$ and $\mathbf{w} = (3-0, 3-4, -3-2) = (3,-1,-5)$.

Find their cross product to get an equation of a vector mutually perpendicular to the two vectors above.

$$\mathbf{v} \times \mathbf{w} = (1,-3,-4) \times (3,-1,-5) = \begin{vmatrix} -3 & -4 \\ -1 & -5 \end{vmatrix} \mathbf{i} + \begin{vmatrix} 1 & -4 \\ 3 & -5 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -3 \\ 3 & -1 \end{vmatrix} \mathbf{k} = (11,7,8)$$

Then we have the coefficients for our equation of the plane. Now, to guarantee that this plane intersects one of these points (thereby intersecting all of them with our choice of coefficients) we can express the plane as: 11x+7(y-6)+8(z-2) = 0 (plug in the given points and see if they satisfy this equation to verify)

11

Let A, B be two n×n matrices. Then we know that $tr(A+B) = \sum_{i=1}^{n} a_{ii} + b_{ii}$ (as matrix addition is element wise).

$tr(A+B) = \sum_{i=1}^{n} a_{ii} + b_{ii} = \sum_{i=1}^{n} a_{ii} + \sum_{i=1}^{n} b_{ii} = tr(A) + tr(b)$ as wanted.

And also note that scalar multiplication of matrices multiplies every entry. So we know that for matrix A and scalar c :

$tr(cA) = \sum_{i=1}^{n} c \cdot a_{ii} = c \cdot \sum_{i=1}^{n} a_{ii} = c \cdot tr(A)$ as wanted.