Communication Through Coherently Controlled Quantum Channels

Alastair A. Abbott

partially based on joint work with
Julian Wechs, Dominic Horsman, Mehdi Mhalla and Cyril Branciard

LIG, Grenoble, 30 January 2020

[arXiv:1810.09826]

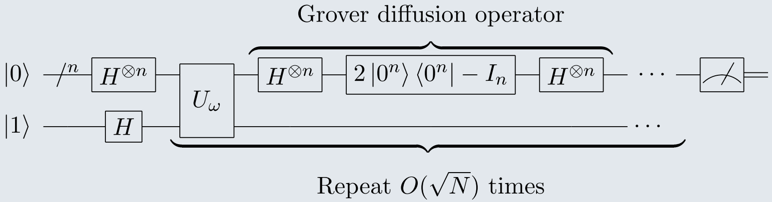
UNIVERSITÉ DE GENÈVE

Quantum Circuits and Black Boxes

Quantum circuit architecture is behind much of quantum computing and information

Grover's algorithm

Given U_ω s.t. $U_\omega |x\rangle |y\rangle = |x\rangle |y \oplus f_\omega(x)\rangle$, $f_\omega(x) = 1 \iff x = \omega$. Find ω.



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- Interested in query complexity and not the rest of the circuit

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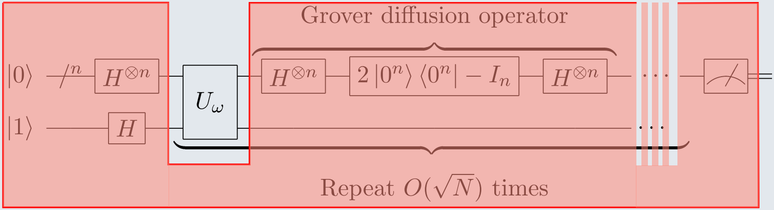

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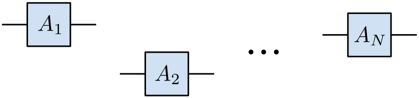
Computing with Black Boxes

Given some unknown operations as black boxes, how can they be composed or transformed?

- Standard approach is to place them in quantum circuits



- Formalised under many different names: quantum circuit architecture, quantum combs, process tensors, ...

[Chiribella, D'Ariano, Perinotti, PRL 101 (2008) and PRA 80 (2009)], [Pollock et al., PRA 97 (2018)]

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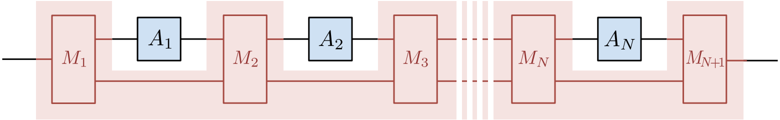

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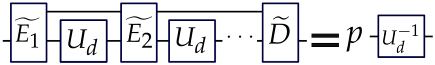
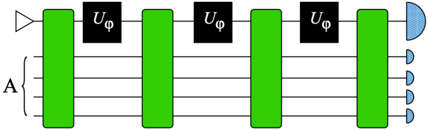
Applications of the Framework

The black box framework is a powerful approach to many problems, e.g.:

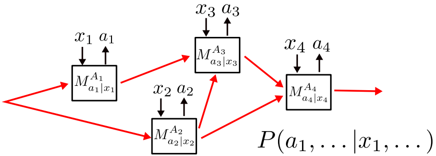
- Inverting unknown unitaries
  [Quintino *et al.*, PRL 123 (2019)]
- Quantum algorithm-learning
  [Bisio *et al.*, PRA 81 (2010)]



- Quantum metrology
  [Giovannetti *et al.*, PRL 96 (2006)]
- Optimal quantum tomography
  [Bisio *et al.*, PRL 102 (2009)]



- Quantum networks
  [Chiribella *et al.*, PRA 80 (2009)]
- Quantum causal models
  [Barrett, Lorenz, Oreshkov, arXiv:1906.10726]



Outline

Can we compose unknown operations in a more general way, and gain advantages by doings so?

- By exploiting coherent control, yes!

Quantum Channels

Most generally, assume boxes are quantum channels rather than unitary transformations

- Most general transformation of quantum states to quantum states
  - Recall: Quantum states represented by density matrices $\rho \in \mathcal{L}(\mathcal{H})$:

$$\rho = \rho^\dagger, \quad \rho \succcurlyeq 0, \quad \mathrm{Tr}(\rho) = 1$$

- Mathematically channels represented as completely-positive trace-preserving (CPTP) maps

$$\rho \;—\; \boxed{\mathcal{C}} \;—\; \mathcal{C}(\rho)$$

Can represent (non-uniquely) by Kraus operators $\{K_i\}_i$ satisfying $\sum_i K_i^\dagger K_i = \mathbb{1}$ with

$$\mathcal{C}(\rho) = \sum_i K_i \rho K_i^\dagger$$

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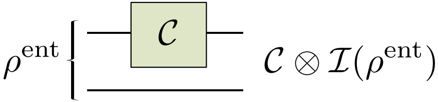

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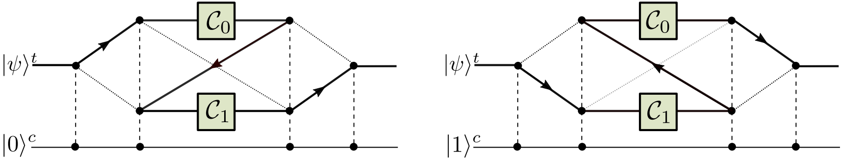
The Quantum Switch

The quantum switch coherently controls the order in which two quantum channels $\mathcal{C}_0$ and $\mathcal{C}_1$ are applied to a target system

- In contrast, a quantum circuit always composes them in a *fixed order*



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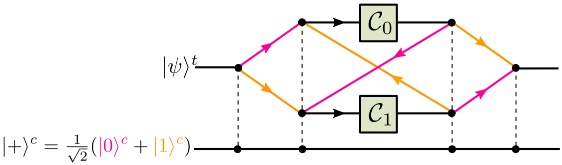

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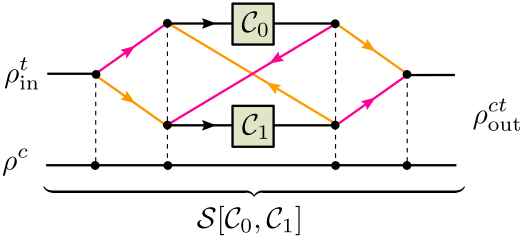
Quantum Switch Superchannel

The quantum switch is a "superchannel": it maps arbitrary $\mathcal{C}_0, \mathcal{C}_1$ to a new channel $\mathcal{S}[\mathcal{C}_0, \mathcal{C}_1] : \mathcal{L}(\mathcal{H}^c \otimes \mathcal{H}^t) \to \mathcal{L}(\mathcal{H}^c \otimes \mathcal{H}^t)$



- If $\{K_i\}_i$ and $\{L_j\}_j$ are Kraus operators for $\mathcal{C}_0$ and $\mathcal{C}_1$, Kraus operators of $\mathcal{S}[\mathcal{C}_0, \mathcal{C}_1]$ are $\{W_{ij}\}_{ij}$, with

$$W_{ij} = |0\rangle\langle 0|^c \otimes L_j K_i + |1\rangle\langle 1|^c \otimes K_i L_j$$

Advantages from the Quantum Switch

Why is the quantum switch interesting?

- Need an example of a black-box problem for which it provides an advantage over standard (causally ordered) quantum circuits

Commuting/Anticommuting Unitary Discrimination

Given unitaries U_A and U_B with the promise that either $[U_A, U_B] = 0$ or $\{U_A, U_B\} = 0$, determine which of these cases is true.

$$\frac{1}{\sqrt{2}}(|0\rangle^c + |1\rangle^c) \otimes |\psi\rangle^t \xrightarrow{\mathcal{S}} \frac{1}{\sqrt{2}}(|0\rangle^c \otimes U_B U_A |\psi\rangle^t + |1\rangle^c \otimes U_A U_B |\psi\rangle^t)$$
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Measuring the control in the $\{|+\rangle, |-\rangle\}$ basis allows perfect discrimination; with a quantum circuit either U_A or U_B must be used twice to achieve this

- In a generalised version of the problem, $O(N^2)$ advantage using "N-switch" between $N!$ orders

[Deutsch, Jozsa, PRSA 439 (1992)], [Chiribella, D'Ariano, Perinotti and Valiron, PRA 88 (2013)], [Araújo, Costa, Brukner, PRL 114 (2014)]

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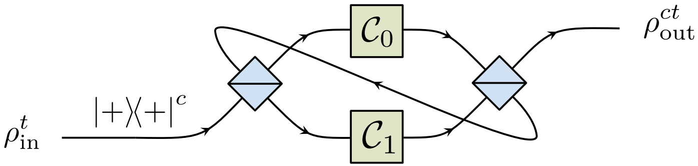
Interest in the Quantum Switch

The quantum switch has helped grow interest in indefinite causal structures as quantum information resources

- Numerous applications for the quantum switch have been considered

- Several experimental implementations highlighting its relevance as a resource for quantum information
  - Groups of Walther (Vienna), White (Brisbane), Pan (Shanghai), Guo (Hefei)

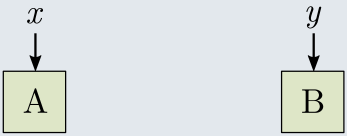


Communication Complexity

The quantum switch can also be used as a quantum communication resource

Communication complexity

In communication complexity problems, parties receive some inputs $x, y, \ldots$ and must compute a distributed function $f(x, y, \ldots)$ while communicating as little information as possible



$f(x,y)?$

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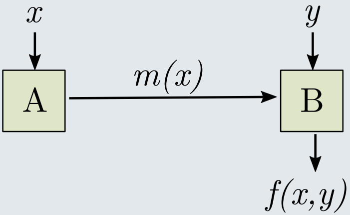

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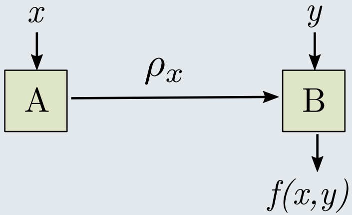

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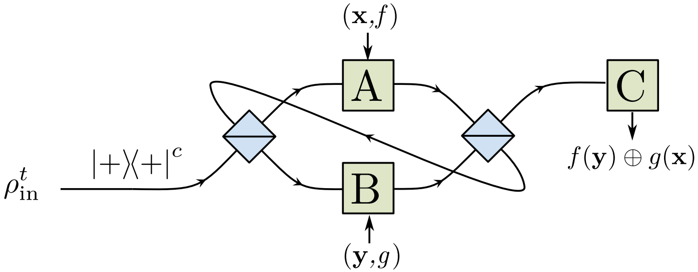
Communication Complexity Advantage

Quantum switch provides $O(2^n)$ advantage in communication complexity over any causally ordered communication strategy

- Scenario in which a referee must output $f(x,y)$

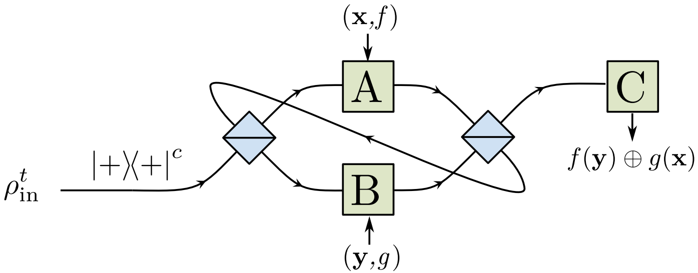


- Game and proof based on the commuting/anticommuting unitary discrimination task

[Guérin Feix, Araújo, Brukner, PRL 117 (2016)]

Communicating Through the Switch

- Communication complexity perspective views that quantum switch as a resource which parties use to communicate
- A complementary approach recently proposed is to take a Shannon-theoretic perspective, viewing the quantum switch as a way to compose communication channels one wishes to communicate through
  - Fundamental problem is studying the capacity of quantum channels



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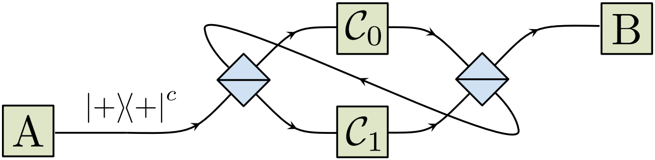

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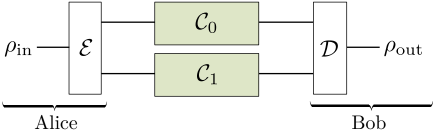
Composition of Communication Channels

Imagine Alice and Bob wish to communicate and have access to some noisy channels $\mathcal{C}_0$ and $\mathcal{C}_1$

- Parallel composition



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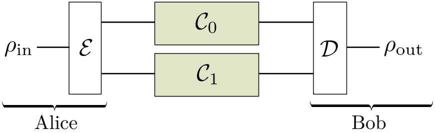

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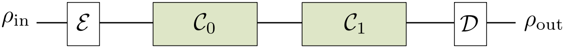

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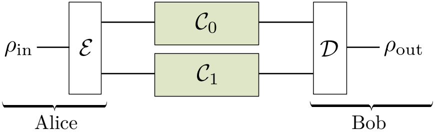

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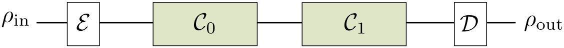

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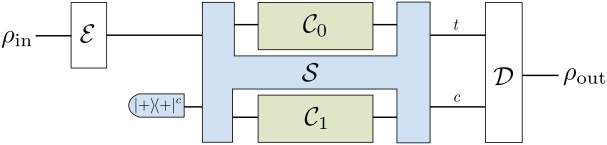
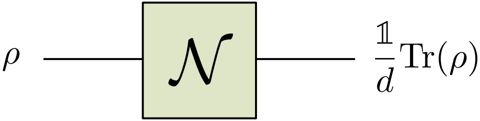


Depolarising Quantum Switch

Ebler, Salek and Chiribella [PRL 120 (2018)] showed the quantum switch enables "causal activation" of channel capacity in this scenario

- Extreme case: classical information can be transmitted through two completely depolarising channels



- Depolarising channels can transmit no information, even when composed in a standard, causal manner
- When placed in a quantum switch, $\mathcal{S}[\mathcal{N}_0, \mathcal{N}_1]$ has nonzero classical capacity!

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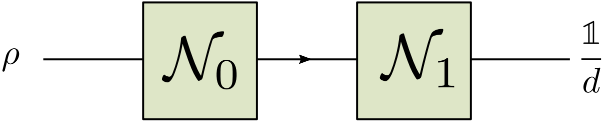

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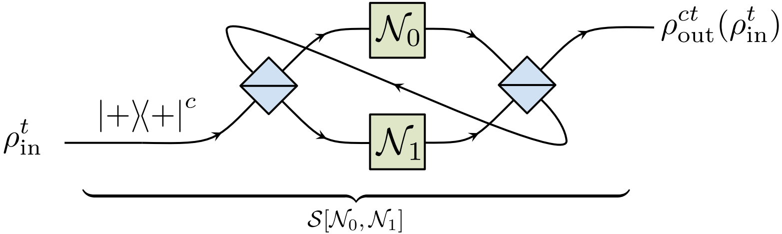

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Causal Activation in Detail

$$\rho \longrightarrow \boxed{\mathcal{N}} \longrightarrow \frac{\mathbb{1}}{d}\mathrm{Tr}(\rho)$$

- For random choice (U_i, U_j) system evolves under unitary

$$W_{ij} = |0\rangle\langle 0|^c \otimes U_j U_i + |1\rangle\langle 1|^c \otimes U_i U_j$$

- Output of global channel is

$$\begin{aligned}\mathcal{S}[\mathcal{N}_0, \mathcal{N}_1](\rho_{\text{in}}^t) &= \frac{1}{d^4} \sum_{ij} W_{ij} (|+\rangle\langle +|^c \otimes \rho_{\text{in}}^t) W_{ij}^\dagger \\ &= \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{1}{2} \big[\, |0\rangle\langle 1|^c + |1\rangle\langle 0|^c \,\big] \otimes \frac{1}{d^2} \rho_{\text{in}}^t\end{aligned}$$

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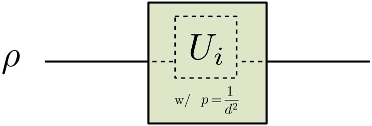

$$\frac{1}{d^2}\sum_i U_i \rho U_i^\dagger = \frac{\mathbb{1}}{d}$$

$\{\frac{1}{d}U_i\}_i$ Kraus operators for $\mathcal{N}$ (with $\{U_i\}_i$ orthogonal unitaries)

- For random choice (U_i, U_j) system evolves under unitary

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Dependence on ρ_{in}^t implies transmission of information

- Can bound the capacity of $\mathcal{S}[\mathcal{N}_0,\mathcal{N}_1]$

Holevo information

A single use of a quantum channel $\mathcal{C}$ can transmit information at best at the rate

$$\chi(\mathcal{C}) = \max_{\{p_x,\rho_x\}} I(X:B)_\sigma$$

where $I(X:B)$ is the von Neumann mutual information evaluated on the state $\sigma = \sum_x p_x\,|x\rangle\langle x|^X \otimes \mathcal{C}(\rho_x)^B$

$\chi(\mathcal{C})$ is a lower bound on the classical capacity of $\mathcal{C}$, $\lim_{n\to\infty}\frac{1}{n}\chi(\mathcal{C}^{\otimes n})$

- For qubits, Holevo information is $\chi(\rho_{\text{out}}^{ct}) = -\frac{3}{8} - \frac{5}{8}\log_2\frac{5}{8} \approx 0.05$

Causal Activation in Detail

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- Relies on coherent control of order: if the control is depolarised, the dependence on ρ_{in}^t disappears
- Tracing out (discarding) either control or target also removes any dependence on ρ_{in}^t
  - Bob needs access to both control and target to decode information
  - Alice can only access target: scenario doesn't make sense from Shannon-theoretic viewpoint if she can encode information directly in the control
  - Information in the *correlations* between the control and target systems

[Kristjánsson, Salek, Ebler and Chiribella, arXiv:1910.08197]

Activation of Quantum Capacity

$\mathcal{S}[\mathcal{N},\mathcal{N}]$ cannot generate entanglement: it has zero quantum capacity

Coherent information

Number of qubits that can be transmitted by a single use of a channel $\mathcal{C}$:

$$I(\mathcal{C}) = \max_{\rho} I(A\rangle B)_{(\mathcal{I}\otimes\mathcal{C})(\rho)},$$

where $I(A\rangle B)_{\sigma_{AB}} = H(\mathrm{Tr}_A[\sigma_{AB}]) - H(\sigma_{AB})$.

$I(\mathcal{C})$ lower-bounds the quantum capacity of $\mathcal{C}$, $\lim_{n\to\infty} \frac{1}{n} I(\mathcal{C}^{\otimes n})$

The quantum switch can activate quantum capacity *maximally*!

- Let $\mathcal{E}_{XY}(\rho) = \frac{1}{2}(\sigma_x\rho\sigma_x + \sigma_y\rho\sigma_y)$
- $I(\mathcal{E}_{XY}) = I(\mathcal{E}_{XY}\circ\mathcal{E}_{XY}) = 0$
- $I(\mathcal{S}[\mathcal{E}_{XY},\mathcal{E}_{XY}]) = 1!$

[Chiribella et al., arXiv:1810.10457]

Causal Activation Summary

- By coherently controlling order of channel use, the quantum switch activates capacity of noisy quantum channels
- Free resource for an extension of quantum Shannon theory in which not only information carriers, but also their *propagation* is treated quantum mechanically
  - Is it reasonable to take it as a free resource?
  - Does the control system act as a sidechannel?

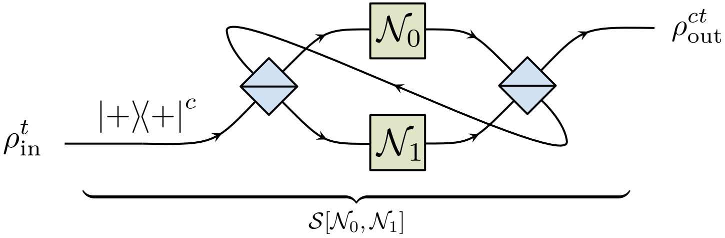
Are there other interesting ways in the same spirit in which coherent control can be used to activate capacity?

[Kristjánnson, Chiribell, PRSA (2019)], [Kristjánsson, Salek, Ebler and Chiribella, arXiv:1910.08197]

Outline

Cutting The Switch in Half

Why not use the control to simply control which channel used?



- Coherently control sending the target through either $\mathcal{N}_0$ or $\mathcal{N}_1$
- What are the Kraus operators of the induced global channel?

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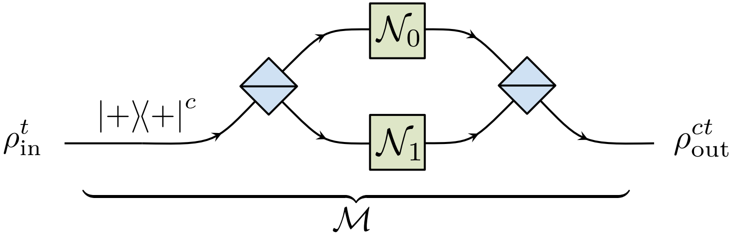

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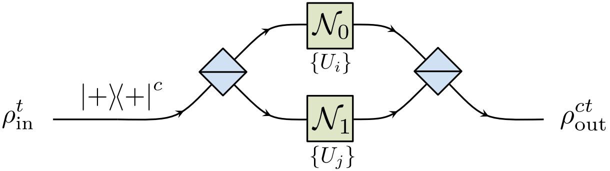
Analysing the "Half-Switch"



Take $\mathcal{N}_0, \mathcal{N}_1$ again as randomisation of U_i, U_j

- For random choice of unitaries (U_i, U_j), system evolves under unitary

$$W'_{ij} = |0\rangle\langle 0|^c \otimes U_i + |1\rangle\langle 1|^c \otimes U_j$$

- The output $\rho_{\text{out}}^{ct} = \frac{1}{d^4} \sum_{ij} W'_{ij}(|+\rangle\langle+|^c \otimes \rho_{\text{in}}^t) W'^{\dagger}_{ij}$ is then

$$\rho_{\text{out}}^{ct} = \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{1}{2}\left[|0\rangle\langle 1|^c + |1\rangle\langle 0|^c\right] \otimes T \rho_{\text{in}}^t T^\dagger$$

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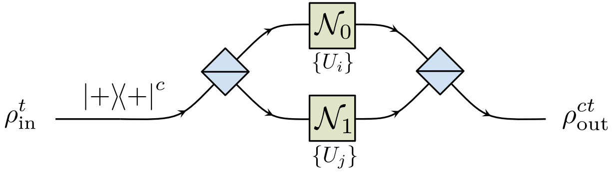

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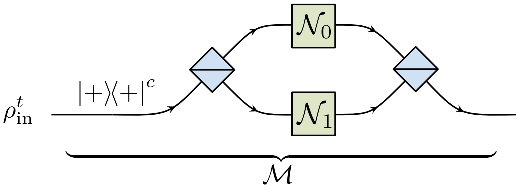
Communicating through the Half-Switch



$$\frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{\sigma_x}{2} \otimes T\rho_{\text{in}}^t T^\dagger$$
with $T = \frac{1}{d^2}\sum_i U_i$

- For orthogonal U_i, $T\rho_{\text{in}}^t T^\dagger \neq 0$ and depends on ρ_{in}^t, so some information is again transmitted!

- But $T = \frac{1}{d^2}\sum_i U_i$ depends on the orthonormal set $\{U_i\}_i$ chosen!
  - To speak about "coherently controlled channels", need further information about their "implementation"
  - What freedom is there in controlling quantum channels?

cf. Araújo, Feix, Costa, Brukner, NJP (2014); Thompson, Modi, Vedral, Gu, NJP (2018); Chiribella, Kristjánnson, PRSA (2019).

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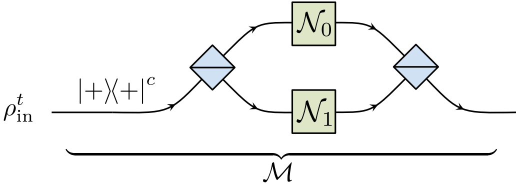

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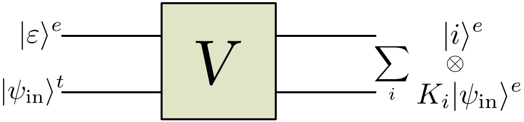
Unitary Dilation of Channels

Any channel $\mathcal{C}$ can be purified to a unitary interacting with some local environment $|\varepsilon\rangle^e$

- Given Kraus operators $\{K_i\}_i$ for $\mathcal{C}$, Stinespring purification:



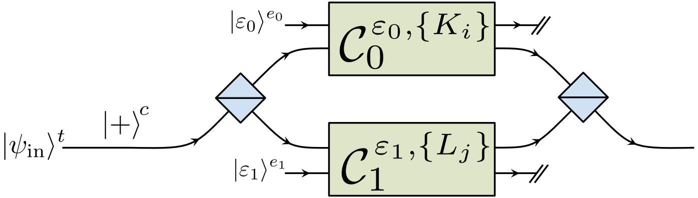
giving joint target-environment evolution:

$$|\psi_{\text{in}}\rangle^t \otimes |\varepsilon\rangle^e \to \sum_i K_i\,|\psi_{\text{in}}\rangle^t \otimes |i\rangle^e := |\Phi_{\text{out}}\rangle^{te}$$

- Tracing out environment gives $\mathrm{Tr}_e\,|\Phi_{\text{out}}\rangle\langle\Phi_{\text{out}}|^{te} = \mathcal{C}(|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|^t)$

Calculating the Channel Dependence

- Coherently control the unitary purifications then trace out the environment:



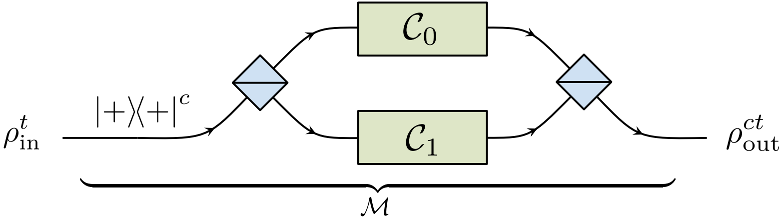
$$\begin{aligned}\rho_{\mathsf{out}}^{ct} =& \frac{1}{2}\left[\, |0\rangle\!\langle 0|^c \otimes \mathcal{C}_0(\rho_{\mathsf{in}}^t) + |1\rangle\!\langle 1|^c \otimes \mathcal{C}_1(\rho_{\mathsf{in}}^t)\right] \\ &+ \frac{1}{2}\left[|0\rangle\langle 1|^c \otimes T_0 \rho_{\mathsf{in}}^t T_1^\dagger + |1\rangle\langle 0|^c \otimes T_1 \rho_{\mathsf{in}}^t T_0^\dagger\right]\end{aligned}$$

with $T_0 \coloneqq \sum_i \langle\varepsilon_0|i\rangle K_i$ and $T_1 \coloneqq \sum_j \langle\varepsilon_1|j\rangle L_j$.

Coherent Control of Quantum Channels

Output depends on the transformation matrices T_0 and T_1

- Induced global channel is thus $\mathcal{M}[\mathcal{C}_0, T_0, \mathcal{C}_1, T_1]$



- Fact that output not a function of the CPTP maps $\mathcal{C}_0$ and $\mathcal{C}_1$ perhaps not surprising
  - Think about "global" phases becoming "relative" in interferometers: unitaries U and $-U$ give same CPTP map since $U\rho U^\dagger = (-U)\rho(-U)^\dagger$, but give different controlled operations
- Here we have a deeper dependence on the full purification
  - The quantum switch has no such dependence (it's a quantum superchannel)

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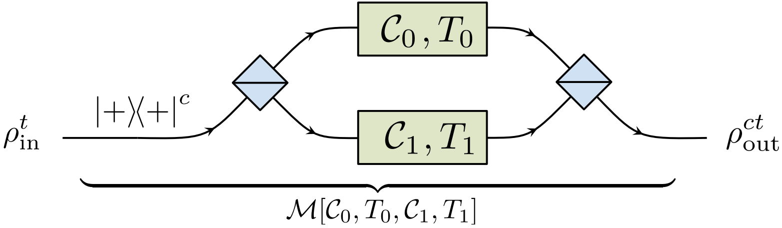

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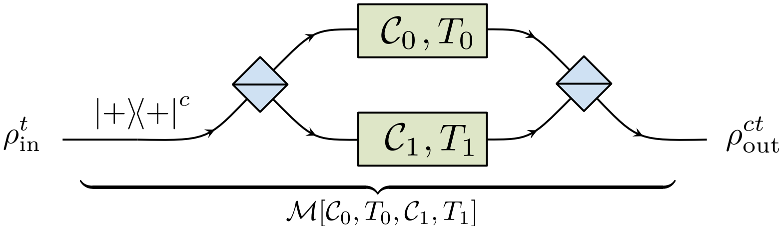
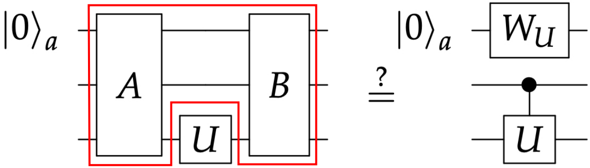


CPTP map $\mathcal{C}$ must be supplemented by T to meaningfully describe the coherent control of the "black box" – i.e., it's description must be given by pair $(\mathcal{C}, T)$

- An equivalent approach: work with vacuum-extended channels $\tilde{\mathcal{C}}$ and apply one channel on target, other on vacuum degree of freedom [Kristjánsson and Chiribella PRSA, (2019)]

Coherent control of operations

- Inability to universally control unknown unitaries previously proved [Araújo *et al.*, NJP 16 (2014)]



- For channels, even the notion of a controlled channel is ill-defined without further qualification
- Many relevant questions nevertheless remain, e.g.:
  - Given a black box that locally performs $\mathcal{C}$, can one optimise the implementation to provide advantages when coherently controlled?
  - What if the implementation (e.g., purification) is chosen randomly from some pertinent class?

Characterising Possible Transformations

For a given CPTP map $\mathcal{C}$, what transformation matrices T can one have?

For a unitary $\mathcal{U} : \rho \mapsto U\rho U^\dagger$:

- One can have $T = \alpha U$ with $\alpha \in \mathcal{C}$, $|\alpha| \leq 1$
- "Implementation details" are just the phase w.r.t. some reference

For arbitrary $\mathcal{C}$, can characterise set of T obtainable for some purification of $\mathcal{C}$ in terms of the Choi state C of $\mathcal{C}$ and its (pseudo)inverse

- For a (completely depolarising channel), one can have any T satisfying $\mathrm{Tr}[T^\dagger T] \leq \frac{1}{d}$
- Allows optimisation over possible "implementations" of a channel

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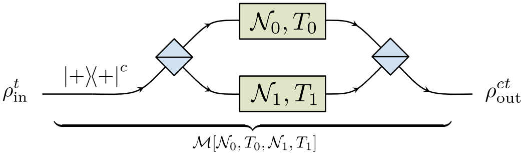
Coherent Control of Depolarising Channels



How much information can one communicate through coherently controlled depolarising channels?

- Three cases of interest saturating $\mathrm{Tr}[T^\dagger T] \leq \frac{1}{d}$:

- Taking $K_i = \frac{1}{d}U_i$ and $|\varepsilon\rangle = \sum_{i=0}^{d^2-1} \frac{1}{d}|i\rangle$ gives $T = \frac{1}{d^2}\sum_i U_i$
  - Recover result of randomisation over (U_i, U_j)
  - For qubits gives Holevo information $\chi(\mathcal{M}[\mathcal{N}_0, T_0, \mathcal{N}_1, T_1]) \approx 0.12$

- Taking $|\varepsilon\rangle = |0\rangle$ and $U_0 = \frac{1}{d}\mathbb{1}$ for each channel gives $T = \frac{1}{d}\mathbb{1}$
  - One recovers precisely the output of the depolarising quantum switch
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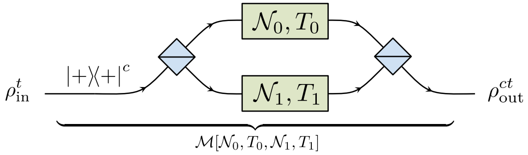

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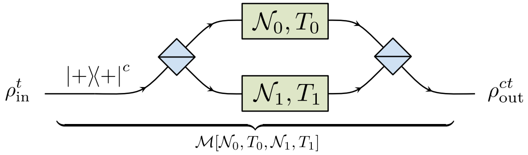

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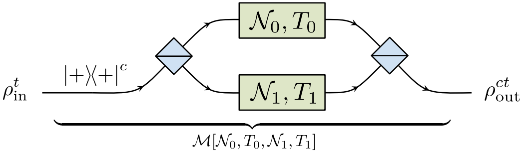


What is the very best one can do?

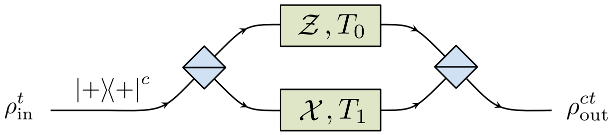
- Numerically, optimal obtained for $T_0 = T_1 = \frac{1}{\sqrt{d}} |0\rangle\langle 0|$
  - Obtain $\chi(\mathcal{M}[\mathcal{N}_0, T_0, \mathcal{N}_1, T_1]) = \frac{1}{d} \log_2 \frac{5}{4}$, which is ≈ 0.16 for qubits
    - Notice that this decreases with d!

$$\rho_{\text{out}}^{ct} = \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{\sigma_x}{2} \otimes T \rho_{\text{in}}^t T^\dagger$$

- *Coherent* control essential: if control decohered, no information transmitted
- Control only carries some information: $\rho_{\text{out}}^c = \frac{1}{2}(\mathbb{1}^c + \text{Tr}(T\rho_{\text{in}}^t T^\dagger)\sigma_x^c)$
  - Rest in control-target correlations

Activation of Quantum Capacity

- What about quantum capacity?
  - Coherent control of channels can't reproduce the maximal activation of $\mathcal{E}_{XY}$ obtained with the quantum switch
  - But it can provide advantages in other situations, e.g., communicating through complementary dephasing channels



- Again, coherent control of channels activates quantum capacity
  - The quantum switch only shown to activate capacity for partially dephasing channels
  - Coherent control activates capacity even for maximally dephasing channels

[Salek, Ebler and Chiribella, arXiv:1809.06655], [AA, Wechs, Horsman, Mhalla, Branciard, arXiv:1810.09826]

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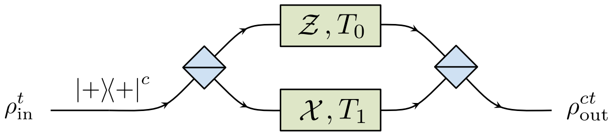

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Summary of Communication Advantages

Both the coherent control of order (in the quantum switch) and between different channels can provide surprising activation of classical and quantum communication capacities

- In the latter case, activation depends on the transformation matrices T_i in addition to the channels $\mathcal{C}_i$
- Subtly different scenarios, and ongoing debate as to how much – and in what scenarios – can they be compared

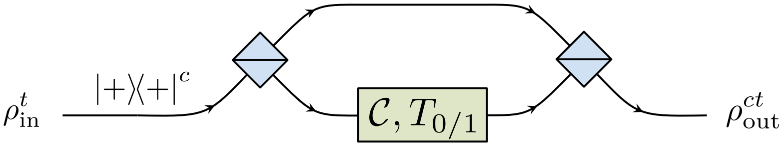
- How much of these effects are due to coherent control, how much to causal indefiniteness?
- To what extent do these types of coherent control generate communication sidechannels?

Ultimately, relevance these coherent control strategies depends on the scenario of interest and the questions one wishes to ask

Exploiting Coherent Control of Channels

What other ways can the coherent control of channels be exploited?

- Implementation dependence a subtlety, but opens up new possibilities
  - E.g., discrimination of different implementations of a channel



- Two implementations of $\mathcal{C}$ with transformation matrices T_0 and T_1 induce two different global channels $\mathcal{M}_{T_0}$ and $\mathcal{M}_{T_1}$
- If chosen with equal priors, can discriminate with probability

$$\frac{1}{2}\left(1+\tfrac{1}{2}\|\mathcal{M}_{T_0}-\mathcal{M}_{T_1}\|_{\diamond}\right)=\frac{1}{2}\left(1+\tfrac{1}{2}\|T_0-T_1\|_2\right)$$

  - For unitary $\mathcal{C}$, one recovers known perfect discrimination
  - For depolarising channels $\mathcal{N}$ best is $\frac{1}{2}(1+\frac{1}{\sqrt{d}})$

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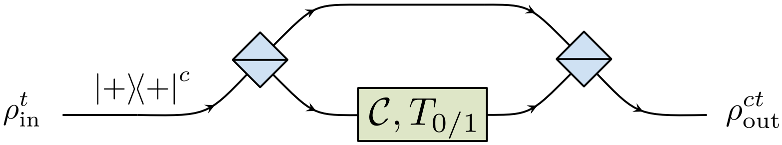

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Exploiting Coherent Control of Channels

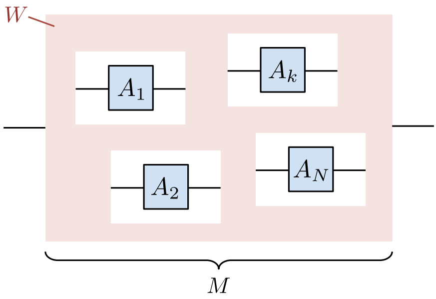
What can we gain from treating coherently controllable channels as an operational primitive?

- Much more work to be done to understand what advantages such an approach could entail
- Adds to more general call to extend the standard circuit approach to experimentally conceivable situations
  - E.g., Araújo at al., NJP 16 (2014); Portmann et al., IEEE Trans. IT 63 (2017); Thompson, Modi, Vedral, Gu, NJP (2018)
  - Chiribella and Kristjánsson's extension of quantum Shannon theory a first step [Proc. R. Soc. A 475 (2019)]
- Can even consider higher order coherent control, e.g., control of quantum combs [Dong, Nakayama, Soeda and Murao, arXiv:1911.01645]

Beyond the Quantum Switch

Quantum switch highlights possibility of going beyond the standard circuit approach for computing with black boxes

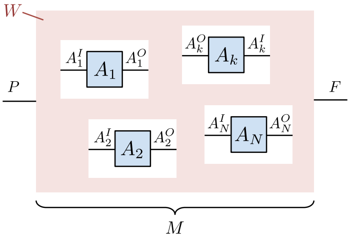
- Can one go further?
- Most general composition is given my superchannels



Process Matrices

Supermaps and their (non)causal structure have been studied within the process matrix formalism

- Represented as positive semidefinite W matrices obeying additional linear constraints ensuring validity of supermap



$$M = \mathrm{Tr}_{A_1^I A_1^O \dots A_N^I A_N^O}[W \cdot A_1 \otimes \cdots \otimes A_N \otimes \mathbb{1}^{PF}]$$

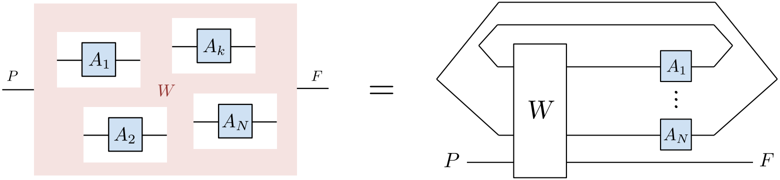
- Well defined notion of compatibility with causal composition
- Although the quantum switch can be represented in this way, most causally indefinite supermaps have no clear interpretation

[Oreshkov, Costa, Brukner, Nat. Commun. (2012)], [Araújo et al., NJP (2014)], [Wechs, AA, Branciard, NJP (2019)]

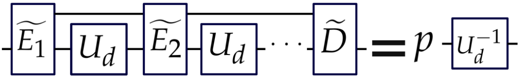
Quantum Information with Supermaps

Nonetheless, quantum information processing capabilities of general supermaps are interesting

- Computational and query complexity can be studied through equivalence with *linear* closed time-like curves [Araújo, Guérin, Baumeler, PRA (2017)]



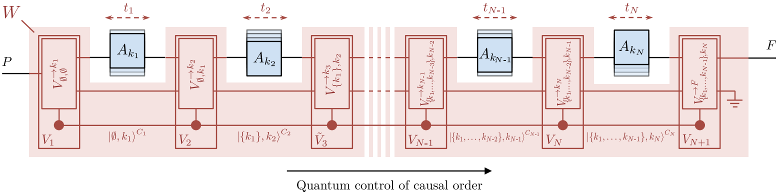
- Can provide advantages in tasks such as inverting unknown unitaries [Quintino, Dong, Shimbo, Soeda, Murao, PRL (2019)]



Generalising Coherent Control

Conversely, can also look for the most general supermaps that coherently control the order of operations

- New effects beyond the quantum switch: dynamical control and interference of causal histories



- Class of more general supermaps with strong structure to study
  - What new advantages do they provide?
  - What is the cost of simulating them with standard circuits?
  - Which processes are "equivalent" to the quantum switch, and which are fundamentally new?

[Wechs, Dourdent, AA, Branciard, *in preparation*.]

The End

Thank you!

[arXiv:1810.09826]

Further reading:

- Causal activation paper: Ebler, Salek, and Chiribella, PRL **120**, 120502 (2018)
- With quantum information: Salek, Ebler, and Chiribella, arXiv:1809.06655
- Activation impossible with control of path only: Chribella et al., arXiv:1810.10457
- Alternative causal control: Guérin, Rubino, and Brukner, PRA **99**, 062317 (2019)
- More general model: Chiribella and Kristjánsson, Proc. R. Soc. A **475** (2019)
- Resource theory approach: Kristjánsson, Salek, Ebler and Chiribella, arXiv:1910.08197