Query Complexity of Noncausal Quantum Computation

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Context

Quantum computers allow certain problems, such as the factorisation of large numbers, to be performed more efficiently than on a classical computer. The study of the computational power of quantum computational models is called quantum complexity theory. The standard model of quantum computing is that of quantum circuits, and an important approach to studying their power is to look at quantum query complexity, where one tries to extract information about a function by evaluating it on different inputs.

Used this way, quantum circuits can be treated as a higher-order computations on the functions being queried. But quantum circuits are not the only higher-order quantum computations one can perform, and it turns out there exist more exotic quantum computations that lack a well-defined causal structure but can provide advantages over quantum circuits in some tasks.

I am interested in these "causally indefinite" higher order computations. While we know they can provide some computational advantages, we still lack a more general understanding of how they compare to quantum circuits in computational complexity.

Description/Main activities

This internship will study the query complexity of general higher order, causally indefinite quantum computations using what is known as the process matrix framework. The goal is to understand better how they compare to quantum circuits. Some possible approaches to be studied include:

1. Generalizing techniques used to place lower bounds on the query complexity of quantum circuits to higher-order quantum computations.
2. Using convex optimisation (semidefinite programming) to compare quantum circuits to higher-order computations on small instances of well-studied problems, and attempting to generalise any advantages found.
3. Transforming known computational advantages due to causal indefiniteness into classical query problems.

Skills required

- General knowledge of computational models (e.g., Turing machines, Boolean circuits) and computational complexity (how classes like P and NP are defined, etc.).
- Prior knowledge of quantum information not necessary, but would be a plus.
- Mathematics: Linear algebra and discrete mathematics. Convex optimisation would be a plus.

Bibliography

1. M. Nielsen and I. Chuang, Quantum Computation and Quantum Information.
2. R. de Wolf, Quantum Computing: Lecture Notes, http://homepages.cwi.nl/~rdewolf/qcnotes.pdf.
3. J. Wechs, H. Dourdent, A. Abbott and C. Branciard, Quantum circuits with classical versus quantum control of causal order, PRX Quantum 2, 030335 (2021).