Causal Indefiniteness from Quantum Control of Processes

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(including joint work with Julian Wechs, Hippolyte Dourdent, Cyril Branciard, Mehdi Mhalla)

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Outline

- Rethinking causality in light of quantum theory
- Quantum causal indefiniteness
  - Process matrices and the quantum switch
- Which causally indefinite processes are *physical* ?
  - Excluding processes using quantum information theory
  - Constructing causal indefiniteness from quantum control
- Speculative comments, questions and discussion

Introduction: Cause and Effect

- What does it mean to say that X causes Y?
- How to *infer* causal structure from, possibly statistical, observations?
- What changes if events or causes are quantum in nature?

- Sun rising and rooster crowing are correlated, but correlation $\neq$ causation
- Intervening on a cause changes the distribution of the effect
  - P(sun rises | rooster crows) = P(sun rises | do(eat rooster for dinner))
  - P(rooster crows at 7am | sun rises at 7am) $\neq$ P(rooster crows at 7am | do(sun rises at 6am))

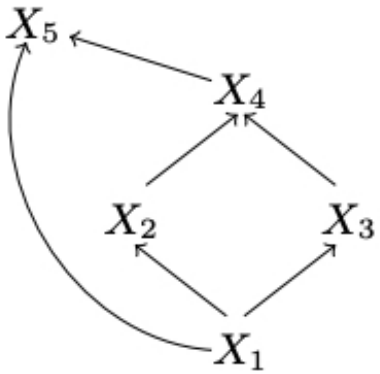
Classical causal models

Causal (Bayesian) models: Conditional independencies and interventions ("do-calculus") allow some information about causal relations to be inferred

[J. Pearl, Causality (2000)]

Causal model:

1) A causal structure given by a directed acyclic graph (DAG)
2) For each node X_i a distribution $P(X_i|Pa(X_i))$ defining $P(X_1, \ldots, X_n) = \prod P(X_i|Pa(X_i))$

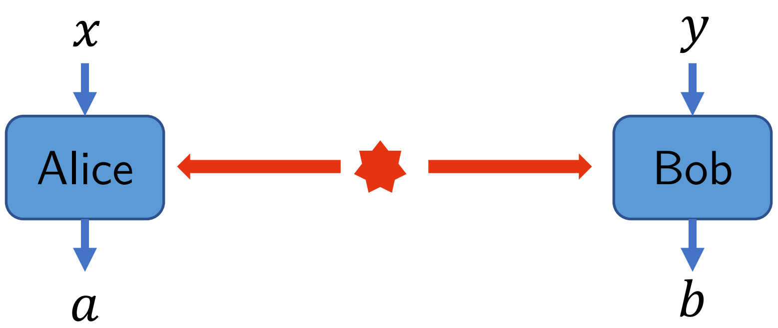


$$P(X_1, X_2, X_3, X_4, X_5) =$$
$$P(X_5|X_1, X_4)\ P(X_4|X_2, X_3)$$
$$P(X_2|X_1)\ P(X_3|X_1)$$
$$P(X_1)$$

- A causal model implies certain conditional independences between variables
- Extensive body of literature on causal inference and applications of causal models in statistics, economics, machine learning, …

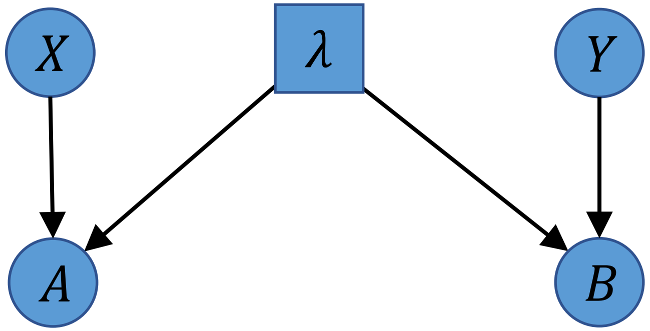
Bell's Theorem and its challenge to causality

Bell scenario



$$P(a, b|x, y)$$

Bell's "local causality"



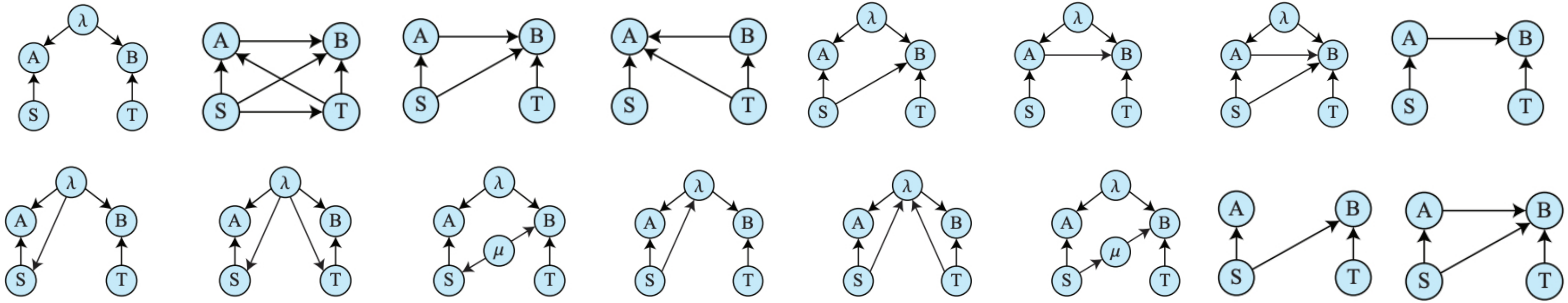
$$P(a, b|x, y) = \sum_{\lambda} P(a|x, \lambda) P(b|y, \lambda) P(\lambda)$$

- **Reichenbach's principle:** All correlations must be explained causally
- **Bell's theorem:** The "natural" causal model cannot explain the observed correlations from entangled states

Causal models fail to explain Bell inequality violation

- Can any causal explanation be given to quantum correlations?

- Wood & Spekkens (NJP 2015): Any causal model that reproduces the Bell-scenario correlations must be *fine tuned* – i.e. observed statistical independencies (e.g., non-signalling) not expected from the model's DAG

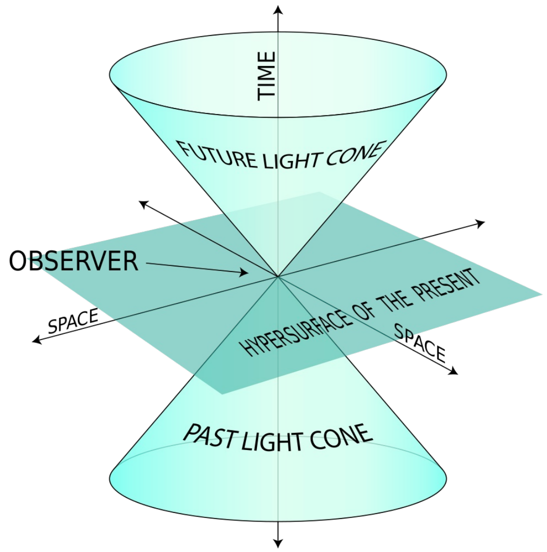


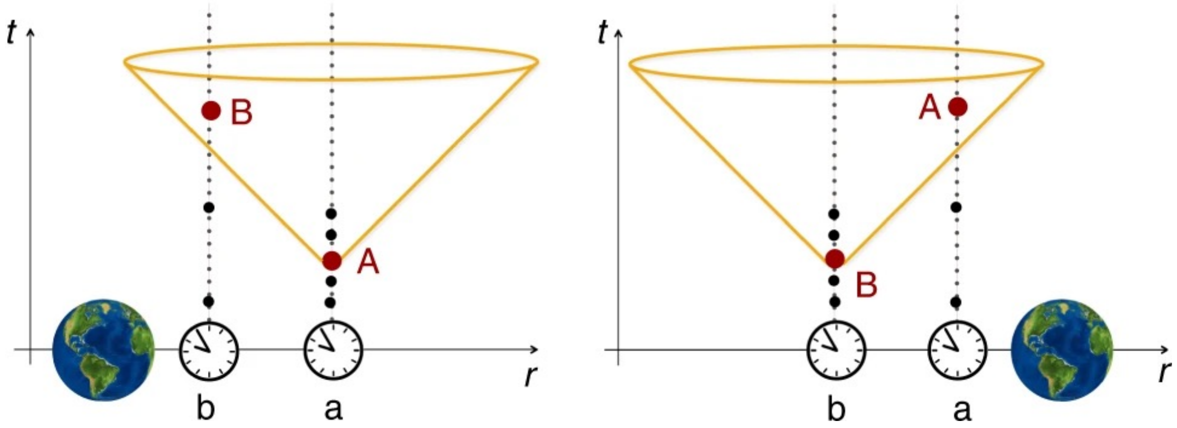
Quantum mechanics seems to require a new notion of causal structure to explain observations

Quantum gravity: A different challenge for causality

- General relativity: causal structure tied to light cone structure of the spacetime metric tensor
- A quantum theory of gravity likely to allow quantum uncertainty in metric tensor
- A quantum superposition of metrics would lead to a superposition of causal structures, and potentially quantum causal indefiniteness

[Hardy, arXiv:gr-qc/0509120 (2005)]





[Zych, Costa, Pikovski, Brukner, Nat. Commun. (2019)]

How can we study and describe such indefinite quantum causal structures?

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- Rethinking causality in light of quantum theory
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  - Process matrices and the quantum switch
- Which causally indefinite processes are *physical* ?
  - Excluding processes using quantum information theory
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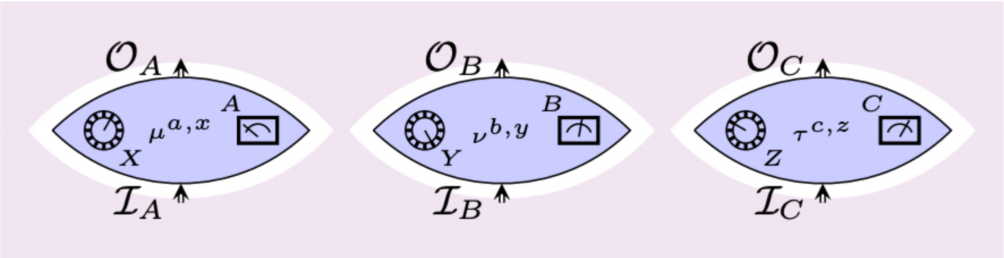
Quantum processes and their causal structure

Goal: to study and formalise quantum processes describing how several events are related without assuming any global causal ordering between the events

- Processes with indefinite causal order?!

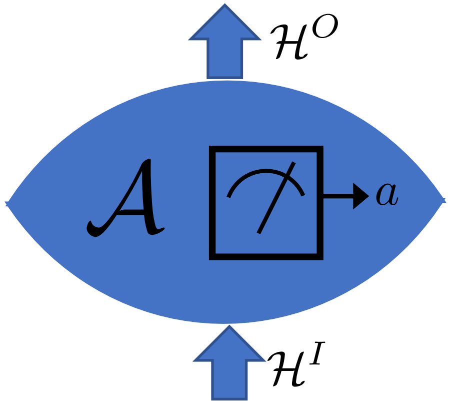
- A framework for such processes should satisfy some conditions:
  - Quantum mechanics should hold *locally:* each local event should be describable as a quantum operation
  - Consistency: A valid process shouldn't allow any logical paradoxes to occur (grandfather paradox, etc.)
  - Multi-linearity: A process composes operations linearly w.r.t. each "event"



[Baumeler, Gilani, Rashid, *Quantum* (2022)]

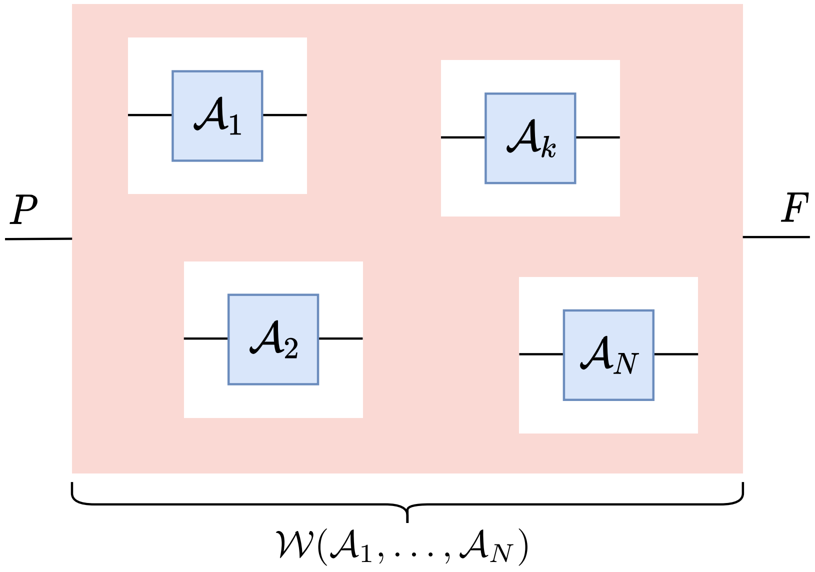
Framework of process matrices or quantum supermaps

The process matrix framework



$$= \quad \rho \;\text{—}\; \boxed{\mathcal{A}} \;\text{—}\; \mathcal{A}(\rho)$$

$$\mathcal{A} : \mathcal{L}(\mathcal{H}^I) \to \mathcal{L}(\mathcal{H}^O)$$



- An event can be thought of as a closed laboratory performing a quantum operation
- Freely chosen action/operation
  - CPTP map = quantum channel
- Need not be localised in time and/or space

- A process combines N events
  - Composition should give a valid CPTP map for any $\mathcal{A}_1, \ldots, \mathcal{A}_N$
  - $\mathcal{W}$ is a higher order operation (a quantum supermap) or a process (matrix)

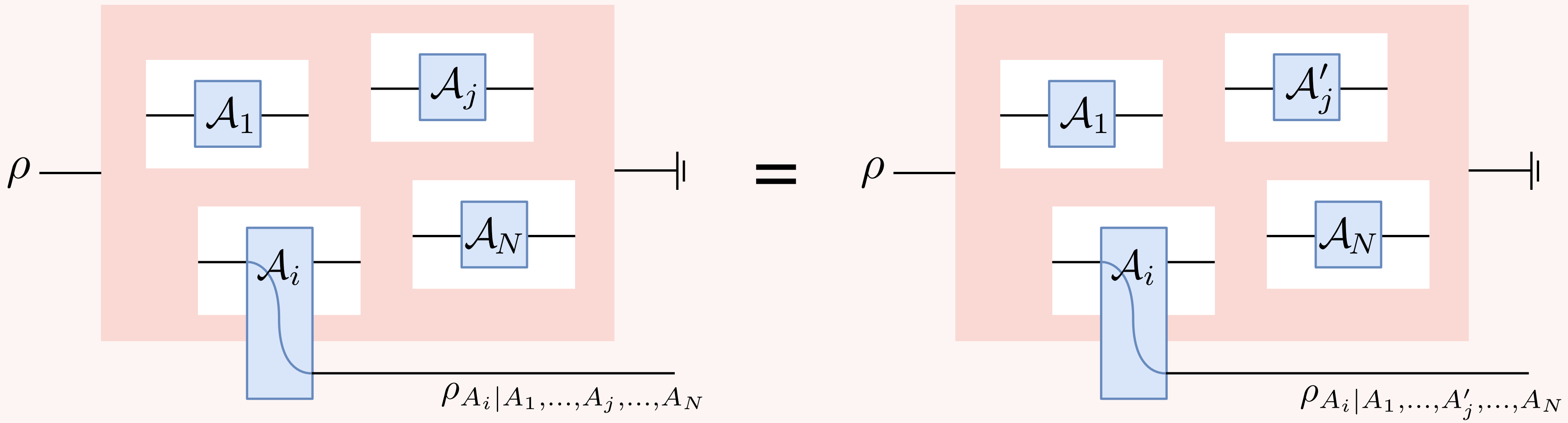
$$(\mathcal{A}_1, \ldots, \mathcal{A}_N) \mapsto \mathcal{W}(\mathcal{A}_1, \ldots, \mathcal{A}_N) : \mathcal{L}(\mathcal{H}^P) \to \mathcal{L}(\mathcal{H}^F)$$

Causal orderings of process

What is the causal structure of a process?

- Causal structure vs. causal order
- Consider operational notion of causal order, as defined by no-signalling properties

$\mathcal{W}$ is compatible with $\mathcal{A}_1 \prec \cdots \prec \mathcal{A}_N$ if, for all $i < j$, $\mathcal{A}_j$ cannot signal to $\mathcal{A}_i$

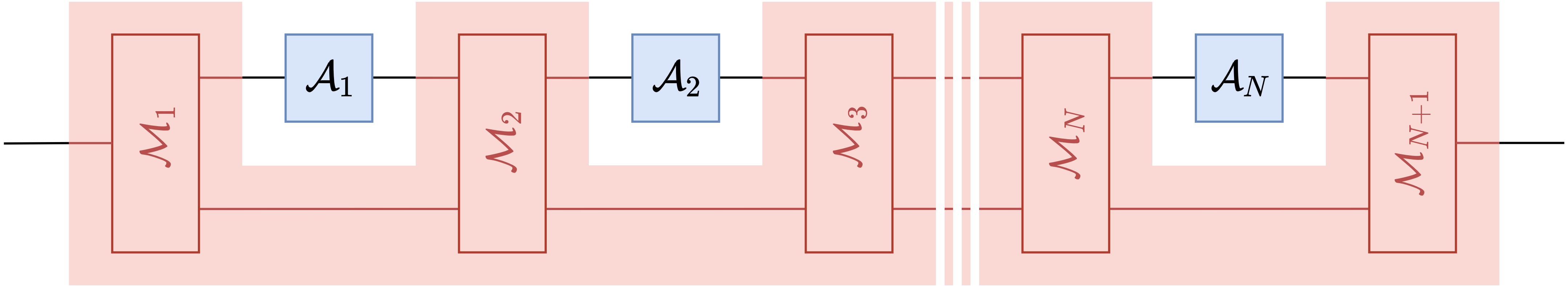


for all $\rho, \mathcal{A}_1, \ldots, \mathcal{A}_{i-1}, \mathcal{A}_{i+1}, \ldots, \mathcal{A}_j, \mathcal{A}'_j, \ldots, \mathcal{A}_N$.

Characterisation of causally ordered processes

Theorem [Chiribella, D'Ariano, Perinotti, PRA (2009)]

$\mathcal{W}$ is compatible with $\mathcal{A}_1 \prec \cdots \prec \mathcal{A}_N$ if and only if it has the form:



- A quantum circuit with N open "slots" – a quantum comb
- Most general process with a fixed causal order

Definite and indefinite causal orders

- Definite causal order beyond *fixed* causal orders?
  - Probabilistic order: causal structure well defined in any instantiation of process
  - Dynamical order: causal order between some events may depend on previous events

Causal separability

[Oreshkov, Costa, Brukner, *Nat. Commun.* (2012)]
[Oreshkov, Giarmatzi, *NJP* (2016)]
[Wechs, Abbott, Branciard, *NJP* (2019)]

"Bipartite" case: $\mathcal{W} = q\mathcal{W}^{A_1 \prec A_2} + (1-q)\mathcal{W}^{A_2 \prec A_1}$

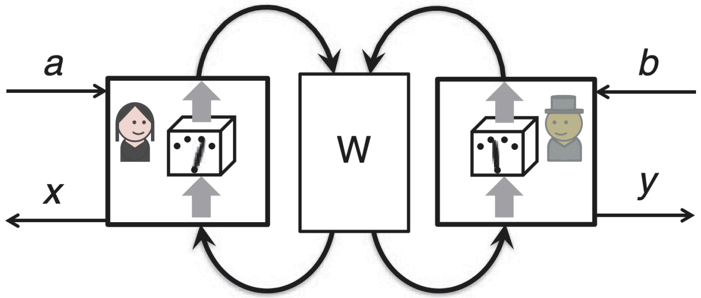
General case: $\mathcal{W} = \sum_k \mathcal{W}^{A_k \prec \{\cdots\}}$

$\forall k, \forall \mathcal{A}_k, \quad \mathcal{W}^{A_k \prec \{\cdots\}}_{|\mathcal{A}_k}$ an $N-1$ partite causally separable process

- Are there any causally *nonseparable* processes?
- Can we formulate explicit quantum processes with such causal indefiniteness?

Causally indefinite processes

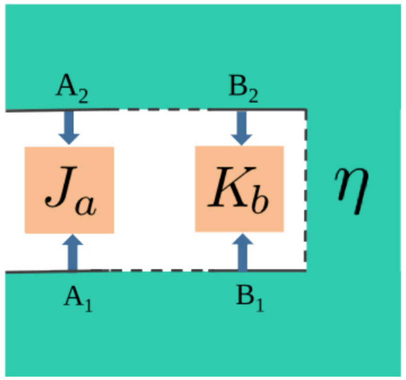
- Oreshkov, Costa & Brukner (2012): Yes, there exist processes incompatible with any definite causal order!
- They can even violate "causal inequalities" – an analogue of Bell inequalities for causal order

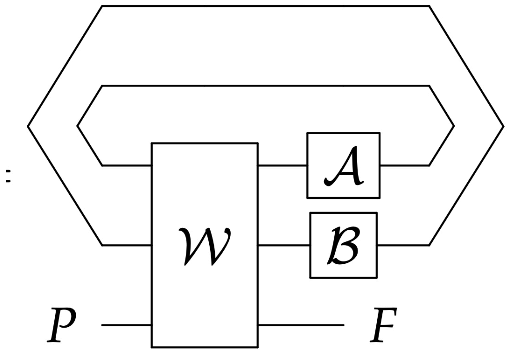


[Oreshkov, Costa, Brukner, *Nat. Commun.* (2012)]

What do such processes look like?

- Logically consistent but no clear physical interpretation ($W^{A_1A_2B_1B_2} = \frac{1}{4}\left[\mathbb{1}^{A_1A_2B_1B_2} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$)
- Understandable with post-selection or CTCs, but is it really physical?



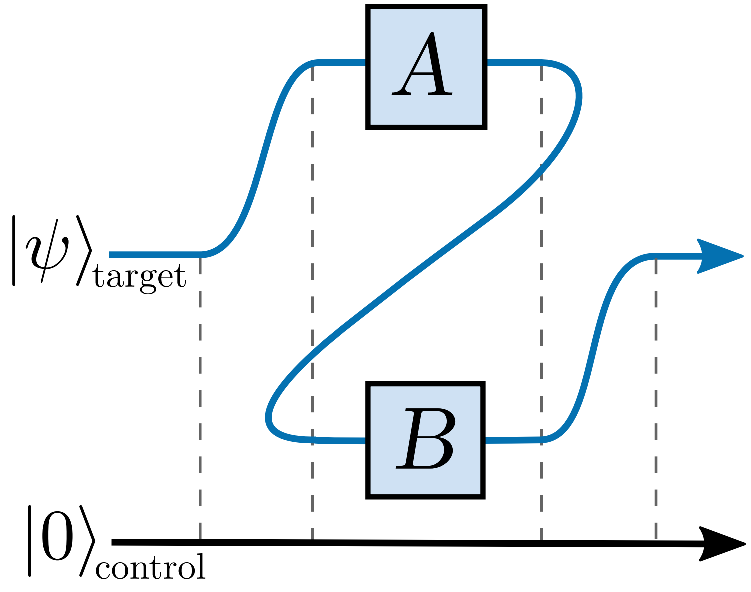


[Silva, Guryanova, Short, Skryzypczyk, Brunner, Popescu, *NJP* (2017)]
[Araújo, Allard Guérin, Baumeler, *PRA* (2017)]

The Quantum Switch

- Can we do better? Is there some physical quantum process with indefinite causal order?

The Quantum Switch [Chiribella, D'Ariano, Perinotti, Valiron, PRA (2013)]



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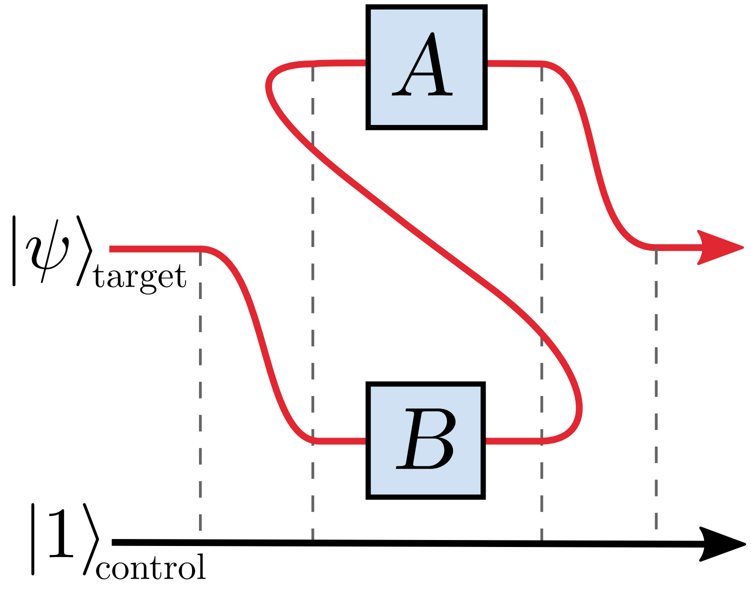

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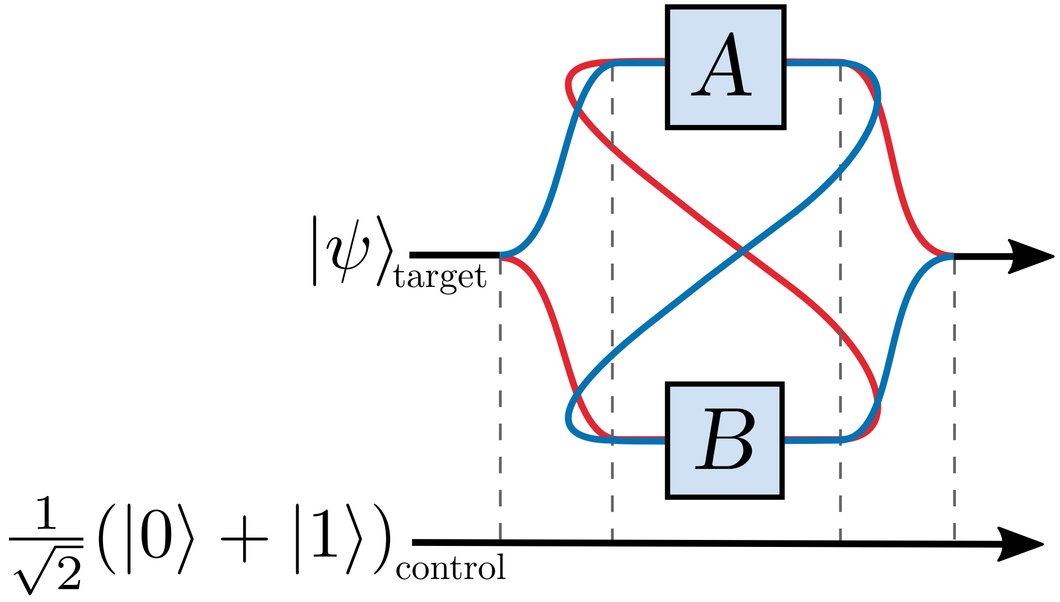
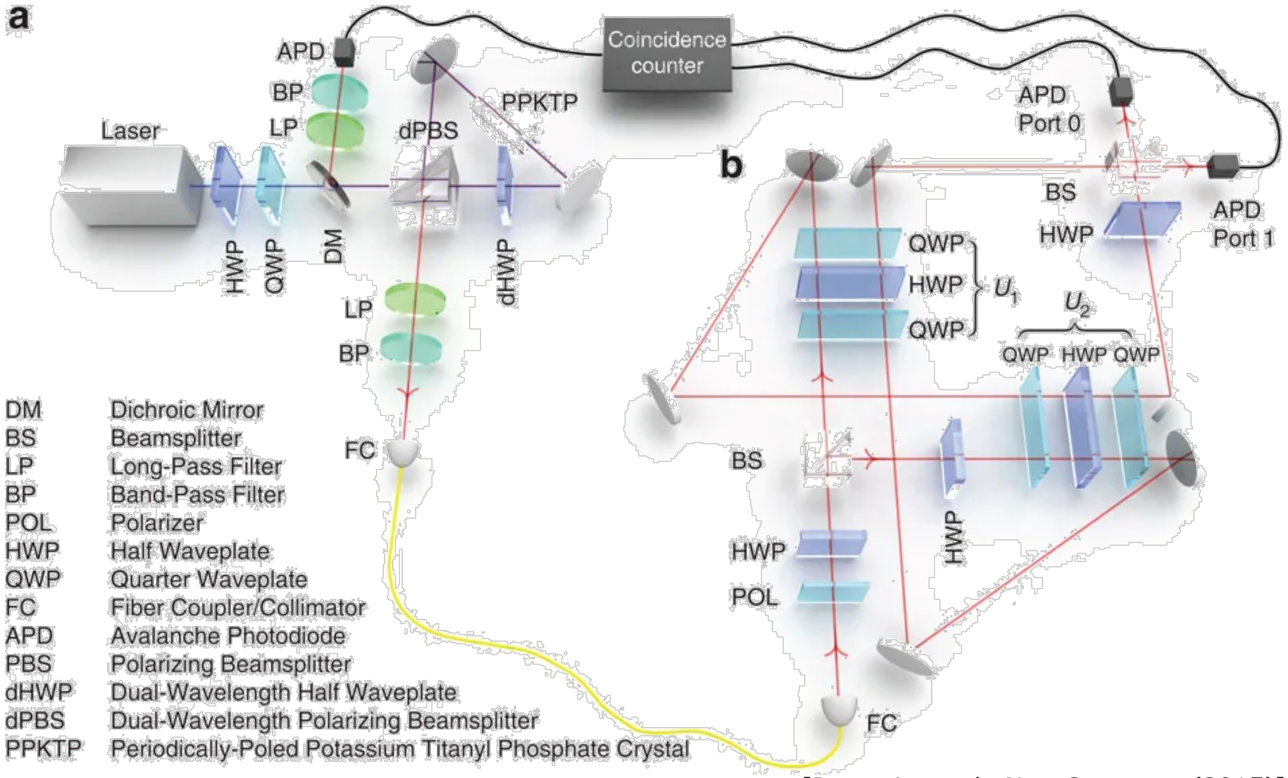


- Causal indefiniteness from quantum *control* of causal order

The Quantum Switch – cont.

- Experimentally realisable?



[Procopio *et al.*, Nat. Commun. (2015)]

- Continued debate as to whether experiments *implement* or just *simulate* quantum switch
- Nevertheless, clear physical interpretation within quantum theory

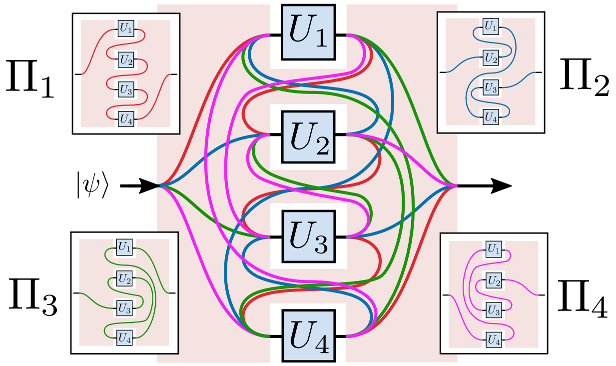
The Quantum Switch – a computational resource?

Quantum information advantages from the (generalised) quantum switch

- N-operation quantum switch use quantum control to determine order N gates applied

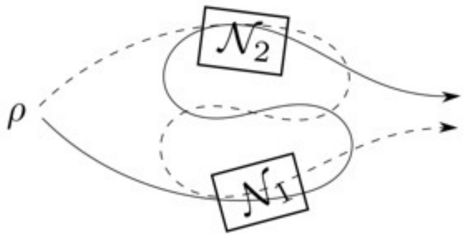
$$|x\rangle \otimes |\psi\rangle \xrightarrow{S_N} |x\rangle \otimes \Pi_x |\psi\rangle$$

- $O(N^2)$ advantage in query complexity in some computational "promise" problems
  [Araújo, Costa, Brukner, *PRL* (2014)]
  [Taddei, Abbott, et al., *PRX Quantum* (2021)]



- Exponential advantage in communication complexity
  [Allard Guérin, Feix, Araújo, Brukner, *PRL* (2016)]

- Reduction in noise for composing (communicating through) noisy channels
  [Ebler, Salek, Chiribella, PRL (2018)]
  [Abbott, Wechs, Horsman, Mhalla, Branciard, *Quantum* (2020)]



Summary of the situation

- Quantum switch cannot violate any causal inequalities
  - A weaker form of causal indefiniteness that some conceivable, consistent processes
- But raises the possibility of causal indefiniteness being a new *resource* for quantum information

A recap of the situation:

- We have a general framework to describe processes that are locally causal and obey quantum theory, but without assuming any global causal order
- Some of the consistent processes have physical interpretations, while others seem not to

- Which causally indefinite processes are physical and why?
- What would a faithful implementation of such a process look like?

Outline

- Rethinking causality in light of quantum theory
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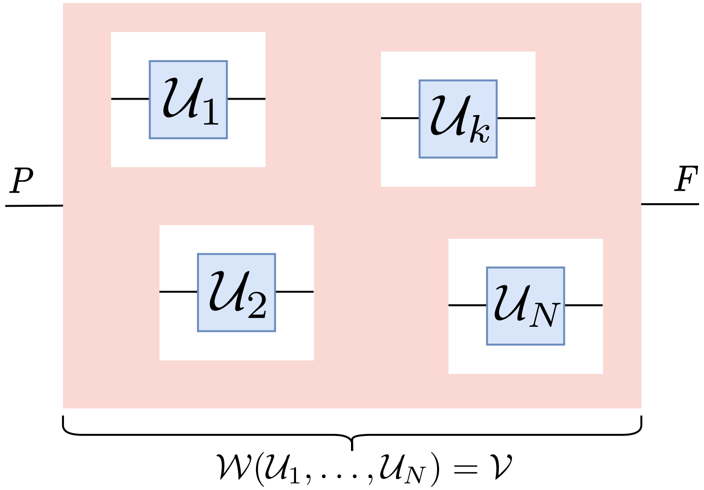
- Which causally indefinite processes are *physical* ?
  - Tow down approach: excluding processes using quantum information theory
  - Bottom up approach: constructing causal indefiniteness from quantum control
- Speculative comments, questions and discussion

A top down approach

- What reasonable principles could be used to rule out the non-physical instances of causal indefiniteness?
  - Violation of causal inequalities? Difficult to motivate *a priori*
  - Precedents in quantum information: information causality, PR boxes, GPT research program…

Pure and purifiable processes

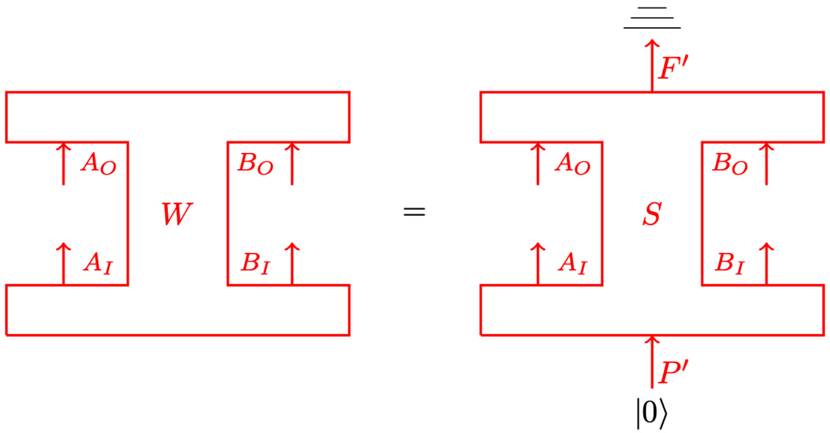
- Motivation: quantum theory governed by unitary dynamics
- Mixed states and non-unitary evolution can always be described as discarding/ignoring information
  - Mixed states can be purified, all quantum channels have unitary dilations
- Not the case for all process matrices!



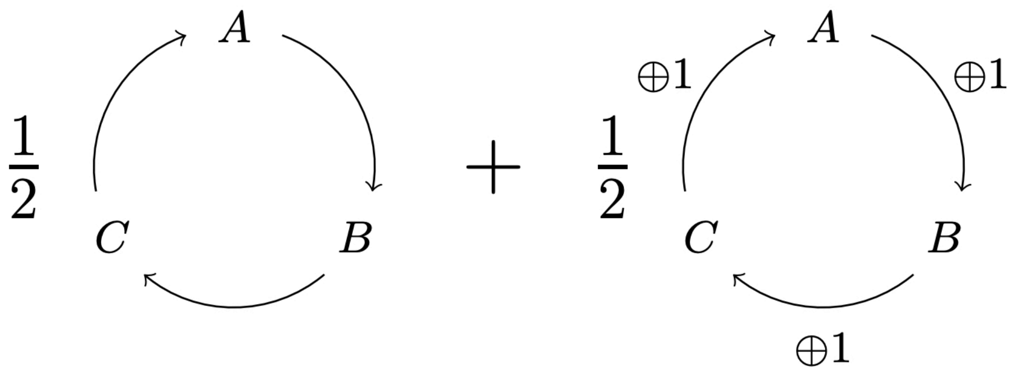
Purification postulate

- **Purification postulate:** All physical processes must be purifiable (unitarily extendible)
  - Rules out the "OCB" process
  - The quantum switch is pure

  [Araújo, Feix, Navascues, Brukner, *Quantum* (2017)]



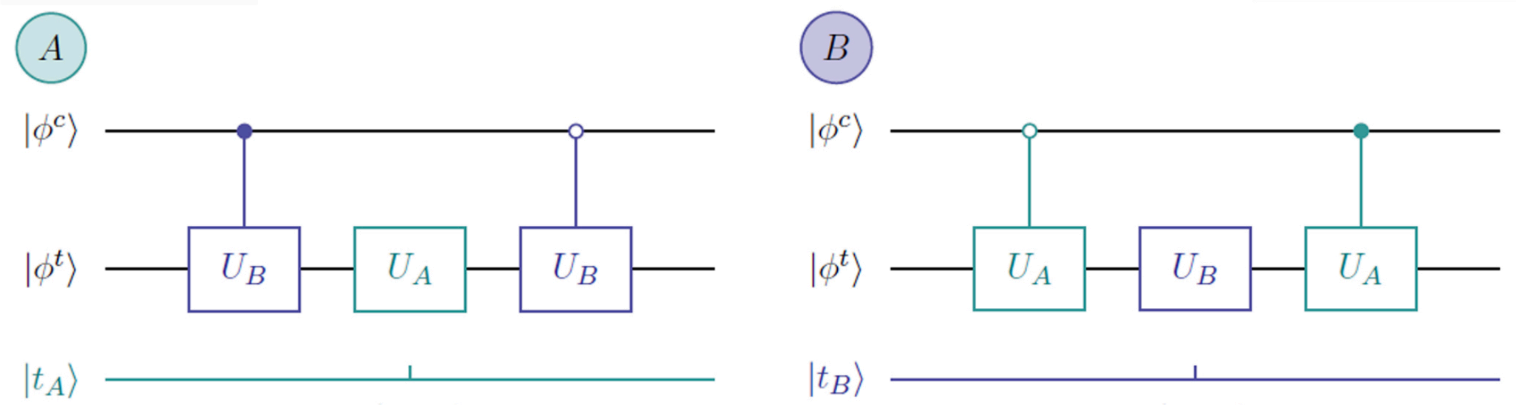
- A sufficient condition?
  - Some seemingly unphysical processes, such as the "Lugano process", are purifiable
    - Violates causal inequalities



[Baumeler & Wolf, *New J. Phys.* (2016)]

Interpretable via quantum causal reference frames

- Posit that each event is associated to a causal reference frame
  - An observer dependent clock
- Each event well localised in its reference frame, while others smeared out over past and future
- Consistency: overall process observer independent

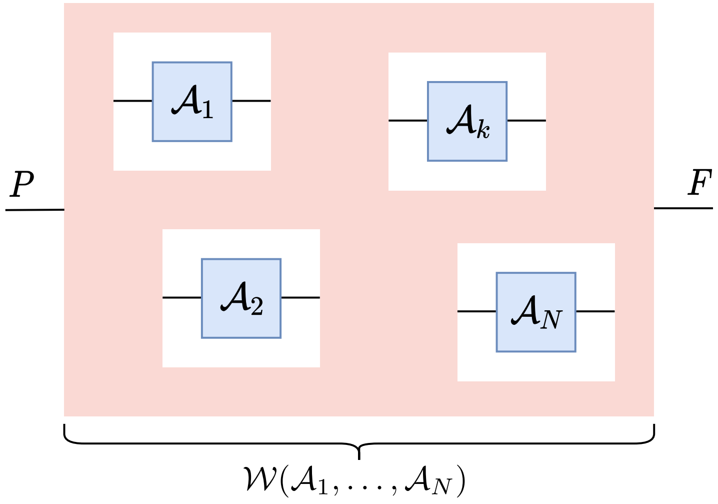


- Equivalent to purification postulate!
- Possible extension: ask that process recoverable from global Page-Wootters history state
  - Tentative evidence it might rule out more processes…

[Allard Guérin & Brukner, *New J. Phys.* (2018)]
[Baumann, Krumm, Allard Guérin, Brukner, Phys. Rev. Research (2022)]

A bottom up approach

- What is the most general concrete model of processes we can envisage?
  - The Quantum Switch uses a quantum control to superpose fixed causal orders. What more can we do?
- Approach motivated by computational view of processes as a compositional object
- Fixed order processes ≈ quantum circuits with open slots

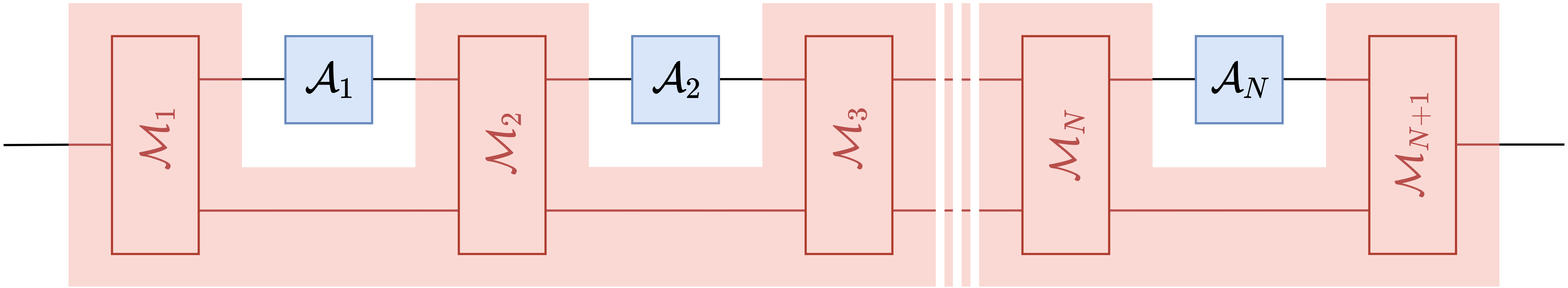


Goal: develop a generalised circuit model with fundamental quantum control of causal order

- Choice of which operation (= event) comes next may be controlled using all available information
- Needs to define a valid process matrix
- A subtle requirement: each event must occur exactly once

Quantum circuits (with fixed causal order)

Processes with fixed causal order can be represented as quantum circuits



- $\mathcal{M}_i$ are arbitrary CPTP maps
  - Can always be purified to unitary operations

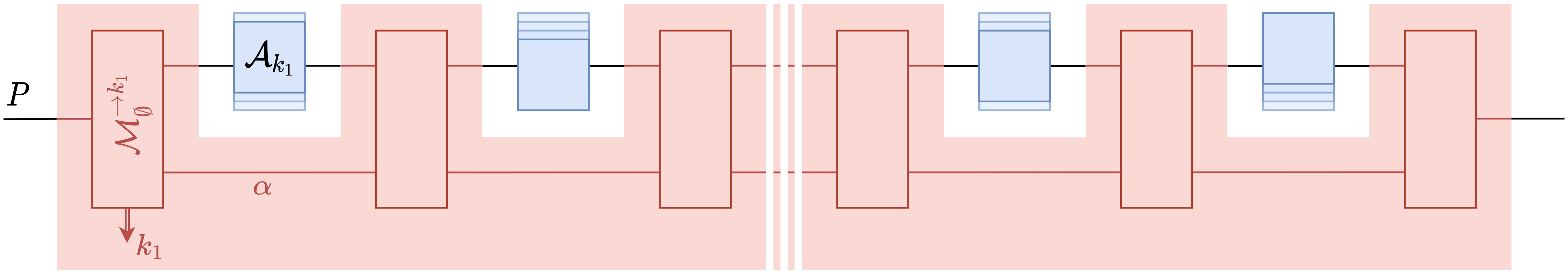
[G. Chiribella, G.M. D'Ariano, P. Perinotti, *Phys. Rev. Lett.* 101 (2008)]

A first step: classical control

- What about probabilistic or dynamical causal structures?

Quantum circuits with classical control of causal order (QC-CCs)

- Determine at each step which (unused) operation to apply next



1) Initial state prepared in global past P
2) Apply a "quantum instrument" $\{\mathcal{M}_{\emptyset}^{\to k_1}\}_{k_1}$ to determine which operation applied first

[Oreshkov, Giarmatzi, *NJP* (2016)]
[Wechs, Dourdent, Abbott, Branciard, *PRX Quantum* (2021)]

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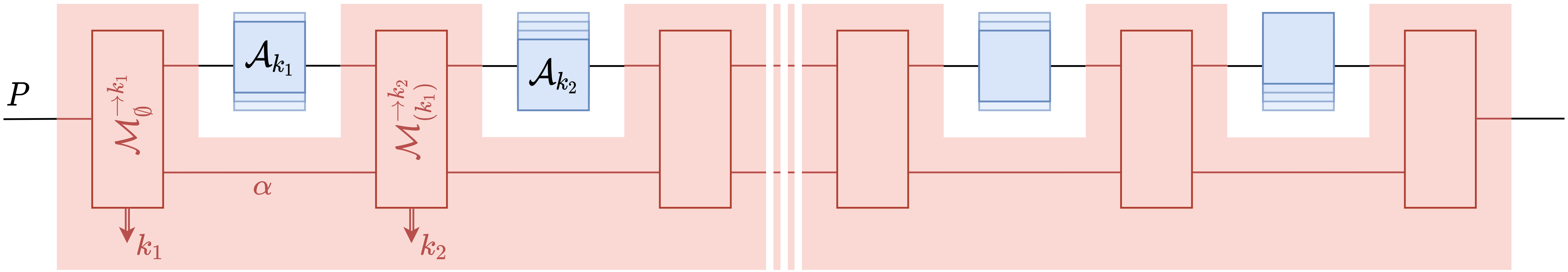

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[Oreshkov, Giarmatzi, *NJP* (2016)]
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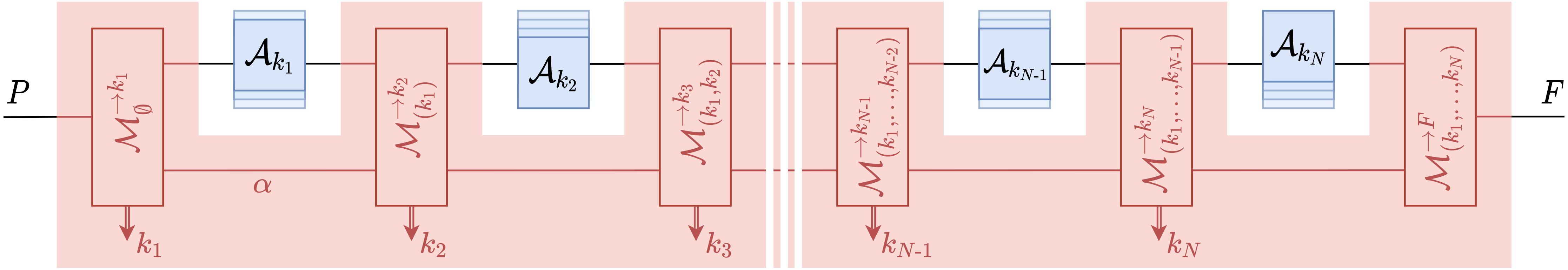

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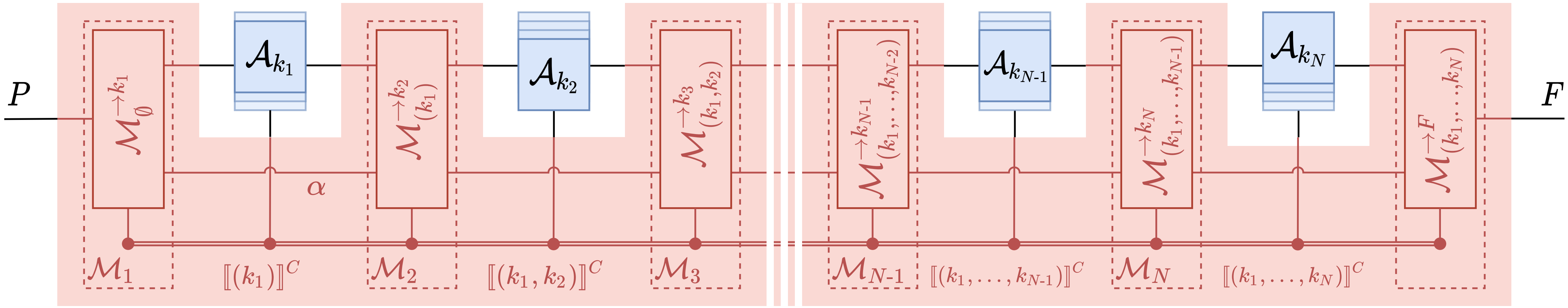
4) Etc.

[Oreshkov, Giarmatzi, *NJP* (2016)]
[Wechs, Dourdent, Abbott, Branciard, *PRX Quantum* (2021)]

Towards quantum control of causal order

- Problem: for quantum control, we don't want to destroy coherence by measuring
- Intermediate step: reformulate classical control with an explicit control system



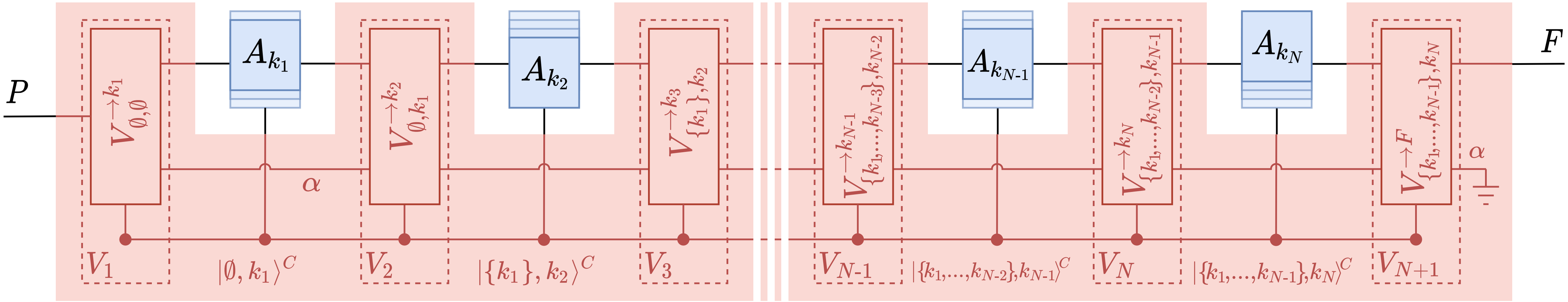
- "Classical" register $[\![(k_1,\ldots,k_n]\!] := |(k_1,\ldots,k_n)\rangle\!\langle(k_1,\ldots,k_n)|$ controls which operation applied
- Clearer path to purify control

[Wechs, Dourdent, Abbott, Branciard, *PRX Quantum* (2021)]

Quantum circuits with quantum control of causal order

Turning the classical control into a quantum one requires a few tweaks:

- Minimal information required in control system to ensure validity: $|\{k_1,\ldots,k_{n-1}\},k_n\rangle$
  - k_n: the operation to apply at "step" n
  - $\{k_1,\ldots,k_{n-1}\}$: history recording which operations already applied
- Internal operations V_i can be taken to be unitary – a purifiable process
- Circuit evolves coherently, exploring causal orders in a quantum superposition

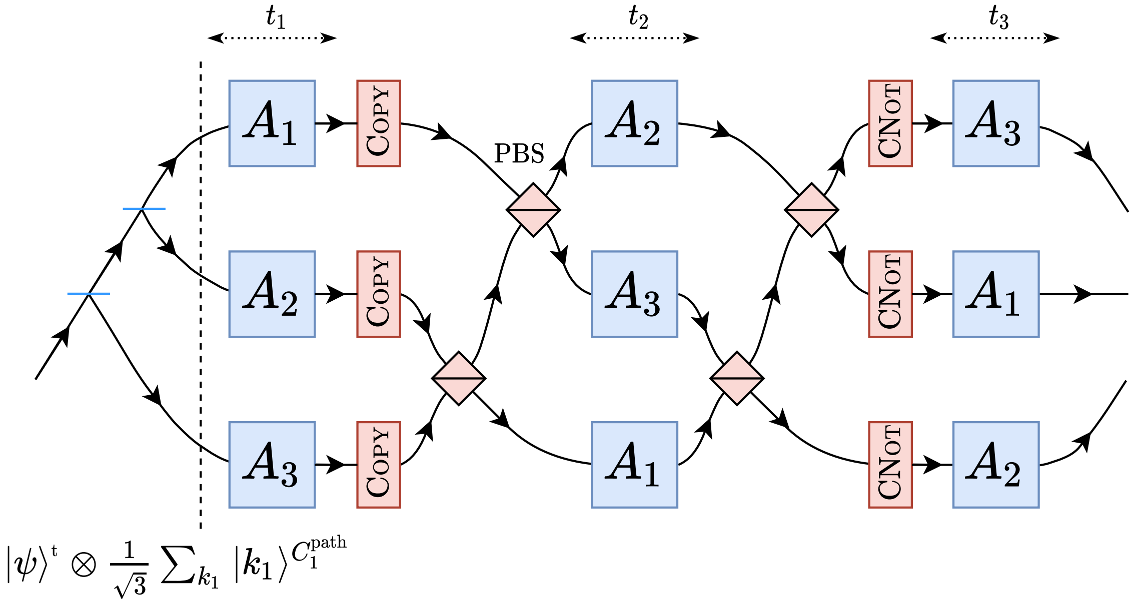


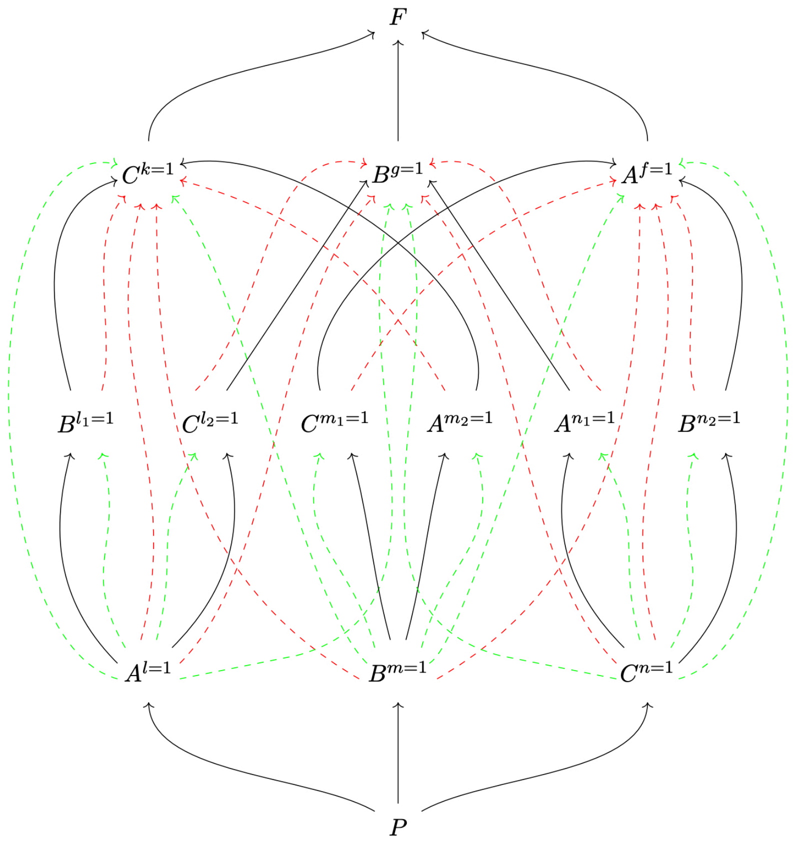
[Wechs, Dourdent, Abbott, Branciard, *PRX Quantum* (2021)]

Beyond the quantum switch

✓ QC-QCs encompass the quantum switch

- Do they go further, or does the quantum switch already capture everything about quantum control of causal order?
  ✓ Yes they do! Causal order can be controlled with dynamical quantum control

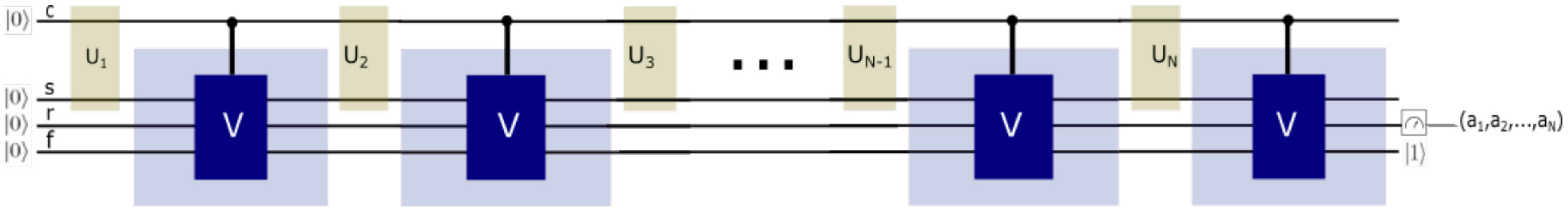




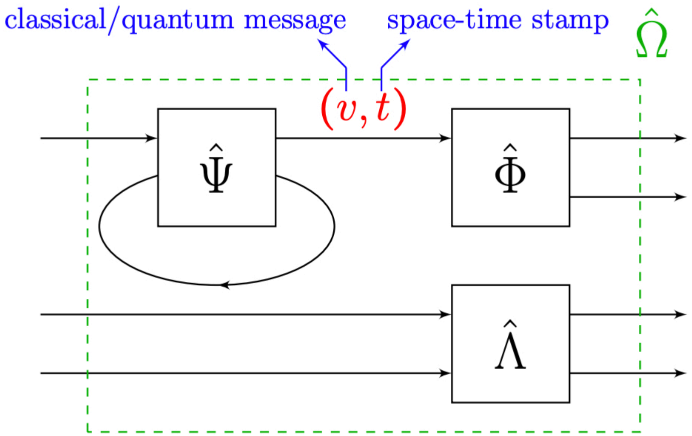
[Wechs, Dourdent, Abbott, Branciard, *PRX Quantum* (2021)]
[Vanrietvelde, Ormrod, Kristjánsson, Barrett, arXiv:2206.10042 (2022)]

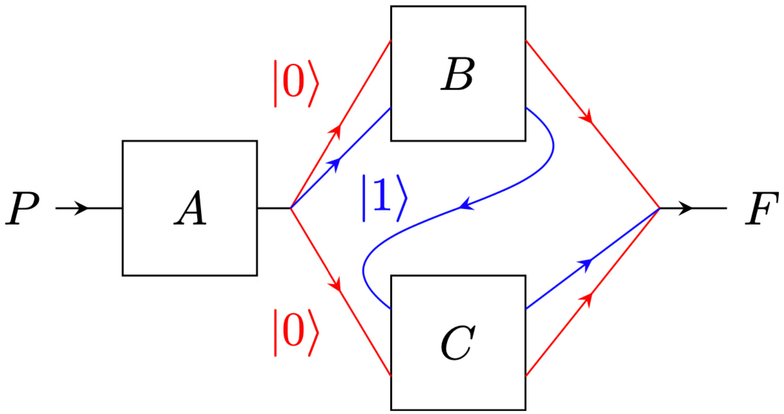
Convergent models

- Are there "physical" causally indefinite processes beyond QC-QCs?
- Purves & Short (PRL, also 2021) independently proposed a similar model of "quantum theory"



- Salzger & Vilasini (2022): Recover QC-QCs by adding a "closed laboratory" assumption to a model of causal boxes





Bottom up meets top down

Bottom up

- Concrete model of quantum circuits with quantum control of causal order (QC-QCs)
- Some convergence of different approaches to QC-QCs beginning to emerge
- Purifiable processes, proven not to violate causal inequalities

Top down

- Restriction to purifiable processes reasonable
- Lacking physical principal to rule out “Lugano process”

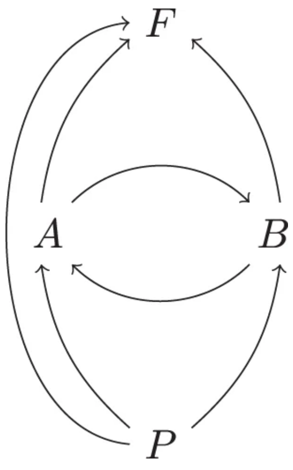
What’s left?

- Principals to close the gap (from physics, information theory, thermodynamics, philosophy)?
- Pure, non-QC-QC processes not violating causal inequalities?

Speculative directions: connecting to causal models

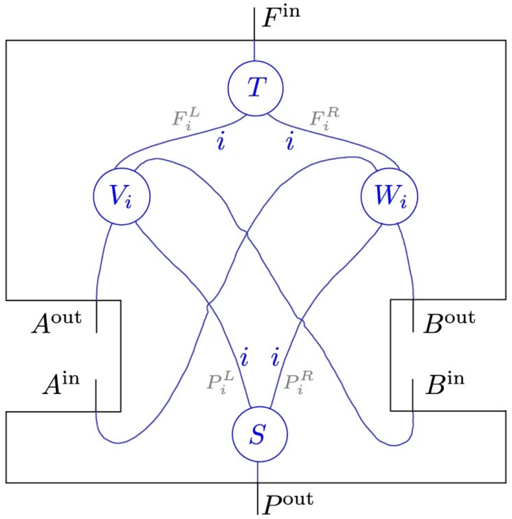
- Here I focused on causal ordering of a process, as defined by (no)-signalling
- What about the underlying causal influences, or causal model?
  - Recent definition of cyclic quantum causal models using purifiable processes
  - Which processes can arise from cyclic causal models?
  - What is the relation to causal indefiniteness?

[Barrett, Lorenz, Oreshkov, arXiv:1906.10726 (2019)]
[Barrett, Lorenz, Oreshkov, *Nat. Commun.* (2021)]



- Finer grained causal structure?
  - "Routed quantum circuits"
  - Physical processes: must have some specific type of fine grained causal structure?

[Vanrietvelde, Ormrod, Kristjánsson, Barrett, arXiv:2206.10042 (2022)]

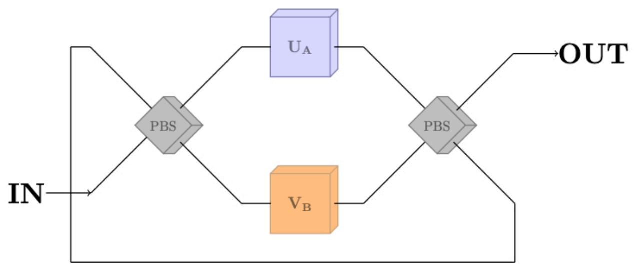


- Towards a fully quantum theory of causal inference

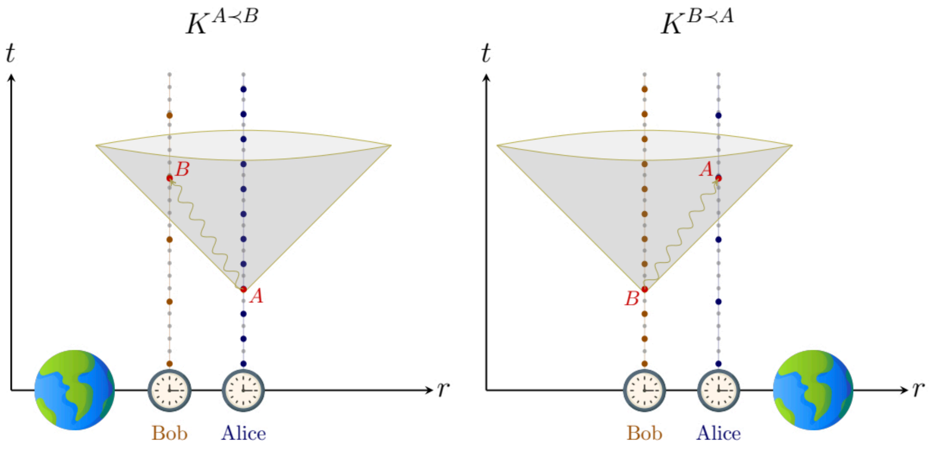
Experimental realisation of causal indefiniteness

- QC-QCs have a clear physical interpretation
- Do experiments "realising" them really do so? In what sense are these processes physical?
  - Experiments have been criticised on the basis that operation performed at different times are different events
- Core of the debate is a disagreement on how to define an "event": as a space-time point or in an operational way

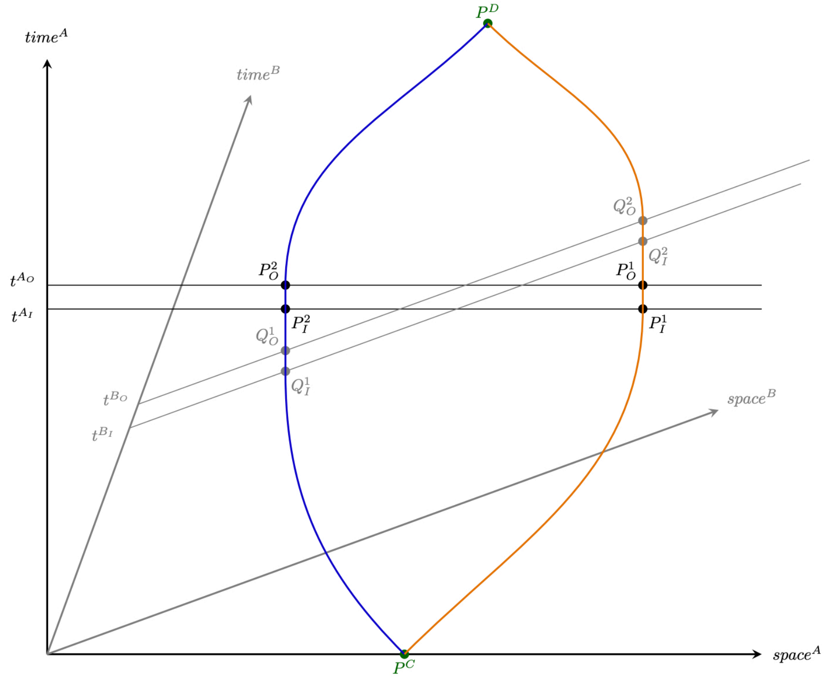
- Vilasini & Renner [arXiv:2203.11245]: it is impossible to implement a causally indefinite process in a fixed spacetime with spacetime localised events
  - OK with time-delocalised systems [Oreshkov, *Quantum* (2019)]
  - Can all purifiable processes be embedded in such a way?



(a) Optical quantum switch



(b) Gravitational quantum switch



Conclusions and some final questions

- Quantum theory forces us to challenge our ideas about causality
  - Need for new notions of fundamentally quantum causes
  - Possibility of indefinite causal order

- Large gap between causally indefinite processes that are consistent with local QM, and those we know how to implement
  - What causal indefiniteness is physical

- Quantum circuits with quantum control of causal order (QC-QCs)
  - Generalises quantum circuits and encompasses causal indefiniteness arising from quantum control
  - Potential new resource for quantum information theory
  - Pushes our understanding of physical/fundamental limits of computation
  - What advantages and applications?

Thank you for your attention!