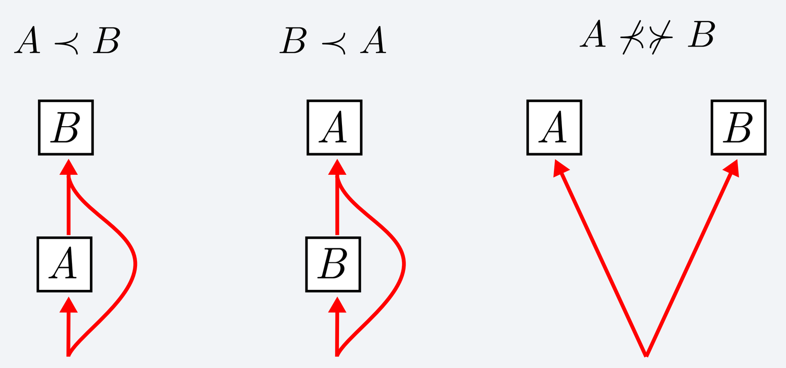
Indefinite Causal Relations in Multipartite Scenarios

Alastair A. Abbott, Julian Wechs and Cyril Branciard

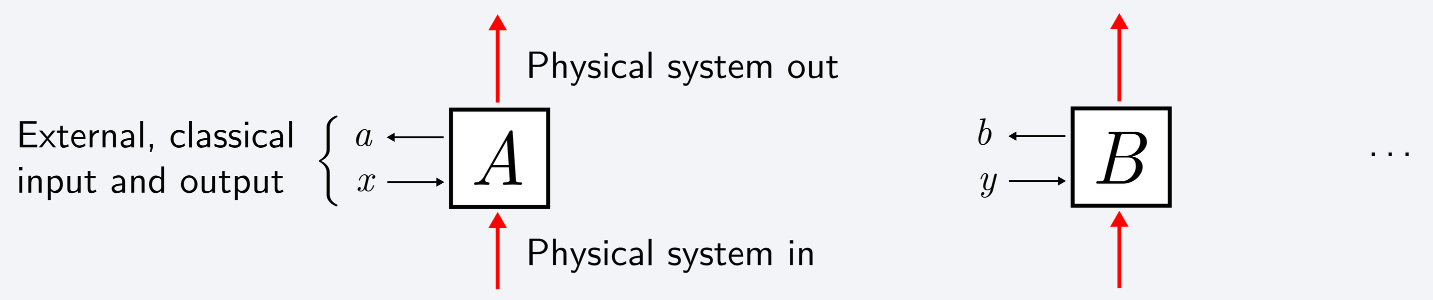
University Grenoble Alpes, CNRS, Grenoble INP, Institut Néel, 38000 Grenoble, France

Introduction

- One generally assumes that there is a well defined causal relation between different events that occur.
  - E.g. for events A and B there are 3 possible relations:



- It is convenient to idealise events as closed "local laboratories":



- The causal order of events imposes constraints on the correlations they can generate.
  - E.g. if $A \prec B$ then B cannot signal to A:

$$P^{A\prec B}(a|x,y) = P^{A\prec B}(a|x,y') \quad \text{for all } a,x,y,y'.$$

- Does quantum mechanics allow us to go beyond this classical notion of causal order, and could resulting *noncausal processes* and *correlations* be observed?

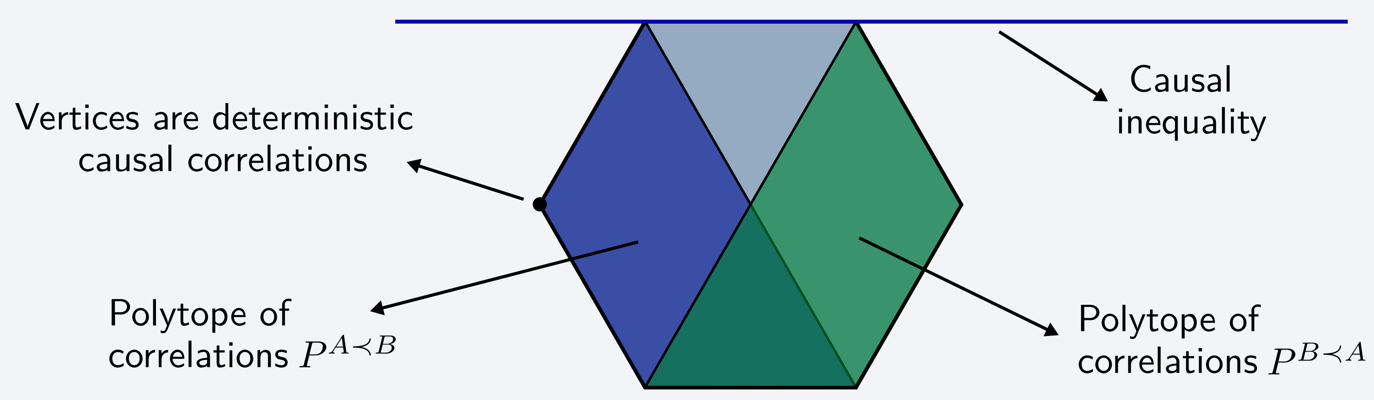
Bipartite Causal Correlations and Inequalities

- What correlations can two parties share if they always have a *definite causal order*, but without assuming any particular such order?

A bipartite correlation $P(a,b|x,y)$ is *causal* [1] iff it can be decomposed as

$$P(a,b|x,y) = q\, \underbrace{P'_A(a|x)\, P'_B(b|x,y,a)}_{A\prec B} + (1-q)\, \underbrace{P''_B(b|y)\, P''_A(a|x,y,b)}_{B\prec A}.$$

- The set of such correlations forms a convex polytope whose facets define *causal inequalities* [2].

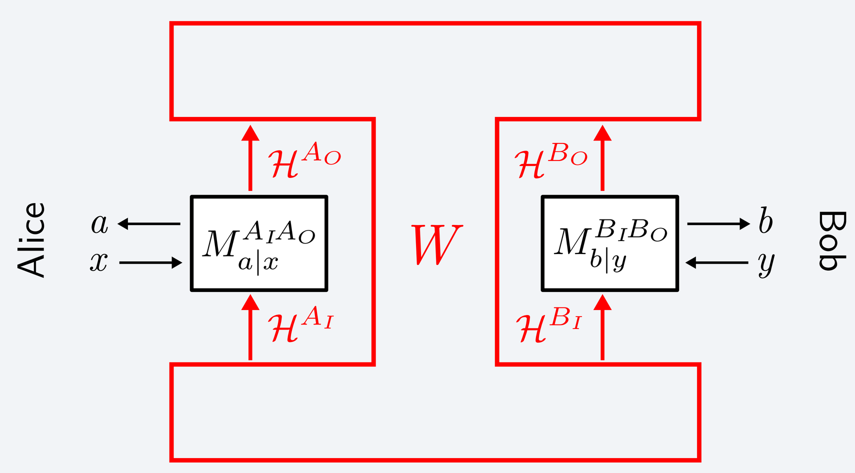


- E.g., for binary inputs and outputs, the "guess-your-neighbour's-input" (GYNI) inequality:

$$p_{\text{win}} = \frac{1}{4}\sum_{x,y} P\left(a=y,\, b=x \middle| x,y\right) \leq \frac{1}{2}.$$

The Process Matrix Formalism

- A framework for processes where parties obey local, causal quantum mechanics, but *no global causal order is assumed* [1].
- Parties process quantum systems and are modeled as *quantum instruments*.
- A *process matrix* W represents the most general way parties can interact under the requirement that the correlations they produce are always well defined.



$$P(a,b|x,y) = \mathrm{Tr}\left[(M_{a|x}^{A_I A_O} \otimes M_{b|y}^{B_I B_O}) \cdot W\right]$$

- Process matrices are Hermitian positive semidefinite operators on $\mathcal{H}^{A_I}\otimes\mathcal{H}^{A_O}\otimes\mathcal{H}^{B_I}\otimes\mathcal{H}^{B_O}$ obeying a finite set of additional linear constraints [1].

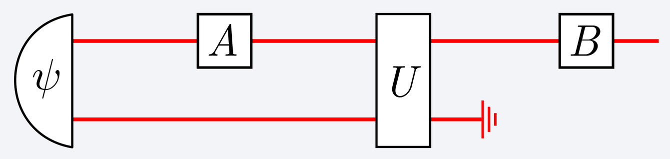
A bipartite process matrix W is *causally separable* iff it can be decomposed as

$$W = q\, W^{A\prec B} + (1-q)\, W^{B\prec A},$$

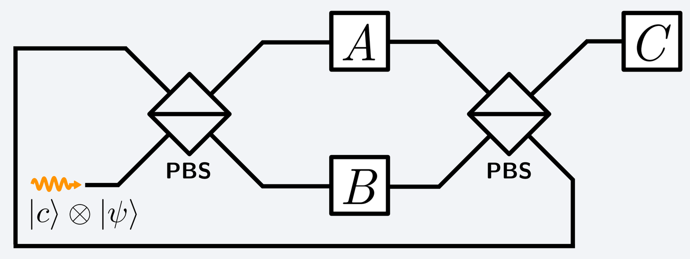
where $W^{A\prec B}$ is compatible with $A \prec B$, etc. Otherwise it is *causally nonseparable*.

Causally Separable, Nonseparable, and Noncausal Processes

- All process matrices compatible with a fixed causal order can be implemented as quantum circuits, e.g. when $A \prec B$:



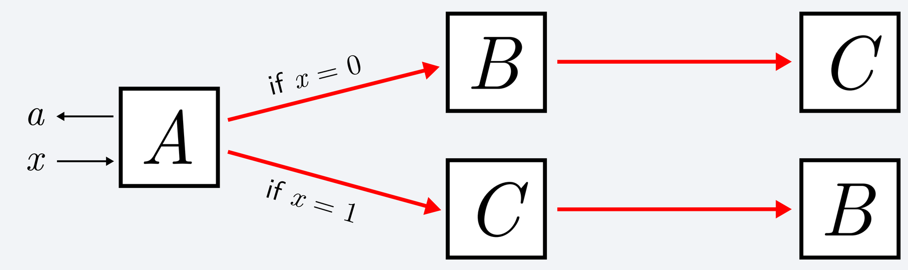
- Using a coherent control qubit, one can create quantum superpositions of different causal orders, thus implementing the *quantum switch*:



- The quantum switch is causally nonseparable but cannot violate any causal inequality [2].
- Process matrices violating causal inequalities exist, but it is an open question whether any of them can be physically implemented.

Multipartite Causality

- Multipartite scenarios offer promise for finding new types of causally nonseparable processes, which may present novel phenomena or even violate causal inequalities.
  - Richer and more complex structure: in addition to fixed causal orders such as $A \prec B \prec C \prec \cdots$, parties can exhibit *dynamical causal orders* [3], e.g.



N-partite causal correlations $P(\vec{a}|\vec{x})$ are defined recursively [4]:

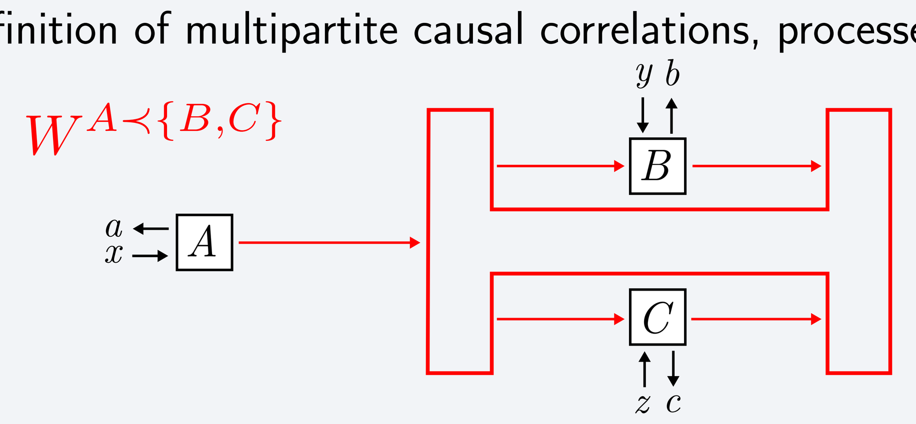
1) For $N=1$, any $P(a_1|x_1)$ is causal.
2) For $N\geq 2$, $P(\vec{a}|\vec{x})$ is causal iff it can be written

$$P(\vec{a}|\vec{x}) = \sum_k q_k\, \underbrace{P_k(a_k|x_k)}_{A_k \text{ acts first}}\, \underbrace{P_{k,x_k,a_k}(\vec{a}_{\mathcal{N}\setminus k}|\vec{x}_{\mathcal{N}\setminus k})}_{\text{(N-1)-partite } causal \text{ correlation}}.$$

- We characterised the simplest tripartite scenario and found all (302) nontrivial causal inequalities, many of which can be understood as "causal games".
  - All 302 causal inequalities can be violated by process matrices.

Genuinely Multipartite Noncausality?

- With this definition of multipartite causal correlations, processes such as



may produce noncausal correlations despite one party having a definite causal relation to the others: this noncausality is *not genuinely multipartite*.

- In [5] we define a notion of *genuinely multipartite noncausality (GMNC)* and characterise the simplest nontrivial scenario, finding some tight inequalities detecting GMNC that appear *not* to be violated by process matrices.

Conclusions

- The process matrix formalism allows one to consider quantum phenomena occurring in the absence of a global causal order, and which may generate noncausal correlations.
- Understanding the structure of such correlations and process matrices, and when they are physically meaningful, is crucial to our understanding of causality.
- Multipartite scenarios have a richer structure than bipartite ones and manifest novel phenomena, but much remains to be understood about their characterisation and the advantages they may provide in quantum information and computing.

References

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alastair.abbott@neel.cnrs.fr