## Problem 1

Consider the boundary-value problem

$$\begin{aligned} \frac{d^2u}{dx^2} + u + x &= 0 \quad \text{in } \Omega = (0,1) , \\ u &= 1 \quad \text{on } \Gamma_u = \{0\} , \\ \frac{du}{dx} &= 0 \quad \text{on } \Gamma_q = \{1\} . \end{aligned}$$

Assume a general three-parameter polynomial approximation to the exact solution, in the form

$$u_h(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 . \tag{$\dagger$}$$

(a) Place a restriction on parameters $\alpha_i$ by enforcing only the Dirichlet boundary condition, and obtain a *Bubnov-Galerkin* approximation of the solution.

(b) Starting from the general quadratic form of $u_h$ in ($\dagger$), place a restriction on parameters $\alpha_i$ as in part (a), and determine a *Petrov-Galerkin* approximation of the solution assuming

$$w_h(x) = \beta_1 \psi_1(x) + \beta_2 \psi_2(x) ,$$

where functions $\psi_1(x)$ and $\psi_2(x)$ are defined as

$$\psi_1(x) = \begin{cases} 0 & \text{for } 0 \le x \le \frac{1}{2} \\ 1 & \text{for } \frac{1}{2} < x \le 1 \end{cases} ,$$

$$\psi_2(x) = \begin{cases} x & \text{for } 0 \le x \le \frac{1}{2} \\ 0 & \text{for } \frac{1}{2} < x \le 1 \end{cases} .$$

Clearly justify the admissibility of $w_h$ for the proposed approximation.