CEE 6513

HW # 4.

Donglai Yang

$$-\frac{d^2u}{dx^2} = 1. \quad , \quad u(0) = 0, \quad u'(l) = 0.$$

Analytical solution.

$$\int -\frac{d^2u}{dx^2}dx = \int dx$$

$$\Rightarrow -\frac{du}{dx} = x + C \qquad u'(l) = -x - c\Big|_l = 0 \Rightarrow c = -L.$$

$$\int -\frac{du}{dx}dx = \int (x+c)dx.$$

$$\Rightarrow -u = \frac{1}{2}x^2 + cx + d.$$

$u(0) = 0.$ Thus $d = 0.$

Therefore $u = -\frac{1}{2}x^2 + Lx$

Numerical solution.

$$-\frac{d^2u}{dx^2} \approx -\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2}$$

Thus $\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} = -1. \Rightarrow u_{i+1} - 2u_i + u_{i-1} = -h^2$

Since we don't know l, let's let $l = 1$ for computation.

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & & 0 \\ 1 & -2 & 1 & \cdots & & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & -4 & 3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} 0 \\ -h^2 \\ -h^2 \\ \vdots \\ 0 \end{bmatrix}$$

Second order backward FD.

$$(3U_N - 4N_{N-1} + U_{N-2})/(2h) = \frac{du}{dx}\Big|_l = 0$$

we solve for $\vec{u}$
see matlab script for details.

The plot is separate andattached. The convergence rate is the slope of loglog plot which gives -1.8. I still have not figured out why the slope is negative.