HW #6

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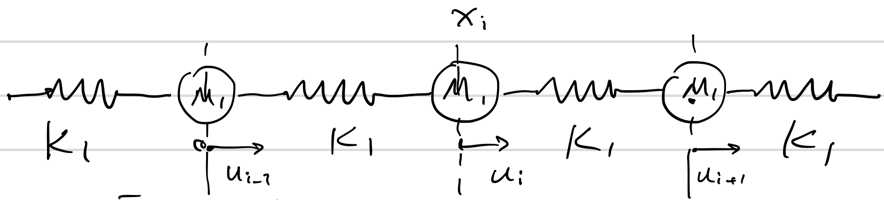
1. when $K_2 = 0$



For each mass (here we use index i)

$$\vec{F} = m\vec{a} \Rightarrow M_1 \ddot{u}_i = (u_{i+1} - u_i)k_1 + (u_i - u_{i-1})k_1$$
$$= u_{i+1}k_1 - u_{i-1}k_1$$
$$= k_1(u_{i+1} - u_{i-1})$$

Turn this into a system equation for $i = 1, 2, 3 \cdots N$

RHS: $$\omega^2 \underbrace{\begin{bmatrix} M_1 & & & & \\ & M_1 & & & \\ & & M_1 & & \\ & & & \ddots & \\ & & & & M_1 \end{bmatrix}}_{N} \Bigg\} N \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

LHS: $$\underbrace{\begin{bmatrix} 0 & k_1 & & & -k_1 \\ -k_1 & 0 & k_1 & & \\ 0 & -k_1 & 0 & k_1 & \\ & & \ddots & \ddots & \ddots \\ -k_1 & \cdots & & k_1 & 0 \end{bmatrix}}_{\text{Stiffness matrix.}} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

Equation 14:
$$\tilde{K}_{mm} \hat{u}_m = \omega_m^2 \tilde{M}_{mm} \tilde{u}_m$$

Equation 15:
$$\tilde{K}_{mm} = \sum_{n=-(N-1)/2}^{(N-1)/2} e^{-ik_m \cdot x_n} \tilde{K}_{0N}, \quad \hat{M}_{mm} = \sum_{n=-(N-1)/2}^{(N-1)/2} e^{-ik_m \cdot x_n} M_{0N}$$

Hence $$\tilde{K}_{mm} = K_{00} + e^{-ik_m L_0} K_{01} + e^{-i2k_m L_0} K_{02} + e^{ik_m L_0} K_{0(-1)} + e^{i2k_m L_0} K_{0(-2)}$$
$$= -k_1 e^{-ik_m L_0} - k_1 e^{ik_m L_0} = -k_1(e^{-ik_m L_0} + e^{ik_m L_0})$$
$$= -2k_1 \cos(k_m L_0)$$

$$\tilde{M}_{mm} = M_1$$

Thus $$\omega_m^2 = \frac{-2k_1}{M_1} \cos(k_m L_0)$$