Problem 1

Consider the initial-value problem

$$\begin{aligned} \frac{du}{dt} + u &= t \quad \text{in } \Omega = (0,1) \ , \\ u(0) &= 1 \ , \end{aligned}$$

and assume a general three-parameter polynomial approximation u_h written as

$$u_h(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 \ ,$$

where α_i, $i = 0, 1, 2$, are scalar parameters to be determined.

(a) Place a restriction on u_h by directly enforcing the initial condition, and, subsequently, obtain an approximate solution to the problem using the point-collocation method. Select the collocation points judiciously.

(b) Place the same restriction on u_h as in part (a), and obtain an approximate solution to the problem using a Bubnov-Galerkin method.

Problem 2

Consider the initial-value problem

$$\begin{aligned} \frac{du}{dt} - u &= 0 \quad \text{in } \Omega = (0,T) \ , \\ u(0) &= 1 \ , \end{aligned}$$

where T is a given positive number, and let a two-parameter polynomial approximation u_h be expressed as

$$u_h(t) = \alpha_0 + \alpha_1 t \ .$$

(a) Obtain a reduced form of $u_h(t)$ by directly enforcing the initial condition.

(b) Determine u_h as a function of t and T using a domain least-squares method in $(0,T)$.

(c) Find the limit of the approximate solution $u_h(t)$ obtained in part (b), as T approaches zero, i.e., as the domain $(0,T)$ of the analysis becomes arbitrarily small. How does the approximate solution compare with the exact solution $u = e^t$ in this limiting case?