CEE 6513
HW#5
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1. $$\begin{cases} u_{tt} = u_{xx} \\ u(x,t) = 0,\ t \le 0,\ 0 \le x \le L \\ u(0,t) = f(t) = \begin{cases} 1-\cos(2t) & 0 \le t \le \pi \\ 0 & \text{o.w.} \end{cases} \\ u(L,t) = 0 \end{cases}$$

use central difference in space and time

$$\cancel{\frac{u_j^{n+1} - u_j^n}{\Delta t} = c\,\frac{u_{j+1}^n - u_{j-1}^n}{}}$$

$$\frac{u_j^{n+1} - 2u_j^n + u_j^{n-1}}{(\Delta t)^2} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta x)^2}$$

j: space index
n: time index

$$\Rightarrow u_j^{n+1} = r^2\left[u_{j+1}^n - 2u_j^n + u_{j-1}^n\right] + 2u_j^n - u_j^{n-1}$$

where $\underbrace{r = c\frac{\Delta t}{\Delta x}}_{\text{CFL number}}$. where $c=1$ and we set $r=1$

This is our update equation.
Since it's a 2-step method, to initialize we add a fictious grid at u_j^{-1} and $u_j^{-1} = [0, 0 \cdots 0]$
because the bar is at rest at $i=0$. thus also at rest at $i=-1$.
The 2 dirichlet Boundary conditions are enforced at each time step.
The plot for $t=20, 40, 60, 80, 100$ are shown, but all lines overlap so you can only see $t=100$. To animate the wave motion, you can run section 2 in the matlab script.
dt or δt is chosen to be < 0.5 to ensure sufficient sampling rate.

An additional plot using L = 6 is shown to illustrate non-overlapping wave at different times