HW#2, MTH 537.

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1. a) $g(x) = 4\mu x(1-x)$. $x_{k+1} = g(x_k)$

fixed point definition $g(x) = x$.

Hence. $x = 4\mu x(1-x)$.

Apparently $z=0$ is one fixed point

if $x>0$. $1 = 4\mu(1-x)$.

$1 = 4\mu - 4\mu x$

$x = \frac{4\mu-1}{4\mu} = 1 - \frac{1}{4\mu}$

for $x>0$ $\frac{1}{4\mu} < 1$, given $\mu \in (0,1)$

$\therefore \mu > \frac{1}{4}$

b) $g'(x) = \frac{d}{dx}(4\mu x - 4\mu x^2) = |4\mu - 8\mu x| < 1$.

fixed point $z=0$. $|4\mu| < 1$. $\mu < \frac{1}{4}$ since $\mu \in (0,1)$

$0 < \mu < \frac{1}{4}$

Non-zero fixed point

① $\begin{cases} 4\mu - 8\mu x > 0. \\ 4\mu - 8\mu x < 1 \\ z = 1 - \frac{1}{4\mu}. \end{cases}$

$4\mu - 8\mu(1 - \frac{1}{4\mu}) = 4\mu - 8\mu + 2 < 1 \Rightarrow \mu > \frac{1}{4}$

also $-4\mu + 2 > 0$

$2\mu < 1 \Rightarrow \mu < \frac{1}{2}$

Hence $\frac{1}{4} < \mu < \frac{1}{2}$

② $\begin{cases} -4\mu + 8\mu x < 1 \\ 4\mu - 8\mu x < 0 \\ z = 1 - \frac{1}{4\mu} \end{cases}$

$-4\mu + 8\mu(1 - \frac{1}{4\mu}) = -4\mu + 8\mu - 2 < 1$

$4\mu < 3$

$\mu < \frac{3}{4}$

Also $4\mu - 8\mu x < 0 \Rightarrow -4\mu + 2 < 0$

$2\mu > 1$

$\mu > \frac{1}{2}$ hence $\frac{1}{2} < \mu < \frac{3}{4}$

③ when $\mu = \frac{1}{2}$ $g'(x) = 0$

It also works since it's second-order convergence

Hence the range of μ is $\frac{1}{4} < \mu < \frac{3}{4}$

c). $\tilde{g}(x) = g(g(x))$ if z is a fixed point of g.

$g(z) = z \Rightarrow 4\mu z(1-z) = z$

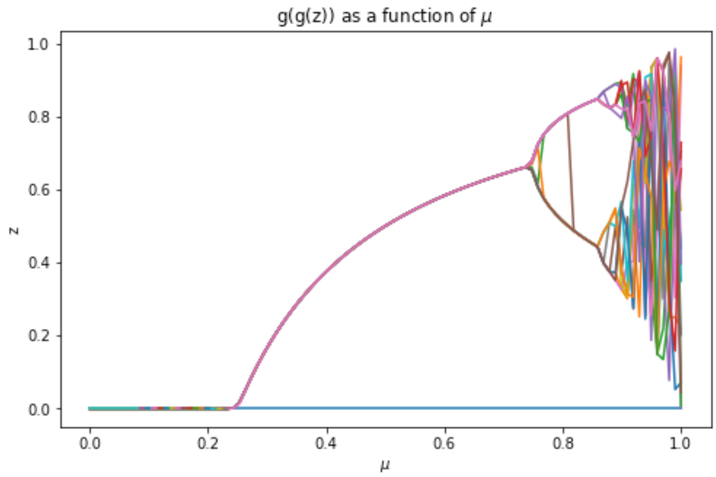
$g(g(z)) = 4\mu[4\mu z(1-z)]\{1 - [4\mu z(1-z)]\}$

$= 4\mu z(1-z) = z$.

Hence $g(g(z)) = z$

fixed points of $\tilde{g}(x)$.

I use fixed point method to solve $\tilde{g}(x)$ on python



lines of different colors represent different initial conditions/seeds.

It shows when $\mu \sim 0.7$ there is bifurcation (period-2 limit cycle)

2.

a) Bisection method.

b).

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Mean Lipschitz constant is 0.67
Bisection is faster since its error upper bound is halved every iteration, where as according to the Lipschitz cons
tant I derived from fixed point method, the error decay rate is more than half, meaning slower.
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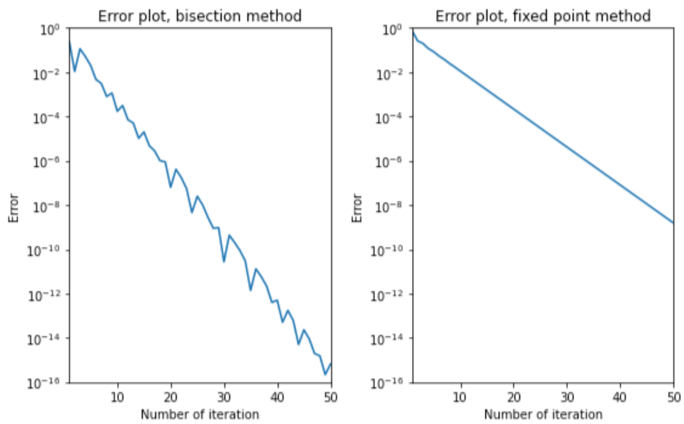

The upper bond of L is found via

$$e^k / e^{k-1}$$

This 0.67 value is the average of all L estimated in this way (between every 2 iterations).

c).

Newton's method is the fastest, since it has quadratic order of convergence. The other two are linear convergence.