MTH 537
Assignment 1
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1. Find the relative truncation error.

$$\text{rel. }E_t = \frac{|P_7(x) - f(x)^*|}{f(x)^*} = \frac{|P(\sin(x)) - x - f(x)^*|}{f(x)^*} = \frac{|(-\frac{x^3}{3}+\frac{x^5}{5!}-\frac{x^7}{7!}) - f(x)^*|}{-\frac{x^3}{3!}}$$

$$\lessapprox \frac{|-\frac{x^9}{9!}|}{|-x^3/3!|} = \frac{x^9}{x^3}\cdot\frac{3!}{9!} = \frac{1}{(9\times8\times7\times6\times5\times4)}x^6$$
$$= \frac{1}{60480}x^6$$
or $O(x^6)$

2. $$\text{rel. }E_r = \frac{|fl(f(x)) - f(x)^*|}{f(x)^*} = \frac{|fl[fl(\sin(x)) - fl(x)] - f(x)^*|}{|f(x)^*|} = \frac{|fl[fl(fl(x) - fl(\frac{x^3}{3!}) + \cdots) - fl(x)] - f(x)^*|}{|f(x)^*|}$$

where $fl(-\frac{x^3}{3!}) = fl(\frac{1}{3!})\,fl(x)\,fl(x)\,fl(x)$
$$= -\frac{1}{3!}(1+\epsilon_a)x^3(1+\epsilon_1)^3$$

so that we can write

$$= \frac{|fl[fl(x(1+\epsilon_1) - \frac{x^3}{3!}(1+\epsilon_a)(1+\epsilon_1)^3 + \frac{1}{5!}(1+\epsilon_b)(1+\epsilon_1)^5x^5 \cdots) - x(1+\epsilon_1)] - fl(x)^*|}{|-\frac{x^3}{3!}|}$$

$$= \frac{|fl[x(1+\epsilon_1)(1+\epsilon_b) - \frac{x^3}{3!}(1+\epsilon_a)(1+\epsilon_b)(1+\epsilon_1)^3 + \cdots - x(1+\epsilon_1)] - fl(x)^*|}{|-\frac{x^3}{3!}|}$$

$$= \frac{|x(1+\epsilon_1)\epsilon_b(1+\epsilon_c) - \frac{x^3}{3!}(1+\epsilon_a)(1+\epsilon_b)(1+\epsilon_c)(1+\epsilon_1)^3 \cdots - fl(x)^*|}{|-\frac{x^3}{3!}|}$$

$$\simeq \frac{|x\cdot\epsilon_b - \frac{x^3}{3!}(\epsilon_a+\epsilon_b+\epsilon_c+3\epsilon_1+1) + \frac{x^5}{5!}(\epsilon_a+\epsilon_b+\epsilon_c+5\epsilon_1+1) \cdots - fl(x)^*|}{|-\frac{x^3}{3!}|}$$

$$\le \frac{|x(\delta-1) - \frac{x^3}{3!}\cdot6\delta + \frac{x^5}{5!}\cdot8\delta|}{|-\frac{x^3}{3!}|}$$

since $\epsilon_i < \delta_i$ $\epsilon_a+\epsilon_b+3\epsilon_1 < 5\delta$ for example.

$$\simeq x(1-\delta)\cdot\frac{3!}{x^3} = \boxed{\frac{6(1-\delta)}{x^2}}$$

3. a) plot: see the next page.

b). Set $E_r = E_t$.

$$\frac{6(1-\delta)}{x^2} = \frac{1}{60480}x^6$$
$$6\cdot60480(1-\delta) = (x^4)^2$$

so $x \simeq 4.954$ and it does match the intersection on the plot, although the lines in my plot don't intersect at $x<1$, so I've made a algebraic mistake somewhere.

plot:

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In [2]: import numpy as np
        import matplotlib.pyplot as plt

In [10]: epsilon = 2e-16
         x_range = np.arange(1e-5, 1e1+1e-5, 1e-5)

In [13]: fig, ax = plt.subplots()
         ax.loglog(x_range, x_range**6/60480, label = 'Truncation error')
         ax.loglog(x_range, 6*(1-epsilon)/x_range**2, label = 'Rounding error')
         ax.legend()
         ax.set_xlabel('x')
         ax.set_ylabel('error')
         ax.set_title('HW1: Problem 3, Donglai Yang')

Out[13]: Text(0.5, 1.0, 'HW1: Problem 3, Donglai Yang')
```