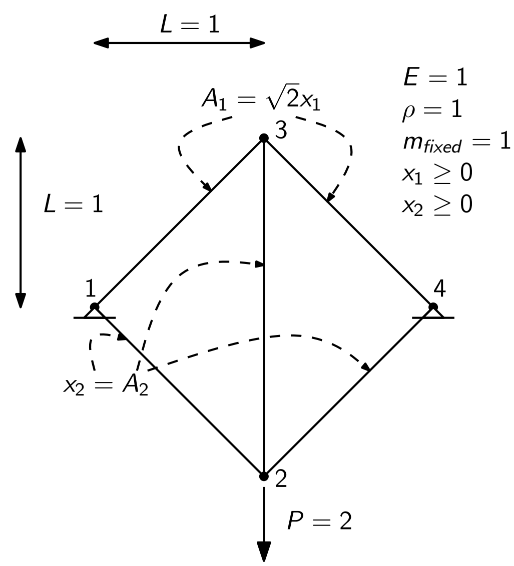
Structural Optimization
Assignment 2

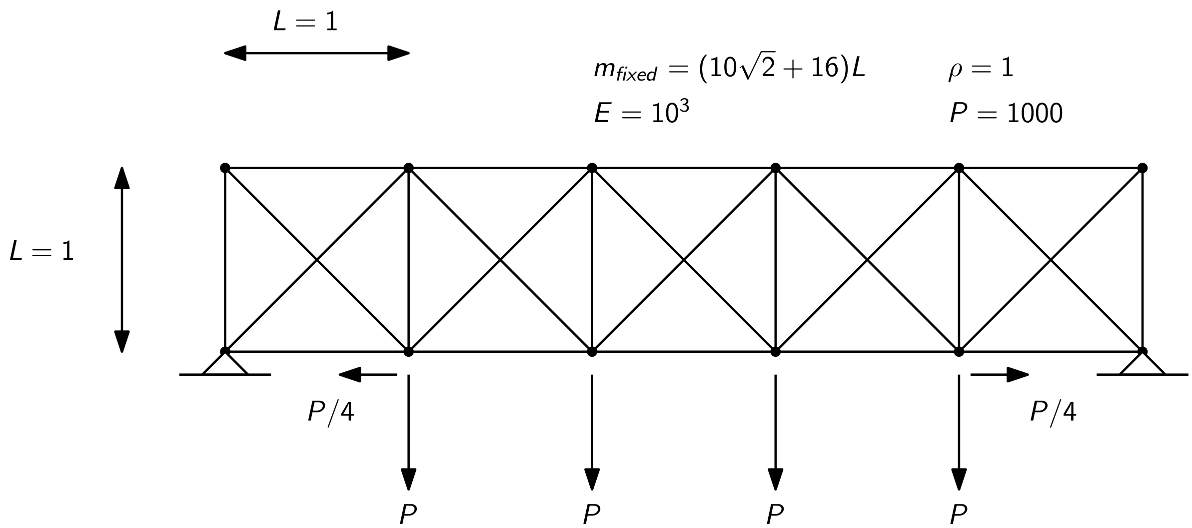
- Organize your work and be careful to properly answer all parts of each question. Points will be deducted for disorganized presentation of results.
- You may use ChatGPT or other AI tools to assist you with these problems. If you do use any AI tools, you must briefly report (1) how you used it to assist you with your assignment and (2) whether you found it useful.

1. (35 points) This question is based on the truss below. Show a derivation for all expressions. It would be a good idea, but is not required, to use your code to verify your answers.



   (a) Find the reduced global stiffness matrix as a function of the design variables.
   (b) Derive an expression for the compliance and mass of the truss.
   (c) Show that the Hessian of the compliance is positive definite for all positive values of the design variables.
   (d) Plot the design space for the mass constrained compliance minimization problem. Illustrate the feasible space and show the constrained minimizer.
   (e) Find the design variable values at the constrained minimizer.

2. (35 points) Write an analysis and optimization code for mass-constrained compliance minimization and apply it on the truss illustrated below. Use the methods presented in class to determine the derivative of the compliance with respect to the design variables. This code should supply the gradients of the objective and constraints to the optimizer to avoid finite-differening.



(a) Check your implementation of the compliance derivative using finite-difference and complex-step method. Make a relative error vs. step size plot that shows that your code computes the correct gradient.

(b) Find an appropriatel scaling for the design variables, the objective and the constraints. Report what you find.

(c) Plot a visualization showing the dimensions of the optimized bars. Plot the deflected structure (exaggerating the deflection if required.)

(d) Consider the effect of the lower bound on the bar areas. Vary the lower bound over the range $A_{\min} \in [10^{-6}, 10^{-2}]$ and plot the optimized compliance vs lower bound.

(e) Vary the lower bound over the range $A_{\min} \in [10^{-6}, 10^{-2}]$ and plot the maximum strain in any of the bars in the truss.

3. (30 points) The following question is based on a ground structure for the set of N nodes given at the following locations for $k = 1, \dots, N$:

$$x_k = \cos\left(\frac{(k-1)}{N-1}\pi\right)$$
$$y_k = \sin\left(\frac{(k-1)}{N-1}\pi\right)$$

In a ground structure, you must connect every node to every other node. The node at $k = 1$ is fixed against movement in the x and y direction, the node at $k = N$ is fixed against movement in the y direction. All nodes from $k = 2, \dots, N-1$ are subject to the load $100/(N-2)$ in the negative y direction. Use the properties $E = 1$ and $\rho = 1$. Use the bounds $A_i \in [10^{-6}, 1]$.

(a) Using the bar areas as design variables, optimize the trusses for the cases $N = 5$, 10, 25, 40, 50 with the mass constraint $m_{fixed} = 1$.

(b) Optimize the same set of trusses using SIMP penalization with $P = 3$. This time use the fixed mass constraint $m_{\text{fixed}} = N/4$.

(c) Plot the compliance of the optimized trusses for both the sizing and SIMP trusses.

(d) Plot the number of optimization iterations for both the sizing optimization and the SIMP optimized trusses.