AE6170 Structural Optimization
Midterm

Answer the following questions and submit them on Canvas as either a pdf document or images. Clearly indicate the question numbers and your answers. The midterm is out of 100 points.

1. (10 points) Show that the following function is convex for $x_1 > 0$ and $x_2 > 0$

$$f(x) = \frac{1}{x_1 + 2x_2}$$

2. (20 points) Consider the compliance evaluated using a SIMP penalization parameter p on the interval $0 \leq x \leq 1$.

$$f(x) = \frac{1}{(2 + (1-x)^p)} + \frac{1}{(2 + x^p)}$$

   (a) For the case $p = 1$, find the minimizer on the interval $0 \leq x \leq 1$.

   (b) For the case $p = 3$, determine whether the point from part (a) is a minimizer still.

3. (40 points) Using a finite-element program, you evaluate the stiffness matrix and force vector for a truss problem and find the result

$$K(x) = \begin{bmatrix} 1 + x_1 + 2x_2 & 0 \\ 0 & 5x_1 + x_2 \end{bmatrix} \qquad f = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$$

   You wish to impose the mass constraint $x_1 + x_2 \leq 1$ with the additional constraints that $x_1 \geq 0$ and $x_2 \geq 0$.

   (a) Find an expression for the compliance as a function of x_1 and x_2

   (b) Write out the mass constrained compliance minimization problem in standard form.

   (c) Compute the gradients of the mass and compliance

   (d) Find the KKT point and state the values of the design variables and multipliers

4. (30 points) You evaluate the stress constraints find that the ratio of stresses obey the following convenient form:

$$s_1 = 2x_1$$
$$s_2 = 3$$

   for the scalar $x_1 \in \mathbb{R}$.

   (a) Find an expression for the KS function with parameter ρ

   (b) At the point $x_1 = 3/2$, show that $c_{KS} \geq 3$ for any value of ρ

   (c) At the point $x_1 = 0$, show that $c_{KS} \geq 3$ for any value of ρ

   (d) At both the points $x_1 = 3/2$ and $x_1 = 1$ show that in the limit as $\rho \to \infty$, $\lim_{\rho \to \infty} c_{KS} = 3$