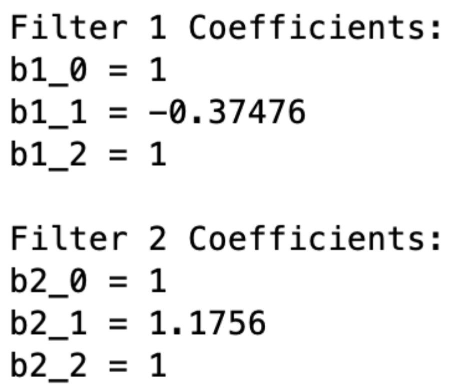
3.1 Nulling Filters for Rejection

A)

```
clear
clc

wn1 = 0.44 * pi; % First nulling frequency
wn2 = 0.7 * pi; % Second nulling frequency

% Coefficients for the first nulling filter
b1_0 = 1;
b1_1 = -2 * cos(wn1);
b1_2 = 1;

% Coefficients for the second nulling filter
b2_0 = 1;
b2_1 = -2 * cos(wn2);
b2_2 = 1;

% Coefficients Displaying of Filters
disp('Filter 1 Coefficients:');
disp(['b1_0 = ', num2str(b1_0)]);
disp(['b1_1 = ', num2str(b1_1)]);
disp(['b1_2 = ', num2str(b1_2)]);
disp(' ');

disp('Filter 2 Coefficients:');
disp(['b2_0 = ', num2str(b2_0)]);
disp(['b2_1 = ', num2str(b2_1)]);
disp(['b2_2 = ', num2str(b2_2)]);
```

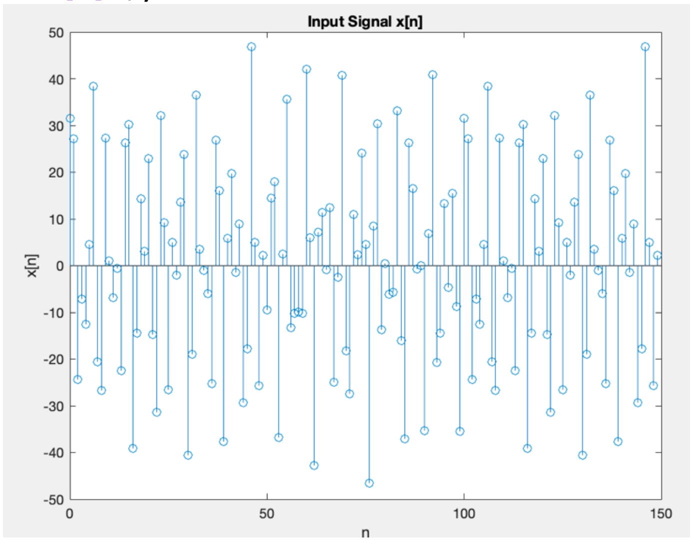


```
Filter 1 Coefficients:
b1_0 = 1
b1_1 = -0.37476
b1_2 = 1

Filter 2 Coefficients:
b2_0 = 1
b2_1 = 1.1756
b2_2 = 1
```

B)

```
clear
clc

N = 150; % Length of the Signal
n = 0:N-1; % Time Index
w1 = 0.3 * pi; % Frequency of first sinusoid
w2 = 0.44 * pi; % Frequency of second sinusoid
```

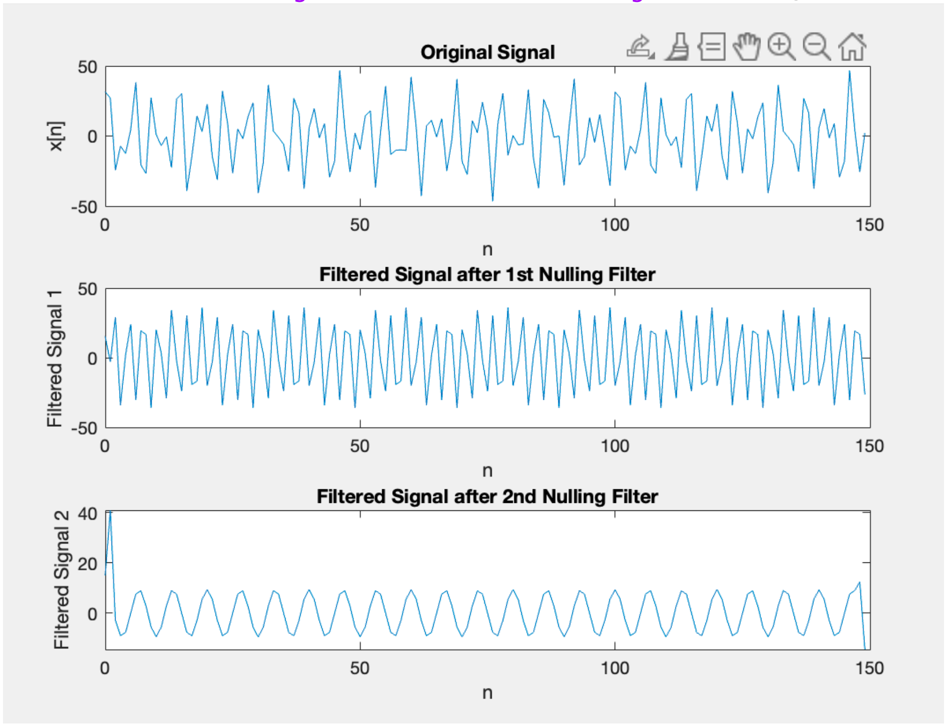
```
w3 = 0.7 * pi; % Frequency of third sinusoid
phi1 = 0; % Phase of first sinusoid
phi2 = -pi/3; % Phase of second sinusoid
phi3 = -pi/4; % Phase of third sinusoid
x1 = 5 * cos(w1 * n); % First sinusodial component
x2 = 22 * cos(w2 * n + phi2); % Second sinusodial component
x3 = 22 * cos(w3 * n + phi3); % Third sinusodial component
x = x1 + x2 + x3; % Input Signal Calculation

% Plotting the Input Signal
figure;
stem(n, x);
xlabel('n');
ylabel('x[n]');
title('Input Signal x[n]');
```



C)

```
clear
clc

wn1 = 0.44 * pi; % First nulling frequency
wn2 = 0.7 * pi; % Second nulling frequency

% Coefficients for the first nulling filter
b1_0 = 1;
b1_1 = -2 * cos(wn1);
b1_2 = 1;

% Coefficients for the second nulling filter
b2_0 = 1;
b2_1 = -2 * cos(wn2);
b2_2 = 1;

% Waveform Properties
N = 150; % Signal length
n = 0:N-1; % Time Index
w1 = 0.3 * pi; % Frequency of first sinusoid
w2 = 0.44 * pi; % Frequency of second sinusoid
w3 = 0.7 * pi; % Frequency of third sinusoid
```

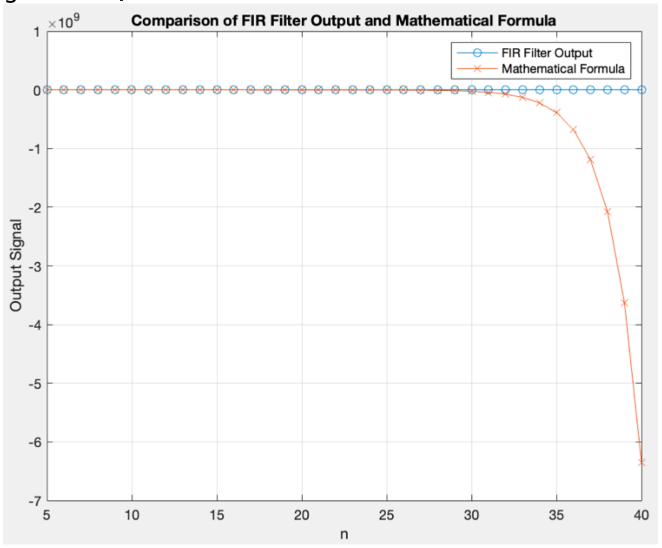
```
phi1 = 0; % Phase of first sinusoid
phi2 = -pi/3; % Phase of second sinusoid
phi3 = -pi/4; % Phase of third sinusoid
x1 = 5 * cos(w1 * n); % First sinusodial component
x2 = 22 * cos(w2 * n + phi2); % Second sinusodial component
x3 = 22 * cos(w3 * n + phi3); % Third sinusodial component
x = x1 + x2 + x3; % Input Signal Calculation

b1 = [b1_0, b1_1, b1_2]; % First nulling filter coefficients
b2 = [b2_0, b2_1, b2_2];  % Second nulling filter coefficients

% Nulling filter implementation
filtered_x1 = conv(x, b1, 'same');
filtered_x2 = conv(filtered_x1, b2, 'same');

% Plotting the signals [Original & Filtered]
figure;
subplot(3,1,1);
plot(n, x);
xlabel('n');
ylabel('x[n]');
title('Original Signal');
subplot(3,1,2);
plot(n, filtered_x1);
xlabel('n');
ylabel('Filtered Signal 1');
title('Filtered Signal after 1st Nulling Filter');
subplot(3,1,3);
plot(n, filtered_x2);
xlabel('n');
ylabel('Filtered Signal 2');
title('Filtered Signal after 2nd Nulling Filter');
```



D & E)

```
clear
clc

wn1 = 0.44 * pi; % First nulling frequency
wn2 = 0.7 * pi; % Second nulling frequency

% Coefficients for the first nulling filter
b1_0 = 1;
b1_1 = -2 * cos(wn1);
b1_2 = 1;

% Coefficients for the second nulling filter
b2_0 = 1;
b2_1 = -2 * cos(wn2);
b2_2 = 1;

% Waveform Properties
N = 150; % Signal length
n = 0:N-1; % Time Index
w1 = 0.3 * pi; % Frequency of first sinusoid
w2 = 0.44 * pi; % Frequency of second sinusoid
w3 = 0.7 * pi; % Frequency of third sinusoid
phi1 = 0; % Phase of first sinusoid
phi2 = -pi/3; % Phase of second sinusoid
phi3 = -pi/4; % Phase of third sinusoid
x1 = 5 * cos(w1 * n); % First sinusodial component
x2 = 22 * cos(w2 * n + phi2); % Second sinusodial component
x3 = 22 * cos(w3 * n + phi3); % Third sinusodial component
x = x1 + x2 + x3; % Input Signal Calculation

% Nulling filter implementations
filtered_x1 = conv(x, [b1_0, b1_1, b1_2], 'same');
filtered_x2 = conv(filtered_x1, [b2_0, b2_1, b2_2], 'same');

% Define the mathematical equation for the output signal
% For n >= 5, y[n] = x[n] - 2 * cos(wn1) * x[n-1] + x[n-2] - 2 * cos(wn2) *
y[n-1] + y[n-2]
y_mathematical = zeros(1, N);
y_mathematical(1:4) = filtered_x2(1:4); % Initial values
for i = 5:N
    y_mathematical(i) = filtered_x2(i) - 2 * cos(wn1) * filtered_x2(i-1) +
filtered_x2(i-2) ...
                         - 2 * cos(wn2) * y_mathematical(i-1) +
y_mathematical(i-2);
end

% Plot both the mathematical formula and FIR filter output over the range 5
<= n <= 40
range = 5:40;
figure;
plot(range, filtered_x2(range), 'o-', 'DisplayName', 'FIR Filter Output');
hold on;
plot(range, y_mathematical(range), 'x-', 'DisplayName', 'Mathematical
Formula');
```

```
xlabel('n');
ylabel('Output Signal');
title('Comparison of FIR Filter Output and Mathematical Formula');
legend;
grid on;
```



F)

The output signal is not same for the first points due to the transient response or "start-up" attribute of the filters. This start-up behavior exists since the filters necessiate time to be balanced and attain to the steady-state after the input signal is applied.

In the FIR filters domain, the start-up points are the first points in the output signal where the filter response has not yet totally balanced. Points are affected by the initial situations and transient impacts of the filtering process.

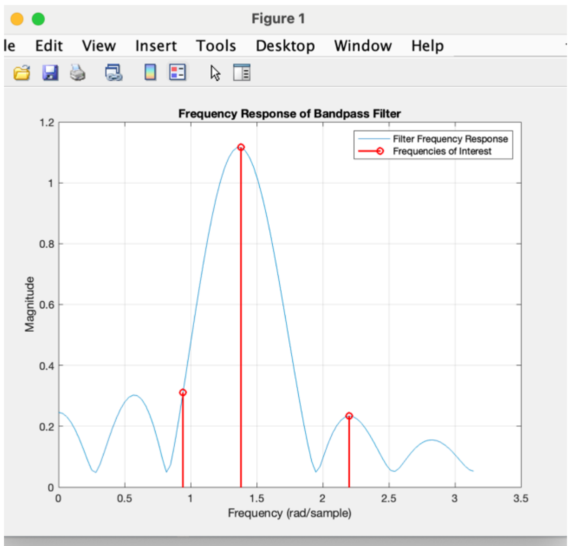
For the cascade of two length-3 FIR filters designed in part (a), each single filter has a length of 3 coefficients. However, when cascaded, the length of the combined filter is the sum of the lengths of the filters minus one. Due to that, when convolving two filters, the length of output is the sum of the individual filter lengths minus one.

Hereby, for two length-3 FIR filters cascaded each other, the length of the combined filter is $3 + 3 - 1 = 5$. Thus, there are 5 start-up points in the output signal, which is relevant with the lengths of the filters designed in part (a). These start-up points correspond to the first transient attributes of the filtering process, and they converge to the steady-state response.

3.2 Simple Bandpass Filter Design

A)

```
clear
clc

L = 10; % Filter length parameter
wc = 0.44*pi; % Center frequency parameter
n = 0:L; % Index
h = 2/L * cos(wc * n); % Impulse response of bandpass filter
w = 0:pi/100:pi; % Angular frequency variable
H = freqz(h, 1, w); % Frequency response of filter

% Gain at interest interval
w_interest = [0.3*pi, wc, 0.7*pi]; % Interest interval
gain_interest = abs(interp1(w, H, w_interest)); % Gain interest value

% Result Displaying & Plotting
disp('Gain at frequencies of interest:');
disp(['w = 0.3*pi: ', num2str(gain_interest(1))]);
disp(['w = 0.44*pi: ', num2str(gain_interest(2))]);
disp(['w = 0.7*pi: ', num2str(gain_interest(3))]);
figure;
plot(w, abs(H));
hold on;
stem(w_interest, gain_interest, 'r', 'LineWidth', 1.5);
xlabel('Frequency (rad/sample)');
ylabel('Magnitude');
title('Frequency Response of Bandpass Filter');
legend('Filter Frequency Response', 'Frequencies of Interest');
grid on;
```



ommand Window

```
Gain at frequencies of interest:
w = 0.3*pi: 0.31132
w = 0.44*pi: 1.1161
w = 0.7*pi: 0.23371
```

B)

```
clear
clc

L_values = [10, 20, 40];  % Filter length parameters
wc = 0.44*pi;              % Center frequency parameter
passband_widths = zeros(1, length(L_values)); % Arrays to store passband
widths

% Frequency response for each of the filter lengths
figure;
for i = 1:length(L_values)
    L = L_values(i);

    n = 0:L; % Index
    h = 2/L * cos(wc * n); % Impulse Response of the bandpass filter
    w = 0:pi/1000:pi; % Angular frequency variable
    H = freqz(h, 1, w); % Frequency response of filter

    % Find frequencies on |H(e^jw)| >= 0.707 * H_max
    passband_indices = find(abs(H) >= 0.707 * max(abs(H)));
    passband_widths(i) = w(passband_indices(end)) - w(passband_indices(1));
    % Plotting of frequency response
    subplot(length(L_values), 1, i);
    plot(w, abs(H), 'LineWidth', 1.5);
    hold on;
    stem(w(passband_indices), abs(H(passband_indices)), 'r', 'LineWidth',
1.5);
    xlabel('Frequency (rad/sample)');
    ylabel('Magnitude');
    title(['Frequency Response for L = ', num2str(L)]);
    legend('Filter Frequency Response', 'Passband Region');
    grid on;
end

% Displaying results
disp('Passband widths at 0.707 level:');
disp(passband_widths);
```

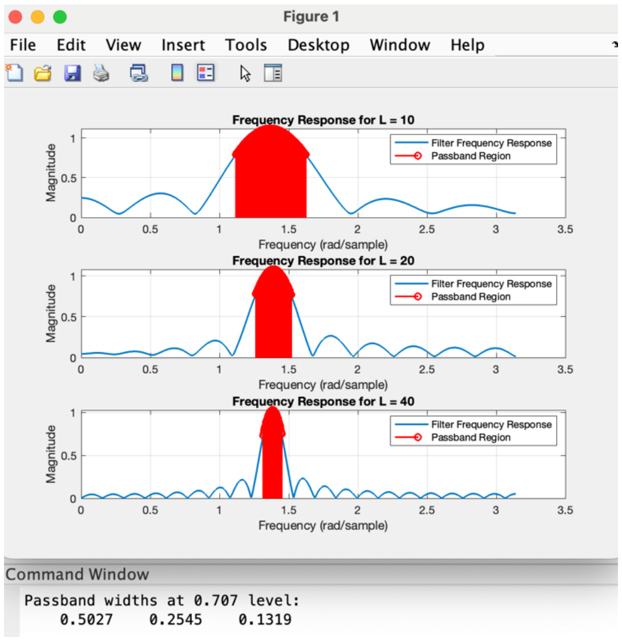


Figure 1
File Edit View Insert Tools Desktop Window Help
Frequency Response for L = 10
Frequency Response for L = 20
Frequency Response for L = 40
Magnitude
Frequency (rad/sample)
Filter Frequency Response
Passband Region
Command Window

```
Passband widths at 0.707 level:
    0.5027     0.2545     0.1319
```

C)

The selectivity of a filter touch on its features to pass particular frequencies while filtering or rejecting others. For the $L = 10$ bandpass filter located at $w = 0.44\pi$, its selectivity can be explained by the attributes of its frequency response.

Inspecting frequency response plot, the filter display a peak in magnitude in the neighborhood of the center frequency of $w = 0.44\pi$. This peak is the passband region where frequencies located neighborhood points of center frequency are admitted to pass. It means, the filter "passes" frequencies close to $w = 0.44\pi$.

At frequencies lower ($w = 0.3\pi$) or higher ($w = 0.7\pi$) than the center frequency, the magnitude of the frequency response reduces reasonably. This filtering represents the filter's properties to reduce or reject frequencies other than the passband region.

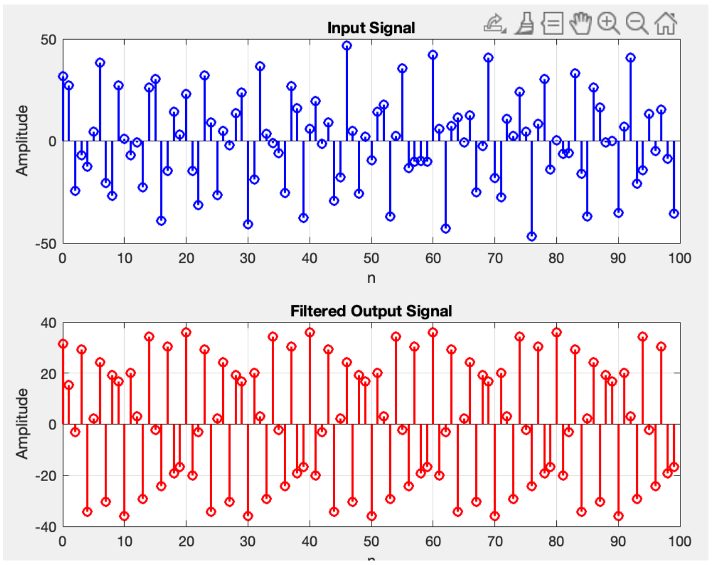
The algorithm of this selectivity depends on the model of the filter's impulse response. By structuring the filter with a certain impulse response that indicates the desired frequency component at $w = 0.44\pi$ while attenuating others, the filter effectively behaves as a bandpass filter, admitting only the requested frequency to "pass through."

D)

```
clear
clc

wc = 0.44*pi; % Center frequency parameter
desired_attenuation = 10; % Desired attenuation factor
L = 5; % Starting value of filter length to initialize
step_size = pi/100000; % Define step size for frequency response calculation
while true
    % Generate impulse response of the bandpass filter
    n = 0:L; % Index
    h = 2/L * cos(wc * n); % Impulse Response of the bandpass filter
    w = 0:step_size:pi; % Angular frequency variable
    H = freqz(h, 1, w); % Frequency response of filter

    % Find frequencies on |H(e^jw)| <= 0.1 * H_max for w <= 0.3*pi
    passband_indices_low = find(abs(H) <= 0.1 * max(abs(H)), 1, 'last');
    % Find frequencies on |H(e^jw)| <= 0.1 * H_max for w >= 0.7*pi
    passband_indices_high = find(abs(H) <= 0.1 * max(abs(H)), 1, 'first');
    if passband_indices_low <= 0.3*pi && passband_indices_high >= 0.7*pi %
Desired attenuation verifier
        break;
    end

    L = L + 1; % Filter length increment
end

% Displaying Smallest Value
disp(['Smallest value of L: ', num2str(L)]);
```

E)

```
clear
clc

n = 0:99; % Time Index
x = 5*cos(0.3*pi*n) + 22*cos(0.44*pi*n - pi/3) + 22*cos(0.7*pi*n - pi/4); %
Signal
b = [1, -2*cos(0.44*pi), 1]; % Filter coefficient of y[n]
a = [1]; % Filter coefficient of x[n]
y = filter(b, a, x); % Input signal filtering

% 100 points plotting of input and output signals
subplot(2,1,1);
stem(n, x(1:100), 'b', 'LineWidth', 1.5);
xlabel('n');
ylabel('Amplitude');
title('Input Signal');
grid on;
subplot(2,1,2);
stem(n, y(1:100), 'r', 'LineWidth', 1.5);
xlabel('n');
ylabel('Amplitude');
title('Filtered Output Signal');
grid on;
```



Explanation:

• The input signal occurs with three sinusoidal elements: a $5\ Hz$ cosine wave, a $22\ Hz$ cosine wave with a phase shift of $\pi/3$, and a $22\ Hz$ cosine wave with a phase shift of $\pi/4$.

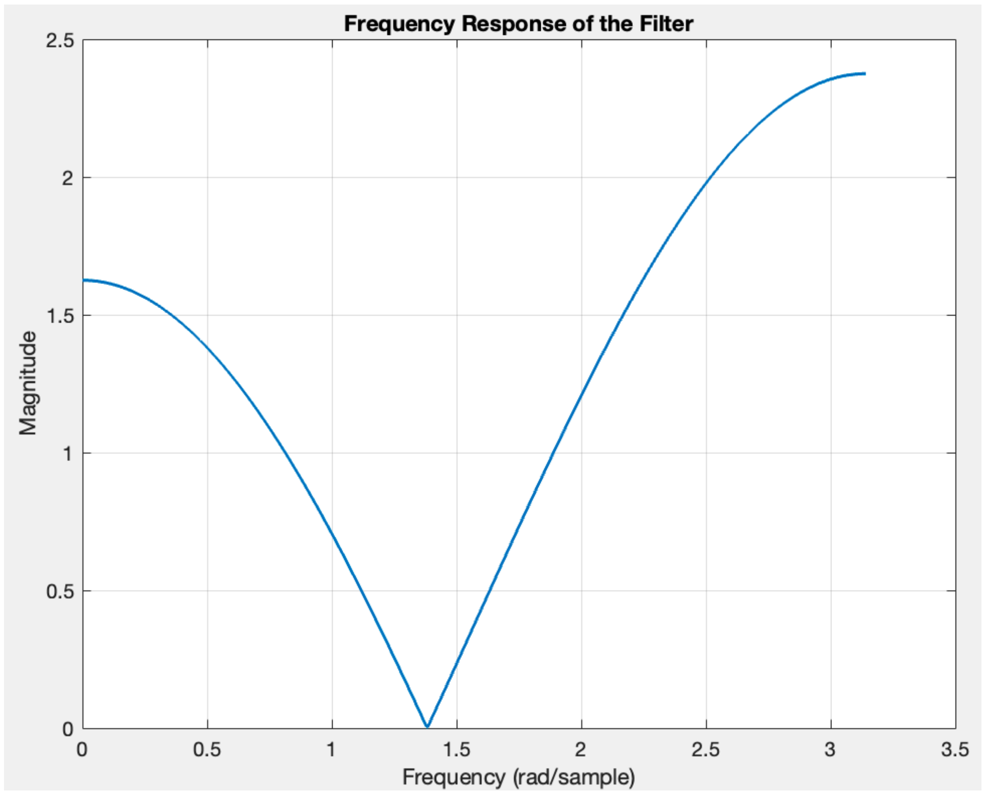
• The filter behaves as a second-order difference equation where the actual output is the difference between the input and twice the previous input. This filter has the impact of indicating differences in the input signal over time.

• By implementing this filter, the output signal is customized for attenuating or removing sinusoidal components with frequencies close to the resonant frequency of filter ($\omega_n = 0.44\pi$).

• In conclusion, the output signal will display reduced amplitudes for sinusoidal components similar to $22\ Hz$ (0.44π), efficiently reducing or removing the $22\ Hz$ cosine waves existing in the input signal. The $5\ Hz$ cosine wave and the $22\ Hz$ cosine wave with a phase shift of $\pi/3$ might exist in the output signal, relying on proximity of their frequencies to the resonant frequency of the filter.

F)

```
b = [1, -2*cos(0.44*pi), 1]; % Filter coefficient of y[n]
a = [1];  % Filter coefficient of x[n]
w = linspace(0, pi, 1000);  % Frequency range from 0 to pi
H = freqz(b, a, w); % Frequency response

% Plot frequency response
plot(w, abs(H), 'LineWidth', 1.5);
xlabel('Frequency (rad/sample)');
ylabel('Magnitude');
title('Frequency Response of the Filter');
grid on;
```



Explanation:
• The frequency response of a filter, demonstrated as $H(e^{j\omega})$, explains how the filter impacts sinusoidal elements of different frequencies.
• The magnitude of the frequency response at a certain frequency ω shows how filter diminished or amplifies sinusoidal elements at that frequency.
• The magnitude response | $H(ej\omega)$ | at desired frequency ω could be utilized to establish the size of sinusoidal elements in the output signal. Sinusoidal components at frequency peaks in the magnitude response would experience less filtering and will be more attenuated in the output signal r for elements at frequencies on the dips or nulls in the magnitude response.
• Hence, by inspecting the frequency response plot, we can conclude which frequencies are amplified or filtered by the filter, and at the end, how the filter impacts the size of each sinusoidal elements in the output signal.