591 K1 Final Project: Exploring Arbitrage and Other Opportunities in the Currency Exchange Market

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Abstract: Triangular Arbitrage is a method to gain risk-free profit in Forex (currency exchange) discrepancies. Our project investigates strategies to profit in the Forex Market by mining for daily exchange rates, stock market indices, and important commodity prices to determine the degree in correlation and leading indicators. We will also search for arbitrage opportunities using traversal algorithms on our exchange rates dataset and identify peak periods of opportunities, clustering of their distribution across time and variance and any other important inferences.

Introduction

Every day, hedge funds and major banking corporations profit from the foreign currency exchange market, also known as Forex. These groups rely mainly on two techniques to find these opportunities. One relies on high frequency discrepancies to generate risk free profit, known as arbitrage. Another technique is to make statistically backed inferences about the market based on other indicators of economic activity, such as stock shares or commodity prices. This relies on the correlation between different factors and the market, and how variables are able to forecast the future movements of currency exchange rates.

Arbitrage is the purchase and then following sale of something in two respective different markets at differing values. The basic principal is meant to exploit different markets where the prices for the same items are different values. In the context of currency markets, this means that any given currency can change in value compared to other currencies at disproportionate rates. Imagine three different currencies: A dollars, B dollars and C dollars. At given moment in time, let's say that the exchange rate of A to B is 2, the exchange rate from B to C is also 2 and the exchange rate from A to C is 3.9. If one trades 1 A dollar to 2 B dollars, and then the 2 B dollars to 4 C dollars, then trades those 4 C dollar back to A dollars, one will have 1.0256 A dollars. This .0256 A dollars gained is your profit from the triangular arbitrage. When such an imbalance exists across multiple exchange rates, it's called an implicit cross exchange rate. Such exchange rates have two important properties. The first is that these implicit cross exchange rates do not exist for a very long time; if there's a significant implicit cross exchange rate, normally the market notices that and exploits that rate to the extent that the values equalize to major trades. The second problem is that these cross exchange rates tend to be very small in size, thus the detection of cycles may be very difficult as they often are very small in terms of percent return.

However, not every cycle is profitable. Limitations in hardware such as network speed and distances between servers that are involved in the Forex market means that there is risk that a trade would take too long to execute before it is gone. Therefore it is important for high frequency traders to make informed decisions to invest into cycles whenever they appear. If a potential relationship between any possible indicator and the duration of arbitrage exists, they would be able to determine whether or not they are able to make the trade in time. Analysis of such indicators involves comparing historical data of the independent and dependent variables for signs of correlation; for example, a positive change in the stock market may mean that people are investing in currencies as well, leading to longer durations of arbitrage. While there rarely is any combination of factors that make reliable indicators for any economic variable, regression of leading indicators have made for statistically significant predictors of variables such as GDP or inflation, usually involving a combination of factors (Banerjee & Massimiliano, 2006).

Another part of our project is to verify findings in other research papers about currency arbitrage. Based on Yukihiro Aiba's paper *Triangular arbitrage as an interaction among foreign exchange rates*, there is a degree of dependence on time for arbitrage. This dependence relates to a day-by-day basis and shows that there is intraday seasonality in fluctuations and magnitude, but not weekly or quarterly: arbitrage occurs at the same rates every day (Aiba et al.). Magnitudes are also dependent on these fluctuations across time of day, depending on which currencies are involved: In Daniel Fenn's paper, the Japanese Yen and the Swiss Franc show inverse levels of magnitude, starting with a Japanese high in the first half of the day and the Swiss in the latter half (Fenn et al., 2009). These relationships suggest that the currencies involved in arbitrage are dependent on the time zones, and that Western currencies between 10am to 10pm show the most instances of arbitrage, a further step forward from Aiba's conclusions. Thus we will attempt to identify these intraday trends and currencies with inverse arbitrage relationships.

Techniques

Using publicly available API calls to finance API's, we can check the values of currency exchange rates extremely rapidly, multiple times over the course of one second. By mapping these each currency to node in a network where the edge weights are exchange rates, and using the Bellman Ford algorithm, we can check if there's any arbitrage cycles during that tick. This algorithm relies on graphs and negative cycles of edges. These edge weights will represent the asking or bidding price for those currencies by using the log of the rate multiplied by -1. The algorithm runs in polynomial time to the number of nodes and edges, and since we'll be working with a limited set of 6 currencies (Euro, US Dollar, Japanese Yen, Canadian Dollar, Swiss Franc and Australian Dollar), this time cost is not significant enough to delay our web calls. By recording arbitrage cycles tick by tick, we can record the magnitude and the duration of each cycle till either the magnitude changes or the cycle is no longer valid.

Using a dataset of exchange rates over a weeklong period, we'll be able to observe the distribution of arbitrage magnitudes and durations, as well as the frequency of arbitrage across that week. Visually, we will be able to determine the distribution of arbitrage attributes and make inferences about the process in which arbitrage arises: for example, a normal distribution may suggest that arbitrage is only likely to exist when duration/magnitude reaches a certain threshold. The time distribution is to visually frequency over time: we may be able to see the same results found by Aiba and Fenn i.e. if there is indeed the same frequencies on average of arbitrage every day.

To identify factors that correlate with duration of arbitrage, we'll use a multitude of factors and identify the highest Pearson correlation coefficient. This may include GMT times, magnitude, stock price changes, or a combination of these factors. This will give us a good idea where to start for a linear regression of values, in order to form a predictive model for arbitrage persistence, and identify statistically important variables that reject the null hypothesis and make predictions about durations of arbitrage within a 95% confidence level.

Finally, to follow up on Fenn's findings, we will be clustering points of arbitrage across a 24 hour axis and respective stock index data across the other axis. This would serve to be a basis for clustering for different values of arbitrage, such that we can identify regions of high and low value, as well as any discernable groups where such points are more or less frequent. If Fenn's findings are verified, then we should be able to see arbitrage cycles that involve specific currencies of their respective time zones emerge after their markets open. We should also see high densities of arbitrage between the opening and closing hours of western markets, since they are the most frequently involved in arbitrage according to Fenn. Lastly, if there were any relationship between arbitrage and stock prices, then we should also observe clear distinctions of clusters across the stock price axis. It would suggest that stock price movements have an effect on arbitrage value, making it a valuable indicator for predictions in the Forex market. However this hypothesis has no support in any of the aforementioned papers, so it is unclear if this would be the case.

Datasets and Experiments

We ran a python function between April 21st to April 27th, which recorded data from the TrueFX web API on near instantaneous updates for the bidding and asking prices for the six currencies we will be focusing on. The six currencies are as follows: US dollar, Euro, Australian Dollar, Japanese Yen, Canadian Dollar, and the Swiss Franc. These six major currencies were picked out the total available of 10 because they offered the most interconnected possible network of trading currencies we could build. Since each one of the 6 currencies can be converted to any other one in the list, in a representative graph every node is directly connected. 30 exchange rates are collected every tick update and recorded to a text file with the current time stamp before the Bellman Ford algorithm is ran separately across the file and recorded in a data frame.

The data for the second part of our project will involve accessing tick data of available stock market indices, sourced from a Bloomberg terminal accessible from the Questrom school of Business. Since Bloomberg records granular data of relevant stock prices ranging back 200 days from the present, we were able to collect relevant data for corresponding market indices for each country that our currencies are based in, namely: S&P 500, Euronext 100, Australian Stock Index, Nikkei 225, Toronto Stock Exchange, and the Swiss Market Index. Each of these indices is representative of the country's economy, as they are primarily comprised of publically owned companies of each geographic market and thus subject to any impact on their local economy, and by extension currencies would be affected by the same forces.

During the collection and processing of data, we ran into a few issues. One major issue was that tick data between currencies and stock data was not aligned: to address this, we had to normalize both to minute-by-minute units. This was done by summing the magnitude and persistence in a minute and aligning that with any corresponding stock price percent change in that minute. Since multiple currencies are involved in arbitrage, we created separate data points for each currency with the corresponding percent change. Finally, certain time ranges were excluded from analysis due to the lack of data from Bloomberg or our API: the ranges of 4/23 to

4/24 was the closing weekend of Forex, and between 22:00 to 00:00 there is no available stock market data as they are the closing hours of most countries.

However, apart from these issues regarding data collection, we managed to run most of our analysis initially laid out in our proposal. We were able to collect almost 7000 different unique arbitrage cycles across the week. Matching stock data across the same time period was also collected and reformatted in a separate data frame for use in clustering. Despite the fact that mismatching market hours meant that we couldn't graph some of the points in our cluster, we can still make important observations in our results. One important thing to note was that on April 27th, a massive market shock occurred in Japan. Investors were surprised when the Bank of Japan defied expectations, leading to widespread buyout of the Yen and a massive jump in yen prices, which led to huge discrepancies in the market. This led to far more arbitrage cycles of greater magnitude and duration than what occurred regularly in the past few days and thus we had to avoid including these points in our analysis. Later on we realized that such economic shocks are in line with our conclusions after we began analysis of our clusters and regression.

Results and Discussion

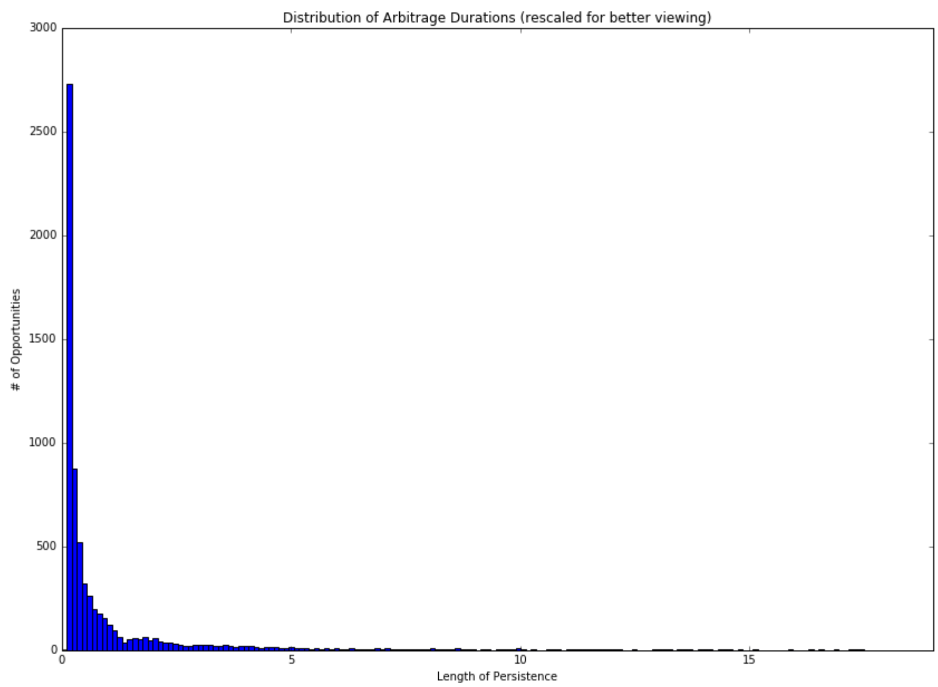

Figure 1: Distribution of Arbitrage Durations

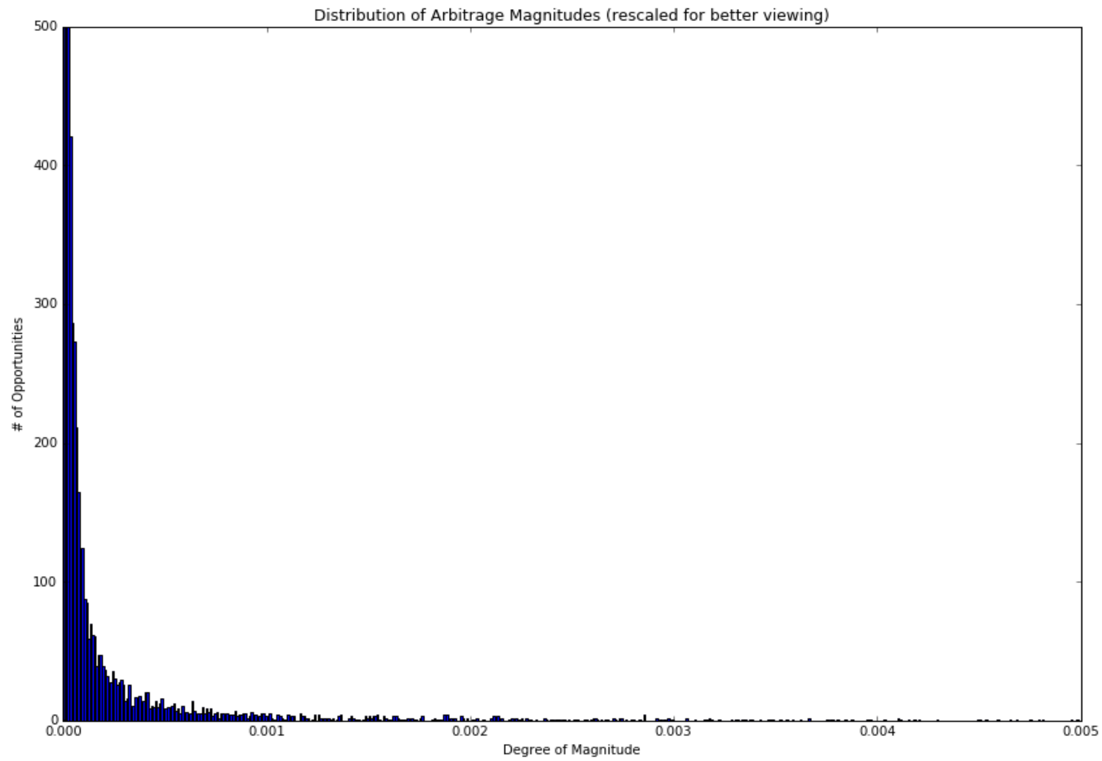

Figure 2: Distribution of Arbitrage Magnitudes

Figure 1 and 2 show the frequency distributions of durations and magnitudes respectively. In total, we collected 6889 instances across 72 hours, which sums up to about 174 minutes of arbitrage in total. On average each cycle made about 0.03% profit for about an average of about 1.523 seconds, with a maximum of 8% return and 2.5 minutes. Both of these distributions show characteristics of an exponential distribution: the majority of events occur with very low values, and the frequencies of higher values drop exponentially. This suggests that arbitrage follows a Poisson process, where events occur at a continuously and independently at a constant average rate. Independence is an important factor to note in our project: since instances of arbitrage do not affect each other, we cannot assume that longer events occur randomly with no correlation of events before them, and the same with events of high magnitude. It is thus more likely that there could be outside factors that affect the rates in which arbitrage occurs, or that there's an indiscernible average rate that affects day-to-day occurrences.

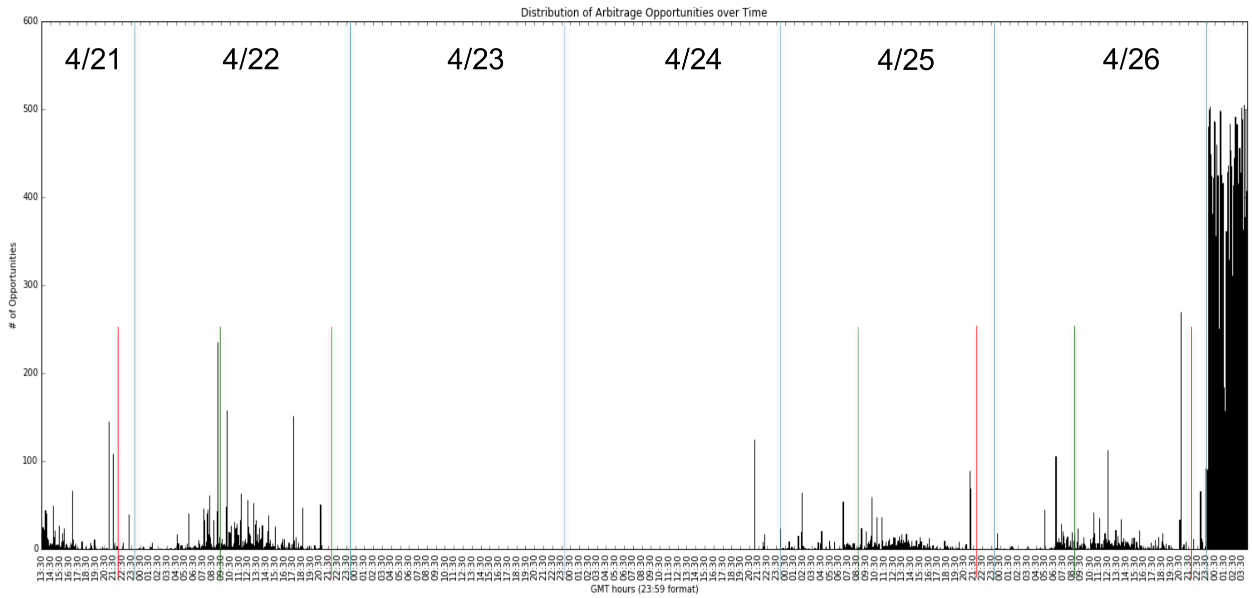

Figure 3: Distribution of Arbitrage across time

Our next distribution is to plot arbitrage events over time to measure its frequency across the several days in which we were recording. Figure 3 shows this distribution with markers for each day as well as closing hours of the Tokyo Forex exchange in Green at 9am, and the New York Forex exchange in Red at 10pm. 4/23 to 4/24 was the weekend in which the exchanges were closed till 8pm on the 24th. Firstly, from the distributions between days, there doesn't seem to be any features of seasonality across different days. Instead, it seems that arbitrage occur throughout each day at fairly regular frequencies, with a growing proportion around midday each day. This is strong evidence for Aiba's findings; intraday trends seem to be independent of one another and seasonality only occurs on a smaller scale of hours not days. Secondly, the markers for closing hours show a degree of causality: the New York close corresponds to a spike in arbitrages before a significant drop, and Tokyo shows another relative increase in frequency before activity begins to peak around noon. This may be related to the currencies used between the hours of regional exchanges. If arbitrage across different hours predominately involve currencies of those specific time zones, then we can also verify Fenn's paper by showing that currencies are only involved in arbitrage when their market is open.

However, there is an important event to note. Due to the events that occurred on April 27th, there was a clear and massive increase in arbitrage on that day when the Tokyo exchange opened at midnight. This scale of increase is unprecedented and clearly classifies as a market shock; unforeseeable events, which in this example is the decision by the Bank of Japan, has lead to market-wide repercussions and is not indicative of normal and regular activity. Our decision to include this data in this distribution is after analysis of this spike; we concluded that our following analysis of correlation effects and clustering were still applicable even in the event of this massive shock. By including these points we had more detailed and extreme data to work with and were able to draw further conclusions.

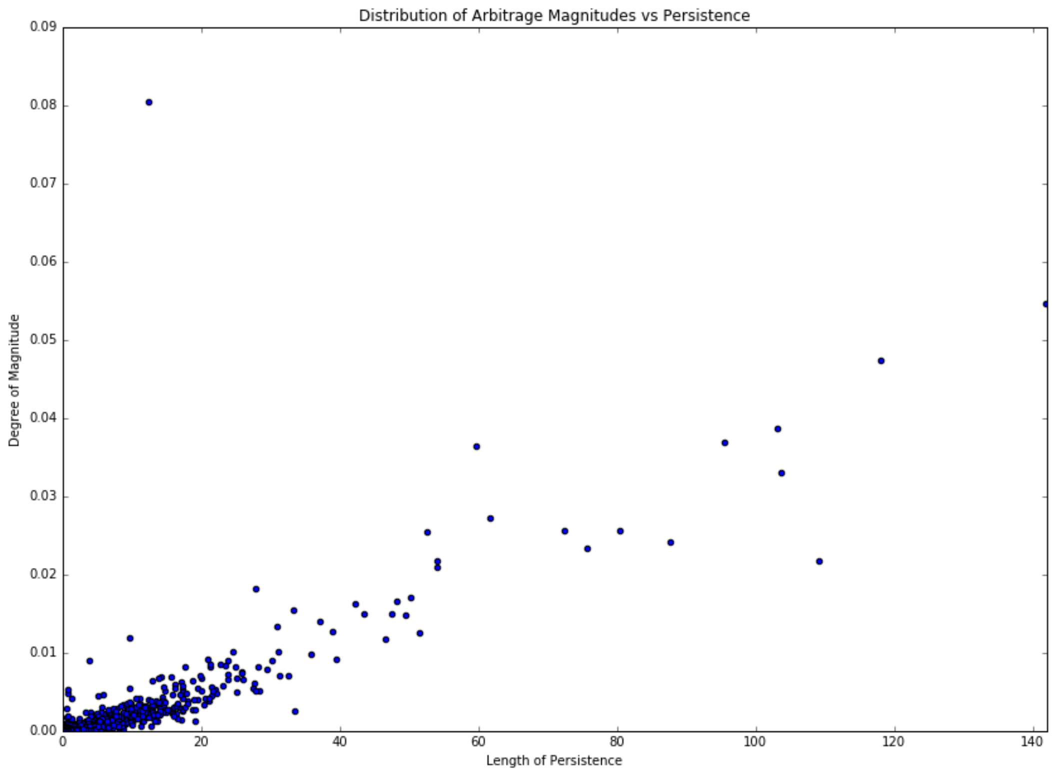

Figure 4: Distribution of Duration vs. Magnitude

The distribution in figure 4 is what we found after multiple plots using a variety of factors against arbitrage duration. Surprisingly, it is magnitude that shows the highest correlation factor of about 0.8430109. Indeed, the scatter plot quite clearly shows that there is a linear trend in the data, with points that show some spread around a centerline. There are a few outliers, but the vast majority of the density in the distribution show that most lower values follow this linear trend, while higher values tend to be much fewer in number and more spread out. Time scales much more quickly than magnitude as well, so very small changes in magnitude seems to have a very large effect on the time it takes for cycles to deteriorate. Given the apparent nature of the plot and the non-binary variable used, we decided that the appropriate model to use in this data is a linear regression with one variable.

```
                             OLS Regression Results
==============================================================================
Dep. Variable:            Persistence   R-squared:                       0.828
Model:                            OLS   Adj. R-squared:                  0.828
Method:                 Least Squares   F-statistic:                 1.258e+04
Date:                Fri, 29 Apr 2016   Prob (F-statistic):               0.00
Time:                        00:29:25   Log-Likelihood:                -8328.9
No. Observations:                2622   AIC:                         1.666e+04
Df Residuals:                    2620   BIC:                         1.667e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
Intercept  -3569.6672     31.878   -111.980      0.000     -3632.175 -3507.159
Magnitude   3571.2150     31.834    112.182      0.000      3508.792  3633.638
==============================================================================
Omnibus:                      859.914   Durbin-Watson:                   0.644
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            47097.544
Skew:                          -0.745   Prob(JB):                         0.00
Kurtosis:                      23.709   Cond. No.                         563.
==============================================================================
```

Figure 5: Linear Regression of Duration using Magnitude

Figure 5 details our results from a univariate linear OLS regression. The R-squared value suggests that the regression model can predict about 82.8% of the variance in the data, an unexpectedly high value. Given that cycles can be instantaneously measured for magnitude, it means that this model can quite accurately predict the duration of its existence. The large coefficient values, however, does mean that small errors in predictions may lead to huge differences. Choosing lower coefficient values within the confidence interval of magnitude for the risk adverse trader can mitigate this risk and erring on the side of caution for predictions. Regardless, a high correlation between magnitude and duration suggests that markets are slow to adapt to large discrepancies in the exchange rates. While participants do eventually equalize prices over time, it is likely that this normalization only occurs after sufficient volume.

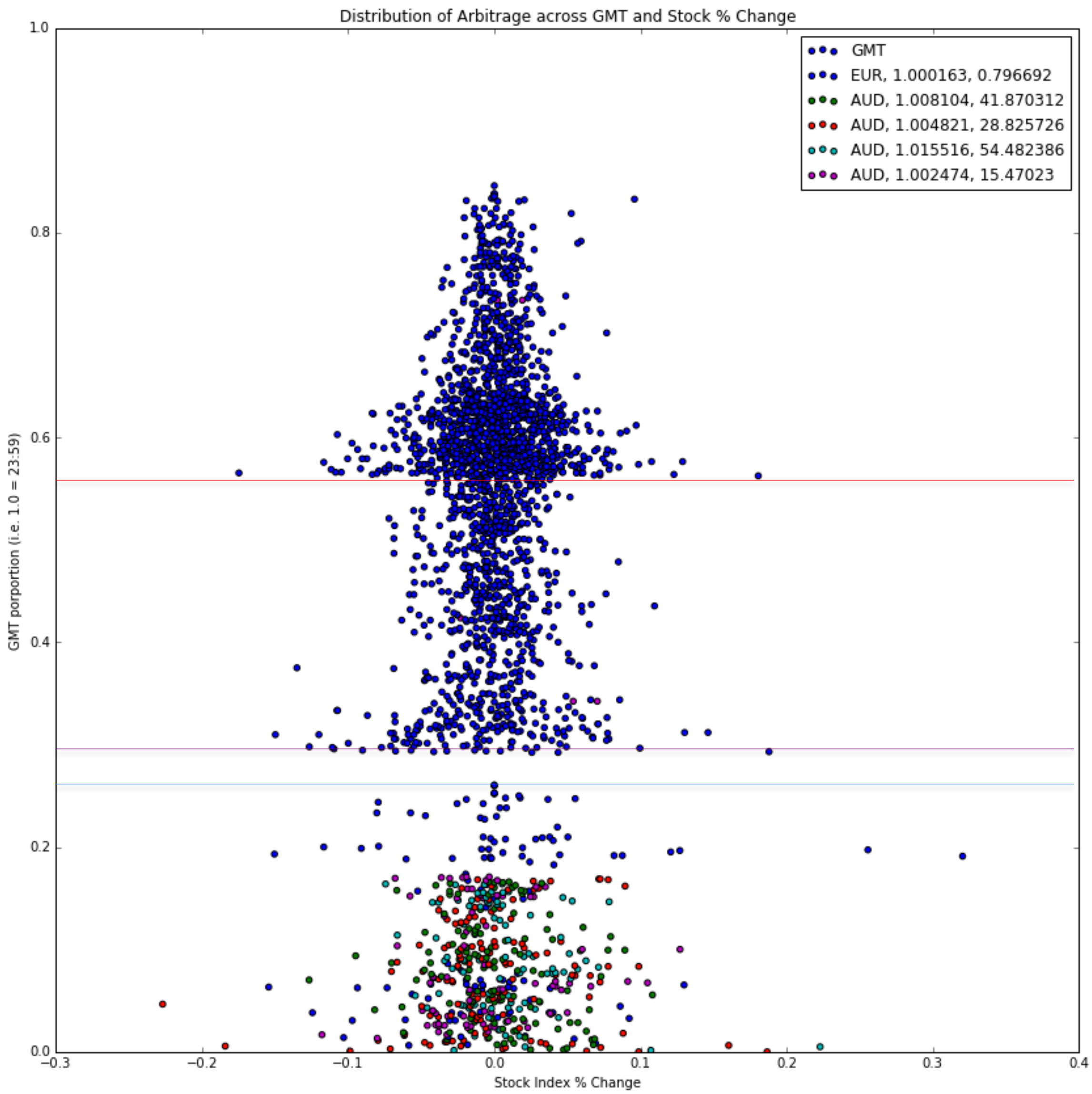

Figure 6: K-Means Clusters of Arbitrage in Time vs. Stock Change

Figure 6 and 7 shows the results of K-means clustering on our specified distribution. By modeling GMT proportion and stock price % change against each other, it is clearer to see how arbitrage emerges across time and the effect stock indices have on arbitrage. We used heavier weights to cluster points based on their value; higher magnitudes and durations are more valuable than points of low magnitude and persistence. Our elbow plot pointed out that the idea number of clusters was five, and the plot above shows that these points do indeed cluster according to a value scale after initial analysis.

```
Number of Clusters: 5

        Cluster #: 0
        Color of cluster: b
        Name of cluster (based on common elements in cluster): EUR, 1.000163, 0.796692
        List of currencies: [('EUR', 683), ('USD', 574), ('CHF', 498), ('CAD', 323), ('JPY', 85), ('AUD', 32)]
        Minimum cosine similarity value: 0.237301189442
        Average cosine similarity value: 0.868608322355
        Cluster #: 1
        Color of cluster: g
        Name of cluster (based on common elements in cluster): AUD, 1.008104, 41.870312
        List of currencies: [('AUD', 74), ('JPY', 54)]
        Minimum cosine similarity value: 0.999972816446
        Average cosine similarity value: 0.999995983608
        Cluster #: 2
        Color of cluster: r
        Name of cluster (based on common elements in cluster): AUD, 1.004821, 28.825726
        List of currencies: [('AUD', 71), ('JPY', 53)]
        Minimum cosine similarity value: 0.999868669799
        Average cosine similarity value: 0.999974281569
        Cluster #: 3
        Color of cluster: c
        Name of cluster (based on common elements in cluster): AUD, 1.002474, 15.47023
        List of currencies: [('AUD', 41), ('JPY', 37), ('EUR', 4), ('CHF', 3), ('USD', 1), ('CAD', 1)]
        Minimum cosine similarity value: 0.997723447353
        Average cosine similarity value: 0.999651001292
        Cluster #: 4
        Color of cluster: m
        Name of cluster (based on common elements in cluster): AUD, 1.015516, 54.482386
        List of currencies: [('AUD', 52), ('JPY', 36)]
        Minimum cosine similarity value: 0.999988428844
        Average cosine similarity value: 0.999998226267
```

Figure 7: Statistics for each Cluster

Blue points represent the lowest amount of value, with an average magnitude of slightly more than 0.001% return and an average persistence of 0.7967 seconds. These points make up the vast majority of the distribution with about 2110 points, more than the other clusters combined. The vast majority of these points occur after the time of day is at around 0.2, or around 4:50 AM. This is right before the closing hours of Sydney's exchange at 6 AM, shown here as the blue line. The vast majority of the weights occur after London opens at 8AM, shown as the purple line. There is a far greater density of points after this point, and another corresponding increase after the New York exchange opens at 1 PM indicated by the red line. The wider spread along those points indicate that arbitrage occur along greater changes to stock price data during that initial period. However most points stay close to the mean 0.0, and thus this observation isn't conclusive beyond the fact that opening hours do affect the frequencies of arbitrage. The majority of the weight in the graph does occur between 10 AM and 10 PM per Aiba's findings. One interesting note is that the majority of these arbitrage points involve the Euro, the USD and the Swiss Franc, with a smaller proportion of Canadian Dollars and Australian Dollars, a point which is expanded upon in further analysis.

These clusters give more credence to Finn's findings that arbitrage of different value and currencies occur in different time zones. The purple, red, green and cyan clusters are of much higher value compared to blue, with time scales ranging from 15 to almost 60 seconds and magnitudes from 0.08% to 1.56% return. These clusters are almost entirely exclusively made up of Australian and Japanese currencies (only exception of the cyan cluster), and occur up to the same time zone mark that marks the beginning of the blue clusters majority: around 4:50 AM. While these clusters are far less dense and populated as the blue cluster, it is important to note that these clusters show that arbitrage value show a dependence on the time of day. Even after we removed data points from 4/27 influenced by the Japanese shock in the market in figure 8, the same cluster are formed with only slightly different values of magnitude and persistence, while the apparent points of delineation and grouping of currencies are preserved. Thus this result does not disprove Finn's conclusions about currency arbitrage and time zones.

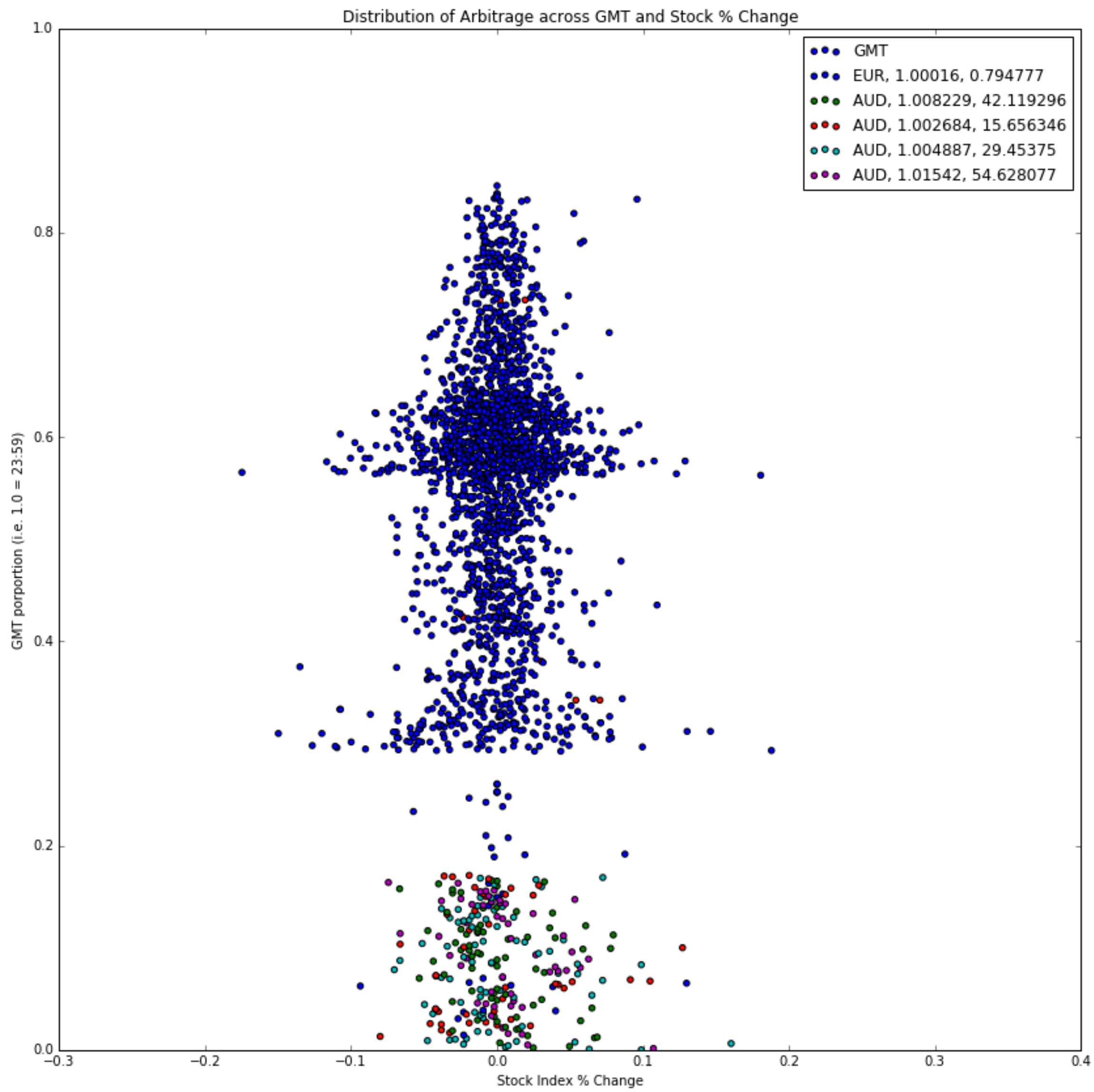

Figure 8: Clustering after removal of data from 4/27

However, no conclusive evidence that arbitrage is affected by stock market price changes can be found. Had there been clear borders of different clusters across the horizontal axis would have suggested that there are effects that the stock market have on arbitrage value; this does not seem to be the case as most of the clusters are interspersed within one another. Thus the important conclusions we are able to draw upon are on the GMT axis.

Conclusion

Our output has definitely demonstrated a strong positive correlation between magnitude of an arbitrage opportunity and the amount of time it persists for. This relationship was not necessarily clear before because it could've been the case that the larger the magnitude of the arbitrage opportunity, the faster market forces would exploit it back to equilibrium. Since we now have a developed relationship between magnitude and persistence, we can reverse engineer this process to determine whether future arbitrage opportunities are valuable. The relation would be imperative for developing this predictive model. It would ideally be able to take the magnitude of present opportunities and return what arbitrage opportunities are most worthwhile in attempting to exploit; this would minimize risk that the opportunity will cease to exist during the trade. Without such a function, many arbitrage opportunities would likely cease midway through the trade; as a consequence significant loss would accrue on the user side.

Our results also indicate that arbitrage happens in very high frequency in correlation with opening and closing times of various stock exchanges. The heavy spur of trading that comes with the end of any given workday likely causes this. The closing and openings of various exchange markets mean different things in regard to arbitrage opportunities. The most significant discovery here is that while there are arbitrage opportunities happening all throughout the day, the distribution of high magnitude opportunities precedes the closing of the Australian market. The less significant, but high frequency moments of arbitrage all tend to follow the London and New York exchange openings. This would suggest the forces exploiting currency arbitrage operating out of London and New York are considerably more adept at reacting to and using up arbitrage opportunities, whereas Australian and Asian markets do not exploit this information as quickly. This is very useful information, in that it would allow for an additional important parameter in a predictive model; telling how the potential for each opportunity should be weighed based on what time of day it appears. Should any given arbitrage opportunity manifest after the openings of the London and New York exchanges, it could have a negative weight applied to, given the high likelihood that it's very ephemeral. Should one occur before the Australian closing time however, a significant positive weight could be applied to the opportunity, given that it has a higher chance of yielding substantial returns.

Unfortunately, there turned out to be no strong correlation between stock pricing changes and arbitrage opportunities. This would indicate that the null hypothesis in regard to currency arbitrage and stock market changes is true. While this might be grounds for additional testing, at the very least a lack of any meaningful correlation means that no processing time need be expended checking the stock data when considering whether to make exploit a triangular arbitrage path. This is useful since Arbitrage opportunities happen in relatively short windows of time, making fast decisions is a high priority.

On the whole, this experiment has yielded important results that can be applied to financial trading algorithms. The actual implementations of this project for the time being might be restricted to banks or firms with high enough degrees of capital to make very fast trades with high enough principal to yield significant profit. However, faster consumer oriented trading software is slowly becoming available, and this information may soon bear meaningful relevance to independent traders.

Works Cited

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