N-Step Factorial

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1 Proving $g(x)$ is Periodic Over the Reals

(BC) This is a nice start. To make it complete, need to add

1 Where the equation $g(x) = \frac{\text{NSF}(x,1)}{\Gamma(x+1)}$ came from

2 Definitions of $\text{NSF}(x, 1)$ and $\Gamma(x + 1)$

Let $g(x) = \frac{\text{NSF}(x,1)}{\Gamma(x+1)}$. Prove $g(x) = g(x + 1)$.
We know $\text{NSF}(x + 1, 1) = (x + 1)\text{NSF}(x, 1)$.
We also know from Gaussian Integrals that $\Gamma(x + 2) = (x + 1)\Gamma(x + 1)$.*

$$\begin{aligned} g(x+1) &= \frac{\text{NSF}(x+1,1)}{\Gamma(x+2)} \\ &= \frac{(x+1)\text{NSF}(x,1)}{(x+1)\Gamma(x+1)^*} \\ &= \frac{\text{NSF}(x,1)}{\Gamma(x+1)} \end{aligned}$$

Thus, we have shown that $g(x) = g(x + 1)$ and $g(x)$ is periodic over the reals.

(BC) I didn't see the $\Gamma(x+1) = x\Gamma(x)$ result in the Gaussian Integrals reference, but I did find it here: https://www.cantorsparadise.com/the-beautiful-gamma-function-and-the-genius-who-discovered-it-8778437565dc

Gaussian Integrals