Mathematic Relations

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Class:

1 I

Absence:

01

Major:

Information Technology

Study Program:

Informatic Engineering

Exercise

Consider these relations on the set of integers
✓ R1 = {(a, b)|a ≤ b}
✓ R2 = {(a, b)|a > b}
✓ R3 = {(a, b)|a = b or a = -b}
✓ R4 = {(a, b)|a = b}
✓ R5 = {(a, b)|a = b + 1}
✓ R6 = {(a, b)|a + b ≤ 3}
Determine whether each of these relations are equivalence

R1 = {(a, b)|a ≤ b} Reflexivity: For all a, (a,a) is in the relation because a ≤ a. Symmetry: If (a,b) is in the relation, then a ≤ b. This means that b ≥ a, so (b,a) is also in the relation. Therefore, R1 is symmetric. Transitivity: If (a,b) and (b,c) are in the relation, then a ≤ b and b ≤ c. This means that a ≤ c, so (a,c) is in the relation. Therefore, R1 is transitive. Since R1 is reflexive, symmetric, and transitive, it is an equivalence relation.

R2 = {(a, b)|a > b} Reflexivity: For all a, (a,a) is not in the relation because a is not greater than itself. Symmetry: If (a,b) is in the relation, then a > b. This means that b < a, so (b,a) is not in the relation. Therefore, R2 is not symmetric. Transitivity: If (a,b) and (b,c) are in the relation, then a > b and b > c. This means that a > c, so (a,c) is in the relation. Therefore, R2 is transitive. Since R2 is not reflexive and not symmetric, it is not an equivalence relation.

R3 = {(a, b)|a = b or a = -b} Reflexivity: For all a, (a,a) is in the relation because a = a. Symmetry: If (a,b) is in the relation, then a = b or a = -b. In the first case, b = a, so (b,a) is also in the relation. In the second case, -b = a, so (-a,-b) is also in the relation. Therefore, R3 is symmetric. Transitivity: If (a,b) and (b,c) are in the relation, then either a = b or a = -b, and either b = c or b = -c.

If a = b and b = c, then a = c, so (a,c) is in the relation.

If a = b and b = -c, then a = -c, so (a,c) is in the relation.

If a = -b and -b = c, then a = -c, so (a,c) is in the relation.

If a = -b and -b = -c, then a = c, so (a,c) is in the relation.

Therefore, R3 is transitive. Since R3 is reflexive, symmetric, and transitive, it is an equivalence relation.

R4 = {(a, b)|a = b} Reflexivity: For all a, (a,a) is in the relation because a = a. Symmetry: If (a,b) is in the relation, then a = b, so (b,a) is also in the relation. Therefore, R4 is symmetric. Transitivity: If (a,b) and (b,c) are in the relation, then a = b and b = c. This means that a = c, so (a,c) is in the relation. Therefore, R4 is transitive. Since R4 is reflexive, symmetric, and transitive, it is an equivalence relation.

R5 = {(a, b)|a = b + 1} Reflexivity: For all a, (a-1,a) is in the relation because a = (a-1) + 1. Symmetry: If (a,b) is in the relation, then a = b + 1. This means that b = a - 1, so (b,a) is also in the relation. Therefore, R5 is symmetric. Transitivity: If (a,b) and (b,c) are in the relation, then a = b + 1 and b = c + 1. This means that a = c + 2, so (a,c) is in the relation. Therefore, R5 is transitive. Since R5 is reflexive, symmetric, and transitive, it is an equivalence relation.

R6 = {(a, b)|a + b ≤ 3} Reflexivity: For all a, (a, a) is in the relation because a + a = 2a ≤ 3. Symmetry: If (a, b) is in the relation, then a + b ≤ 3. This means that b + a ≤ 3, so (b, a) is also in the relation. Therefore, R6 is symmetric. Transitivity: If (a, b) and (b, c) are in the relation, then a + b ≤ 3 and b + c ≤ 3. Adding these inequalities, we get a + b + b + c ≤ 6, or a + c + 2b ≤ 6. Since b is an integer, we know that 2b is even and can be expressed as 0 or 2. Thus, a + c + 2b ≤ 6 implies a + c ≤ 6 or a + c ≤ 4, depending on whether 2b is 0 or 2, respectively. In either case, (a, c) is in the relation. Therefore, R6 is transitive. Since R6 is reflexive, symmetric, and transitive, it is an equivalence relation.