Fractions, ratios and percentages

28 Division of integers

A fraction is a number which is represented by one integer – the *numerator* – divided by another integer – the *denominator* (or the *divisor*). For example, $\frac{3}{5}$ is a fraction with numerator 3 and denominator 5. Because fractions are written as one integer divided by another – a *ratio* – they are called *rational* numbers. Fractions are either *proper*, *improper* or *mixed*:

- in a proper fraction the numerator is less than the denominator, for example, $\frac{4}{7}$
- in an improper fraction the numerator is greater than the denominator, for example $\frac{12}{5}$
- a mixed fraction is in the form of an integer and a fraction, for example $6\frac{2}{3}$

So that $-\frac{8}{11}$ is a ..... fraction?

The answer is in the next frame

Proper

29

Fractions can be either positive or negative.

Now to the next frame

Multiplying fractions 30

Two fractions are multiplied by multiplying their respective numerators and denominators independently. For example:

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

Try this one for yourself.

$$\frac{5}{9} \times \frac{2}{7} = ...$$

$\frac{10}{63}$

31

Because

$$\frac{5}{9}\times\frac{2}{7}=\frac{5\times 2}{9\times 7}=\frac{10}{63}$$

Correct? Then on to the next frame

Of

32

The word 'of' when interposed between two fractions means multiply. For example:

Half of half a cake is one-quarter of a cake. That is

$$\frac{1}{2}\ of\ \frac{1}{2}=\frac{1}{2}\times\frac{1}{2}=\frac{1\times 1}{2\times 2}=\frac{1}{4}$$

So that, for example:

$$\frac{1}{3}\ of\ \frac{2}{5}=\frac{1}{3}\times\frac{2}{5}=\frac{1\times 2}{3\times 5}=\frac{2}{15}$$

So that $\frac{3}{8}\ of\ \frac{5}{7}=$ .....

$\frac{15}{56}$

33

Because

$$\frac{3}{8}\ of\ \frac{5}{7}=\frac{3}{8}\times\frac{5}{7}=\frac{3\times 5}{8\times 7}=\frac{15}{56}$$

On now to the next frame

34 Equivalent fractions

Multiplying the numerator and denominator by the same number is equivalent to multiplying the fraction by unity, that is by 1:

$$\frac{4\times 3}{5\times 3}=\frac{4}{5}\times\frac{3}{3}=\frac{4}{5}\times 1=\frac{4}{5}$$

Now, $\frac{4\times 3}{5\times 3}=\frac{12}{15}$ so that the fraction $\frac{4}{5}$ and the fraction $\frac{12}{15}$ *both represent the same number* and for this reason we call $\frac{4}{5}$ and $\frac{12}{15}$ *equivalent fractions*.

A second fraction, equivalent to a first fraction, can be found by multiplying the numerator and the denominator of the first fraction by the same number.

So that if we multiply the numerator and denominator of the fraction $\frac{7}{5}$ by 4 we obtain the equivalent fraction .....

Check your result in Frame 35

35

$$\boxed{\frac{28}{20}}$$

Because

$$\frac{7\times 4}{5\times 4}=\frac{28}{20}$$

We can reverse this process and find the equivalent fraction that has the smallest numerator by *cancelling out* common factors. This is known as reducing the fraction to its *lowest terms*. For example:

$\frac{16}{96}$ can be reduced to its lowest terms as follows:

$$\frac{16}{96}=\frac{4\times 4}{24\times 4}=\frac{4\times\cancel{4}}{24\times\cancel{4}}=\frac{4}{24}$$

by cancelling out the 4 in the numerator and the denominator

The fraction $\frac{4}{24}$ can also be reduced:

$$\frac{4}{24}=\frac{4}{6\times 4}=\frac{\cancel{4}}{6\times\cancel{4}}=\frac{1}{6}$$

Because $\frac{1}{6}$ cannot be reduced further we see that $\frac{16}{96}$ reduced to its lowest terms is $\frac{1}{6}$.

How about this one? The fraction $\frac{84}{108}$ reduced to its lowest terms is .....

Check with the next frame

36

$\boxed{\frac{7}{9}}$

Because

$$\frac{84}{108} = \frac{7 \times 3 \times 4}{9 \times 3 \times 4} = \frac{7 \times \cancel{3} \times \cancel{4}}{9 \times \cancel{3} \times \cancel{4}} = \frac{7}{9}$$

Now move on to the next frame

Dividing fractions

37

The expression $6 \div 3$ means the number of 3's in 6, which is 2. Similarly, the expression $1 \div \frac{1}{4}$ means the number of $\frac{1}{4}$'s in 1, which is, of course, 4. That is:

$1 \div \frac{1}{4} = 4 = 1 \times \frac{4}{1}$. Notice how the numerator and the denominator of the divisor are switched and the division replaced by multiplication.

Two fractions are divided by switching the numerator and the denominator of the divisor and multiplying the result. For example:

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$$

So that $\frac{7}{13} \div \frac{3}{4} =$ .....

38

$\boxed{\frac{28}{39}}$

Because

$$\frac{7}{13} \div \frac{3}{4} = \frac{7}{13} \times \frac{4}{3} = \frac{28}{39}$$

In particular:

$$1 \div \frac{3}{5} = 1 \times \frac{5}{3} = \frac{5}{3}$$

The fraction $\frac{5}{3}$ is called the *reciprocal* of $\frac{3}{5}$

So that the reciprocal of $\frac{17}{4}$ is .....

$\boxed{\frac{4}{17}}$ **39**

Because

$$1 \div \frac{17}{4} = 1 \times \frac{4}{17} = \frac{4}{17}$$

And the reciprocal of -5 is .....

$\boxed{-\frac{1}{5}}$ **40**

Because

$$1 \div (-5) = 1 \div \left(-\frac{5}{1}\right) = 1 \times \left(-\frac{1}{5}\right) = -\frac{1}{5}$$

Move on to the next frame

41 Adding and subtracting fractions

Two fractions can only be added or subtracted immediately if they both possess the same denominator, in which case we add or subtract the numerators and divide by the common denominator. For example:

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

If they do not have the same denominator they must be rewritten in equivalent form so that they do have the same denominator – called the *common denominator*. For example:

$$\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{10+3}{15} = \frac{13}{15}$$

The common denominator of the equivalent fractions is the LCM of the two original denominators. That is:

$$\frac{2}{3} + \frac{1}{5} = \frac{2 \times 5}{3 \times 5} + \frac{1 \times 3}{5 \times 3} = \frac{10}{15} + \frac{3}{15}$$ where 15 is the LCM of 3 and 5

So that $\frac{5}{9} + \frac{1}{6} =$ .....

42

$$\boxed{\frac{13}{18}}$$

Because

The LCM of 9 and 6 is 18 so that $\frac{5}{9}+\frac{1}{6}=\frac{5\times 2}{9\times 2}+\frac{1\times 3}{6\times 3}=\frac{10}{18}+\frac{3}{18}$

$$=\frac{10+3}{18}=\frac{13}{18}$$

There's another one to try in the next frame

43

Now try $\frac{11}{15}-\frac{2}{3}=$ .....

44

$$\boxed{\frac{1}{15}}$$

Because

$$\frac{11}{15}-\frac{2}{3}=\frac{11}{15}-\frac{2\times 5}{3\times 5}=\frac{11}{15}-\frac{10}{15}$$

$$=\frac{11-10}{15}=\frac{1}{15}$$ (15 is the LCM of 3 and 15)

Correct? Then on to Frame 45

Fractions on a calculator

The calculator we shall use is the Casio *fx-85GT PLUS* and on this calculator the fraction key is denoted by the symbol:

on the key and the symbol for a mixed fraction:

above the key – accessed by using in combination with the SHIFT key. The REPLAY key is used to move the flashing cursor around the screen to enable functions to be entered and manipulated with the results given in fractional form. For example, to evaluate $\frac{2}{3} \times 2\frac{3}{4}$ using this calculator [*note*: your calculator may not have the identical display in what follows, indeed the symbol $a^b/_c$ is often used for the fraction key]:

Press the fraction key
    the fraction symbol is then displayed on the screen
Enter the number 2
    the numerator in the display
Press the down arrow on the REPLAY key
    the flashing cursor moves to the denominator
Enter the number 3
    the denominator in the display
Press the right arrow on the REPLAY key
    the flashing cursor moves to the right of the display
Press the $\times$ key
Press the SHIFT key and then the fraction key
    the mixed fraction symbol is then displayed on the screen
Enter the number 2
Press the right arrow on the REPLAY key
Enter the number 3
Press the down arrow on the REPLAY key
Enter the number 4

The display then should look just like:

$\frac{2}{3} \times 2\frac{3}{4}$

Press the = key and the result appears in the bottom right-hand corner of the screen:

$$\frac{11}{6}$$

Now press the SHIFT key and then the S⇔D key to change the display to the mixed fraction:

$$1\tfrac{5}{6}$$

That is:

$$\tfrac{2}{3} \times 2\tfrac{3}{4} = \frac{2}{3} \times \frac{11}{4}$$
$$= \frac{11}{6}$$
$$= 1\tfrac{5}{6}$$

Now use your calculator to evaluate each of the following:

(a) $\frac{5}{7} + 3\frac{2}{3}$

(b) $\dfrac{8}{3} - \dfrac{5}{11}$

(c) $\dfrac{13}{5} \times \dfrac{4}{7} - \dfrac{2}{9}$

(d) $4\frac{1}{11} \div \left(-\dfrac{3}{5}\right) + \dfrac{1}{8}$

Check your answers in Frame 46

46

(a) $\frac{5}{7} + 3\frac{2}{3} = \frac{92}{21} = 4\frac{8}{21}$

(b) $\dfrac{8}{3} - \dfrac{5}{11} = \dfrac{73}{33} = 2\frac{7}{33}$

(c) $\dfrac{13}{5} \times \dfrac{4}{7} - \dfrac{2}{9} = \dfrac{398}{315} = 1\frac{83}{315}$

(d) $4\frac{1}{11} \div \left(-\dfrac{3}{5}\right) + \dfrac{1}{8} = -\dfrac{589}{88} = -6\frac{61}{88}$

On now to the next frame

RASIO

47 Ratios

If a whole number is separated into a number of fractional parts where each fraction has the same denominator, the numerators of the fractions form a *ratio*. For example, if a quantity of brine in a tank contains $\frac{1}{3}$ salt and $\frac{2}{3}$ water, the salt and water are said to be in the ratio 'one-to-two' – written 1 : 2.

What ratio do the components A, B and C form if a compound contains $\frac{3}{4}$ of A, $\frac{1}{6}$ of B and $\frac{1}{12}$ of C?

Take care here and check your results with Frame 48

48

9 : 2 : 1

Because the LCM of the denominators 4, 6 and 12 is 12, then:

$\frac{3}{4}$ of A is $\frac{9}{12}$ of A, $\frac{1}{6}$ of B is $\frac{2}{12}$ of B and the remaining $\frac{1}{12}$ is of C. This ensures that the components are in the ratio of their numerators. That is: 9 : 2 : 1

Notice that the sum of the numbers in the ratio is equal to the common denominator.

PRESENTASE

49 Percentages

A percentage is a fraction whose denominator is equal to 100. For example, if 5 out of 100 people are left-handed then the fraction of left-handers is $\frac{5}{100}$ which is written as 5%, that is 5 *per cent* (%).

So if 13 out of 100 cars on an assembly line are red, the percentage of red cars on the line is .....

13%

50

Because

The fraction of cars that are red is $\frac{13}{100}$ which is written as 13%.

Try this. What is the percentage of defective resistors in a batch of 25 if 12 of them are defective?

48%

51

Because

The fraction of defective resistors is $\frac{12}{25} = \frac{12 \times 4}{25 \times 4} = \frac{48}{100}$ which is written as 48%.

Notice that this is the same as:

$$\left(\frac{12}{25} \times 100\right)\% = \left(\frac{12}{25} \times 25 \times 4\right)\% = (12 \times 4)\% = 48\%$$

A fraction can be converted to a percentage by multiplying the fraction by 100.

To find the percentage part of a quantity we multiply the quantity by the percentage written as a fraction. For example, 24% of 75 is:

$$24\% \text{ of } 75 = \frac{24}{100} \text{ of } 75 = \frac{24}{100} \times 75 = \frac{6 \times 4}{25 \times 4} \times 25 \times 3 = \frac{6 \times \cancel{4}}{\cancel{25} \times \cancel{4}} \times \cancel{25} \times 3$$
$$= 6 \times 3 = 18$$

So that 8% of 25 is .....

Work it through and check your results with the next frame

2

52

Because

$$\frac{8}{100} \times 25 = \frac{2 \times 4}{25 \times 4} \times 25 = \frac{2 \times \cancel{4}}{\cancel{25} \times \cancel{4}} \times \cancel{25} = 2$$

At this point let us pause and summarize the main facts on fractions, ratios and percentages

Review summary

1 A fraction is a number represented as one integer (the numerator) divided by another integer (the denominator or divisor).

2 The same number can be represented by different but equivalent fractions.

3 A fraction with no common factors other than unity in its numerator and denominator is said to be in its lowest terms.

4 Two fractions are multiplied by multiplying the numerators and denominators independently.

5 Two fractions can only be added or subtracted immediately when their denominators are equal.

6 A ratio consists of the numerators of fractions with identical denominators.

7 The numerator of a fraction whose denominator is 100 is called a percentage.

Review exercise

1 Reduce each of the following fractions to their lowest terms:

(a) $\frac{24}{30}$ (b) $\frac{72}{15}$ (c) $-\frac{52}{65}$ (d) $\frac{32}{8}$

2 Evaluate the following:

(a) $\frac{5}{9} \times \frac{2}{5}$ (b) $\frac{13}{25} \div \frac{2}{15}$ (c) $\frac{5}{9} + \frac{3}{14}$ (d) $\frac{3}{8} - \frac{2}{5}$

(e) $\frac{12}{7} \times \left(-\frac{3}{5}\right)$ (f) $\left(-\frac{3}{4}\right) \div \left(-\frac{12}{7}\right)$ (g) $\frac{19}{2} + \frac{7}{4}$ (h) $\frac{1}{4} - \frac{3}{8}$

3 Write the following proportions as ratios:

(a) $\frac{1}{2}$ of A, $\frac{2}{5}$ of B and $\frac{1}{10}$ of C

(b) $\frac{1}{3}$ of P, $\frac{1}{5}$ of Q, $\frac{1}{4}$ of R and the remainder S

4 Complete the following:

(a) $\frac{2}{5} =$ % (b) 58% of 25 =

(c) $\frac{7}{12} =$ % (d) 17% of 50 =