DECISION SUPPORT SYSTEM

ANALYTICAL HIERARCHY PROCESS (AHP)

TEACHING TEAM

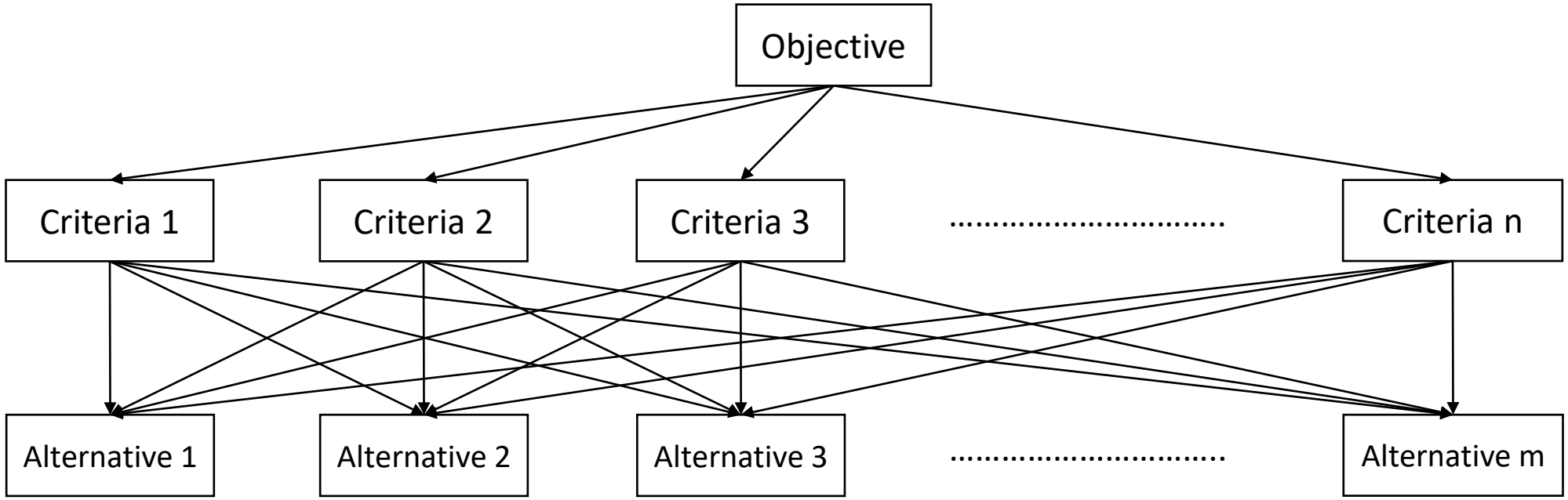
DECISION SUPPORT SYSTEM COURSE

AHP Definition

- An analysis technique that organizes complex problems into levels called hierarchies.
- The hierarchical elements in AHP consist of:
  1. Objectives/Goals
  2. Criteria (subcriteria are included)
  3. Alternatives

AHP Stages

1. Arrange a hierarchy of the problems faced



Goal

Develop a priority scale for ideal home criteria

Criteria

Desire

Budget Priority

Purchasing Power

Urgency of Need

Alternative

PERMATA JINGGA

SAWOJAJAR

GPA

SENGKALING INDAH

GADANG PERMAI

AHP Stages

2. Pairwise comparison

Comparisons are made in pairs between each **criterion** or each **alternative**

Intensity of Interest	Information
1	Both elements are equally important
3	One element is slightly more important than the others
5	One element is more important than the others
7	One element is clearly more important than the other
9	One element is absolutely more important than the others
2,4,6,8	Values between two values of adjacent considerations

(Saaty, 1986)

Intensity of Importance	Definition
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2, 4, 6, 8	For compromises between the above
Reciprocals of above	In comparing elements i and j - if i is 3 compared to j - then j is 1/3 compared to i
Rationals	Force consistency Measured values available

(Saaty, T. Lorie. 1993)

- The **pairwise comparison process** starts from the top hierarchical level which is aimed at selecting criteria, for example A1, then the elements to be compared are taken, namely: A1, A2, and A3.

	A1	A2	A3
A1	1		
A2		1	
A3			1

- Pairwise Comparison Matrix

Kriteria	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5
Alt 1	1	a_{12}	a_{13}	a_{14}	a_{15}
Alt 2	$1/a_{12}$	1	a_{23}	a_{24}	a_{25}
Alt 3	$1/a_{13}$	$1/a_{23}$	1	a_{34}	a_{35}
Alt 4	$1/a_{14}$	$1/a_{24}$	$1/a_{34}$	1	a_{45}
Alt 5	$1/a_{15}$	$1/a_{25}$	$1/a_{35}$	$1/a_{45}$	1

Example

element

X	A	B	C
A	1	3	5
B	1/3	1	5/3
C	1/5	3/5	1

AHP Stages

3. Determination of Priority Weights

a) Divide each cell value by the sum of each corresponding column.

b) Add up and average each row.

c) The average value shows the **Priority Weight (PW)** value for each corresponding row.

AHP Stages

4. Calculate the Consistency Ratio

a) Multiplying the matrix by PW

b) Divide the results of these calculations with PW

c) Calculating *λmax*

$$\lambda_{max} = \frac{\sum calculation\ results\ in\ step\ b}{the\ number\ of\ criteria\ elements}$$

d) Calculating the Consistency Index (CI)

$CI = \frac{(\lambda_{max} - n)}{(n-1)}$, where *n* is the number of criteria elements

e) Calculating the Consistency Ratio (CR) = CI/RI, where *RI* is the random consistency index

n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

If the consistency ratio ≤ 0.1, the data calculation results can be justified

AHP Stages

5. **Calculate pairwise comparisons for each alternative on each criterion** in the same way (steps 1-4)
6. **Decision-making**

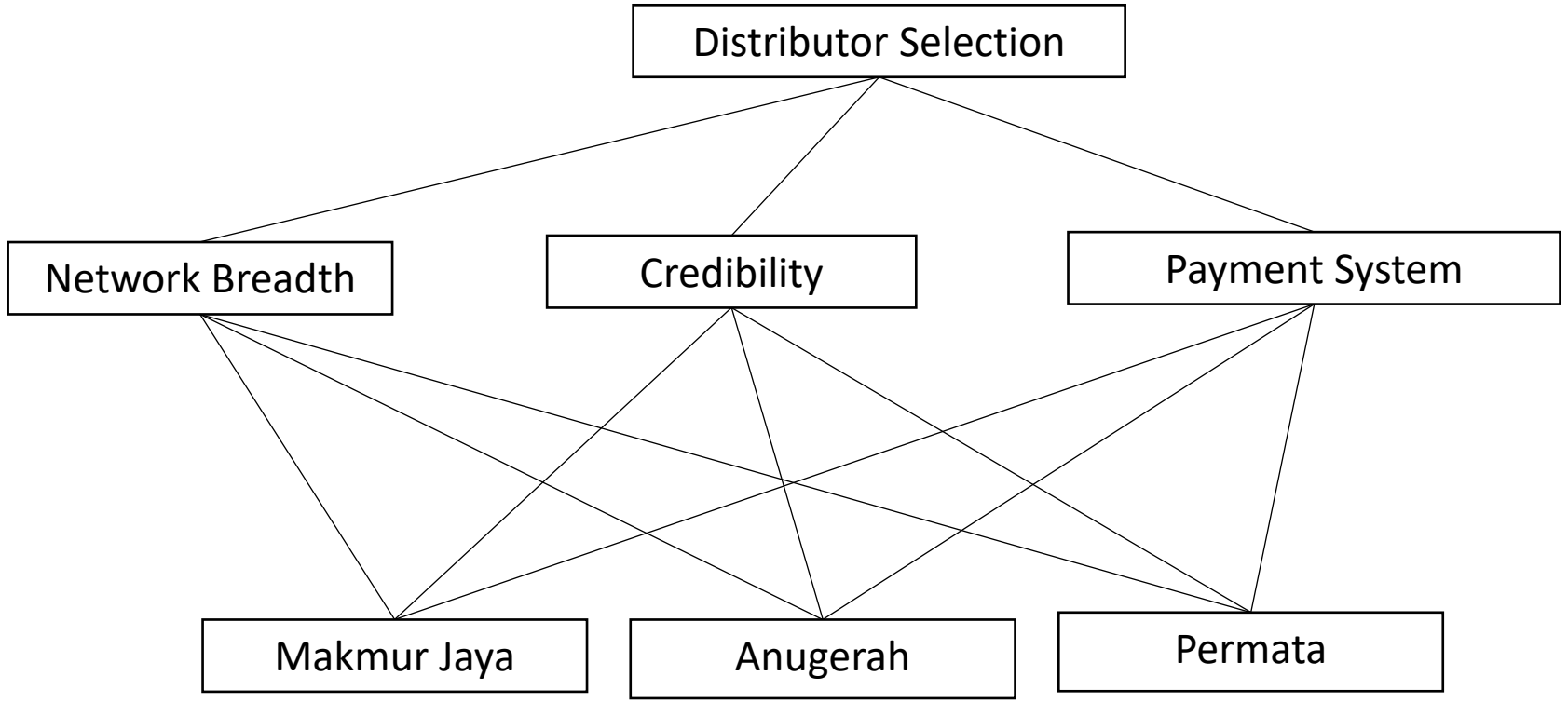
   Decision making is based on a comparison of the multiplication calculation of the **PW of criteria** with the **PW of alternative**.

Case Study

- The case to be resolved is the **selection of distributors**. The alternatives to be selected are 3 distributors (**Makmur Jaya, Anugerah, and Permata**) with 3 criteria used as assessment parameters, namely **network breadth**, **credibility**, and **payment system**.

Completion Steps

1. Arrange a hierarchy of the problems faced



Completion Steps

2. Pairwise Comparison Matrix

Intensity of Interest	Information
1	Both elements are equally important
3	One element is slightly more important than the others
5	One element is more important than the others
7	One element is clearly more important than the other
9	One element is absolutely more important than the others
2,4,6,8	Values between two values of adjacent considerations

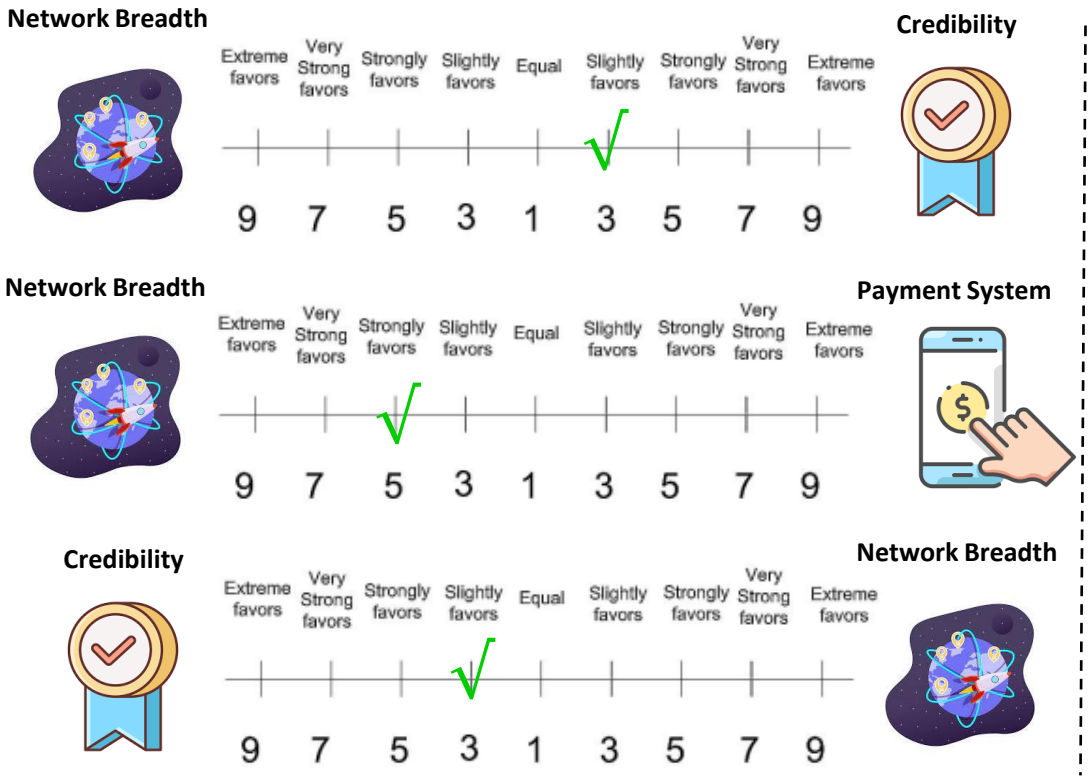
How to read:
Network breadth is 5x more important than the payment system,
or 5 to 1

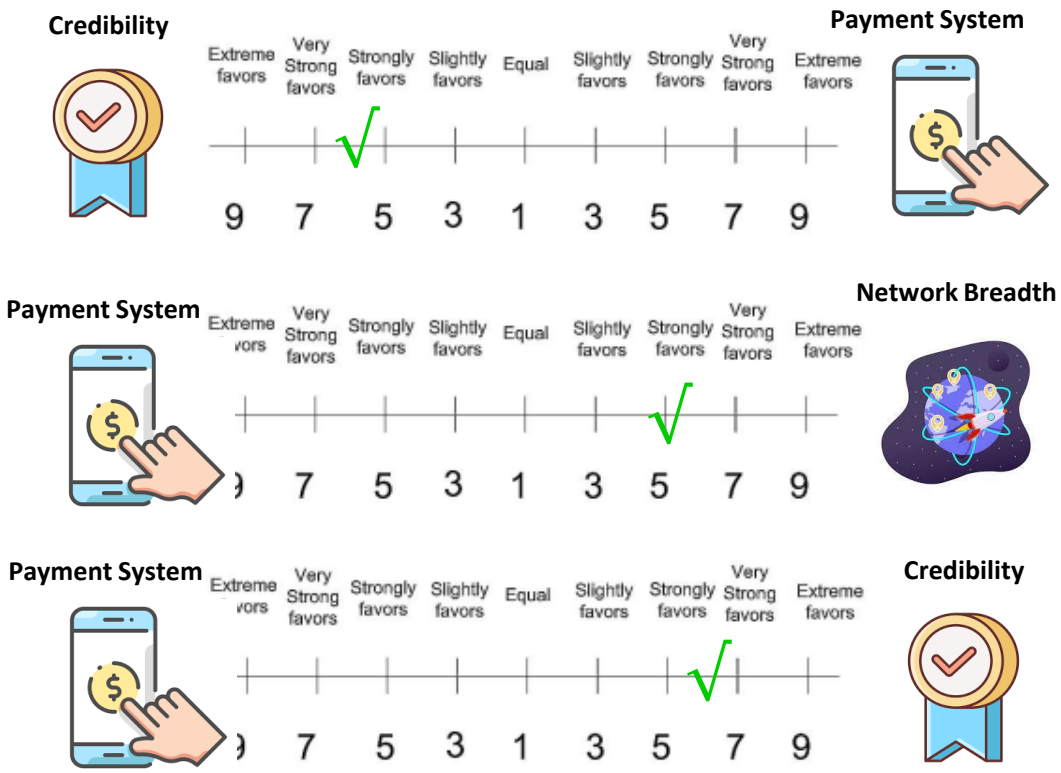
Criteria	Network	Credibility	Payment
Network	1/1	1/3	5/1
Credibility	3/1	1/1	6/1
Payment	1/5	1/6	1/1
Total	4.2	1.5	12

Completion Steps

2. Pairwise Comparison Matrix

Criteria	Network	Credibility	Payment
Network	1/1	1/3	5/1
Credibility	3/1	1/1	6/1
Payment	1/5	1/6	1/1
Total	4.2	1.5	12





Completion Steps

3. Determination of Priority Weights

Criteria	Network	Credibility	Payment
Network	1	0.333	5
Credibility	3	1	6
Payment	0.2	0.167	1
Total	4.2	1.5	12

Normalization 1 : 4.2 = 0.238

Total : Quantity

Eigen Vector

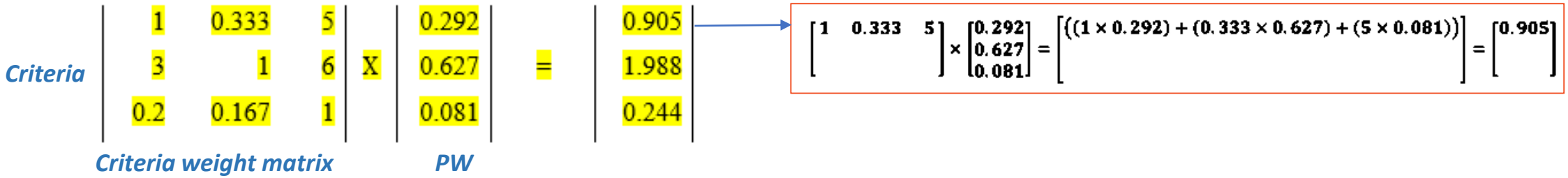
Criteria	Network	Credibility	Payment	Total	Average (Priority Weight)
Network	0.238	0.222	0.417	0.877	0.292
Credibility	0.714	0.667	0.5	1.881	0.627
Payment	0.048	0.111	0.083	0.242	0.0081

PW of Criteria

Completion Steps

4. Calculate the Consistency Rasio

a) Multiplying the matrix by PW



b) Divide the results of these calculations with PW

- **Network Breadth = 0.905/0.292 = 3.095**
- **Credibility = 1.988/0.627 = 3.171**
- **Payment System = 0.244/0.081 = 3.020**

Completion Steps

c) Calculating *λmax* → $\lambda_{max} = \frac{\sum calculation\ results\ in\ step\ b}{the\ number\ of\ criteria\ elements}$

$$\lambda_{max} = \frac{3.095 + 3.171 + 3.020}{3} = 3.09$$

d) Calculating the Consistency Index (CI) → $CI = \frac{(\lambda_{max} - n)}{(n-1)}$

$$CI = \frac{(3.09 - 3)}{(3 - 1)} = 0.047$$

n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

e) Calculating the Consistency Ratio (CR) = CI/RI

Based on the table, **n = 3**, then **RI = 0.58**

Consistency Ratio(CR) = 0.047/0.58 = **0.082** (≤ 0.1, so it is consistent)

Completion Steps

5. Calculate pairwise comparisons for each alternative

Using the **same calculation method**, Consistency Ratio (CR) is calculated for pairwise comparisons between alternatives

a. Consistency Ratio(CR) for Network Breadth

Network Breadth	Makmur Jaya	Anugerah	Permata	Priority Weight	CR
Makmur Jaya	1	5/1	7/1	0.724	
Anugerah	1/5	1	3/1	0.193	**0.057**
Permata	1/7	1/3	1	0.083	

Consistency Ratio (CR) = 0.057 (≤ 0.1, so it is consistent)

Completion Steps

b. Consistency Ratio(CR) for Credibility

Credibility	Makmur Jaya	Anugerah	Permata	Priority Weight	CR
Makmur Jaya	1	5/1	9/1	0.748	
Anugerah	1/5	1	3/1	0.180	**0.025**
Permata	1/9	1/3	1	0.071	

Consistency Ratio (CR) = 0.025 (≤ 0.1, so it is consistent)

c. Consistency Ratio(CR) for Payment System

Payment System	Makmur Jaya	Anugerah	Permata	Priority Weight	CR
Makmur Jaya	1	5/1	7/1	0.724	
Anugerah	1/5	1	3/1	0.193	**0.038**
Permata	1/7	1/3	1	0.083	

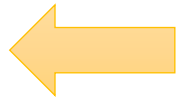
Consistency Ratio (CR) = 0.038 (≤ 0.1, so it is consistent)

Completion Steps

6. Decision-making

Decision making is based on a comparison of the multiplication calculation of the **PW of criteria** with the **PW of alternative**.

	Network	Credibility	Payment	Evaluation Weight
Priority Weight	0.292	0.627	0.081	
Makmur Jaya	0.724	0.748	0.724	**0.739**
Anugerah	0.193	0.180	0.193	0.171
Permata	0.083	0.071	0.083	0.069



Rank 1

Evaluation Weight

= (0.292 x 0.724) + (0.627 x 0.748) +(0.081 x 0.724)

= 0.739 (Makmur Jaya)

Completion Steps

Conclusion:

- From the calculation results, it can be concluded that based on the criteria of network breadth, credibility and payment system, Makmur Jaya was selected as a distributor because it has the highest Evaluation Weight value of 0.739.

Additional Information

- Variations of other ways to calculate eigenvectors

Initial matrix

Criteria	Network	Credibility	Payment
Network	1	0.333	5
Credibility	3	1	6
Payment	0.2	0.167	1

×

Criteria	Network	Credibility	Payment
Network	1	0.333	5
Credibility	3	1	6
Payment	0.2	0.167	1

(1*1)+(0.333*3)*(5*0.2)= 2.999

The matrix used as a multiplier in the 2nd iteration

Criteria	Network	Credibility	Payment
Network	2.999	1.501	11.998
Credibility	7.2	3.001	27
Payment	0.901	0.401	3.002

Result	Eigenvector
= 16.948	= 0.284
= 37.201	= 0.641
= 4.3036 / 58.0026 +	= 0.074 / = 1.000 +

The matrix multiplication will be iterated between the old orange matrix and the new blue matrix until the difference in the eigenvectors resulting from n iterations compared to n-1 is equal to zero or does not change. If these conditions are met, the final eigenvector value is selected

Additional Information

- Cost Criteria in AHP
  a) Separate the hierarchy of costs and benefits
  b) Combine the results

  Example:

Final Value for AHP calculation (benefit)

	Evaluation Weight
Makmur Jaya	0.686
Anugerah	0.171
Permata	0.069

Final Value for AHP calculation (cost)

	Evaluation Weight
Makmur Jaya	0.549
Anugerah	0.176
Permata	0.035

Notes:
Values are in final calculation form

Final Value for AHP calculation (benefit-cost)

	Calculation	Evaluation Weight
Makmur Jaya	0.686/0.549	1.249
Anugerah	0.171/0.176	0.971
Permata	0.069/0.035	**1.971**

Rank 1: Permata