Midyear Exam - 24 octobre 2016

Exercice 1. Give a basis for null space and the column space of the following matrix

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 & -1 \\ -1 & 1 & 0 & -2 \\ 2 & -3 & -1 & 5 \end{pmatrix}.$$

Exercice 2. Find the eigenvalues of the following matrix

$$\boldsymbol{A} = \begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \\ 1 & -1 & 3 \end{pmatrix}.$$

Exercice 3.

1. Project $\boldsymbol{b}$ onto the column space of $\boldsymbol{A}$ by solving the system $\boldsymbol{A}^T\boldsymbol{A}\widehat{\boldsymbol{x}} = \boldsymbol{A}^T\boldsymbol{b}$ and then by computing $\boldsymbol{p} = \boldsymbol{A}\widehat{\boldsymbol{x}}$ with
$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}.$$
2. Compute the corresponding projection matrix.
3. What conditions does this matrix have to check ?

Exercice 4. We consider the following matrix

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

1. Using the Gram-Schmidt process, write the $\boldsymbol{QR}$ factorization of the matrix $\boldsymbol{A}$.
2. Using this $\boldsymbol{QR}$ decomposition, find $\boldsymbol{x}$ such that
$$\boldsymbol{Ax} = \begin{pmatrix} -5 \\ -1 \\ 1 \end{pmatrix}.$$