Evolution with constraints

2/ Introduction to constraint programming

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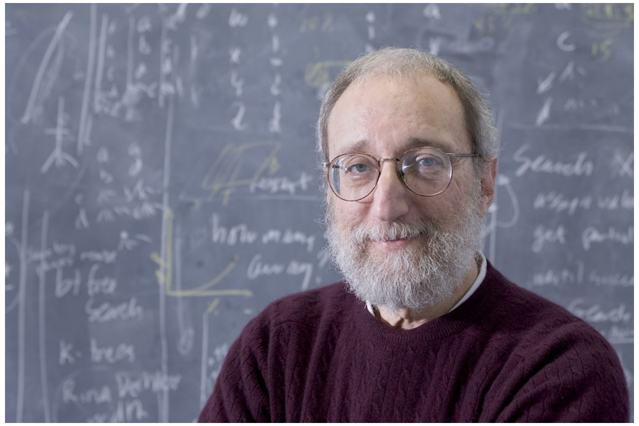
SUPAERO

Agenda / 2. Introduction to constraint programming

2.1. General, definitions & concepts

2.2. Solving/Optimizing algorithms

2.3. Filtering algorithms



Eugène Freuder, Constraints, April 1997:

"Constraint Programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming : The user states the problem, the computer solves it".

2.1. General, definitions & concepts

Definition :

A CSP is defined by a triplet (V, D, C) where :

- V is a set of variables of the problem. These variables can be grouped to formalize specific items, such as tasks, components, etc.
- D is a set of domains of the variables, one per variable. These domains can be symbolic, numerical, discrete or continuous depending on the knowledge they are reprenting (time, performance, reference, etc).
- C is a set of constraints on variables V where a constraint describes the allowed or excluded combinations of variable values.

Solution

$$\forall_{i=1}^{k} v_i \in \mathbb{V}, |D_{v_i}| = 1 \land \nexists_{j=1}^{m} c_j \in \mathbb{C}, c_j = \bot$$

Variable types

Discrete variables

Symbolic variables

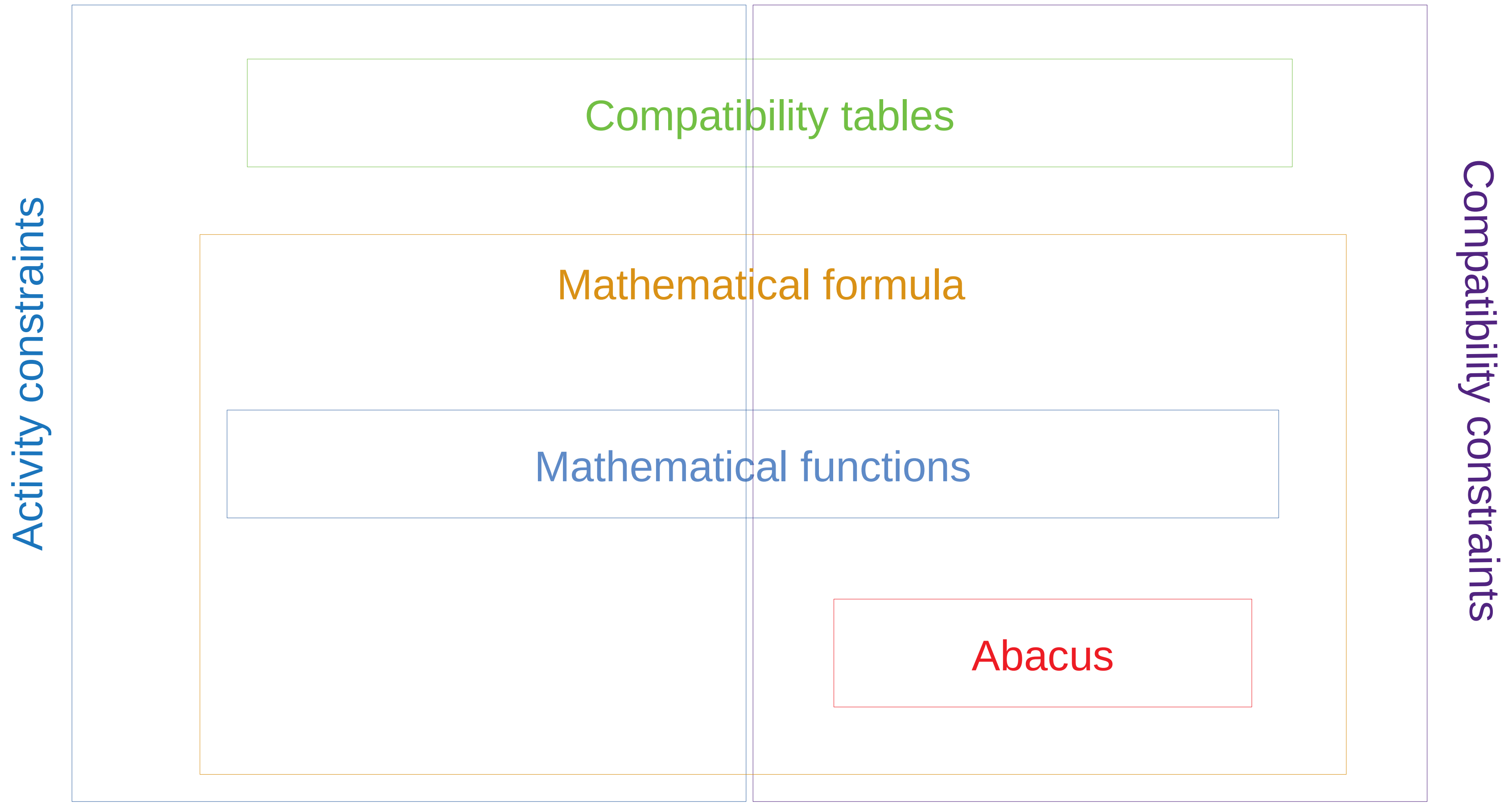
- List of symbols
- List of integers
- Range of integers
- List of reals

Continuous variables

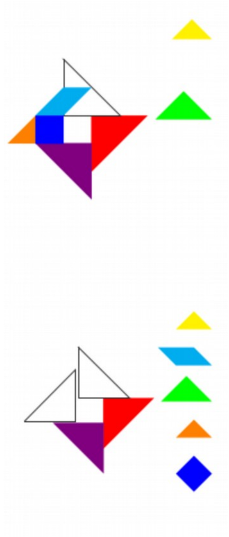
- Range of reals

2.1. General, definitions & concepts

Constraint types



CSP types

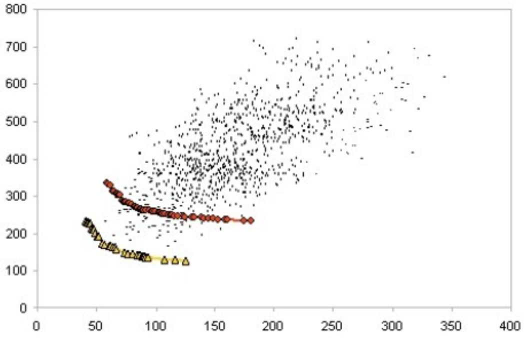


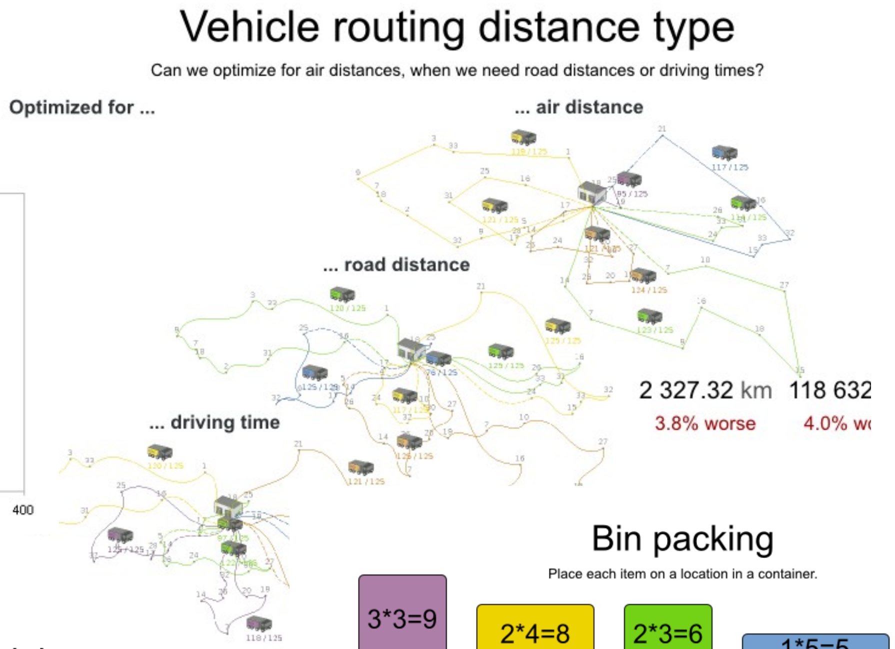
Static versus dynamic

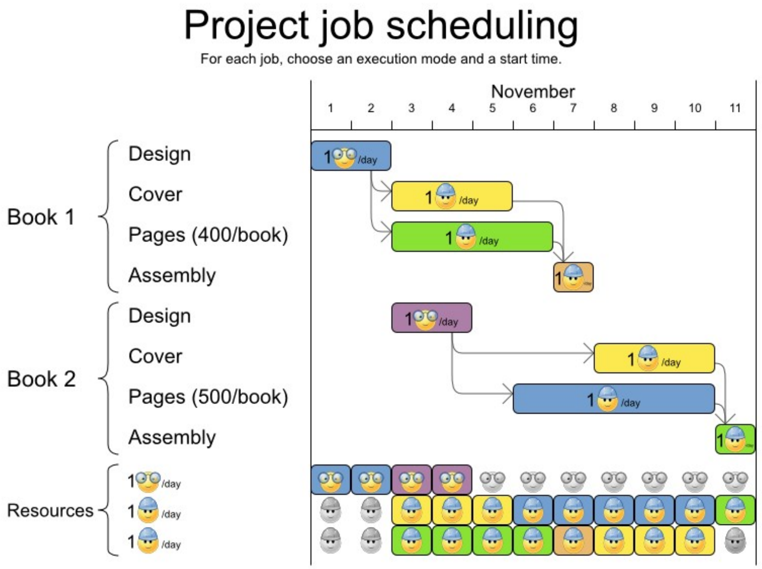
	CSP * CSP e	Conditional CSP Composite CSP Generative CSP
	Discrete CSP Continuous CSP Temporal CSP Mixed CSP	

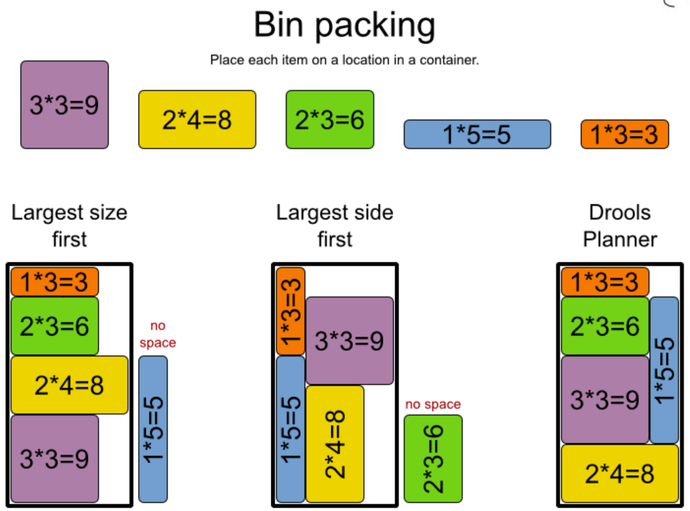
Static versus dynamic

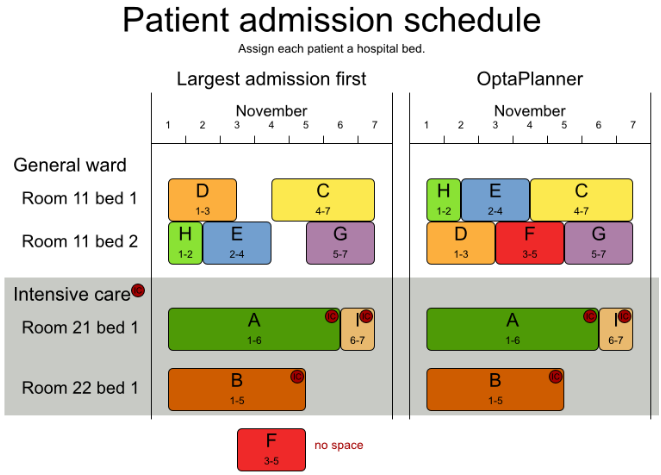
Applications

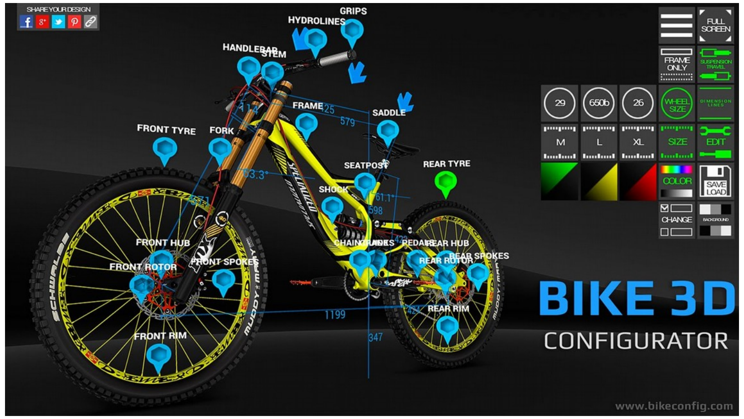












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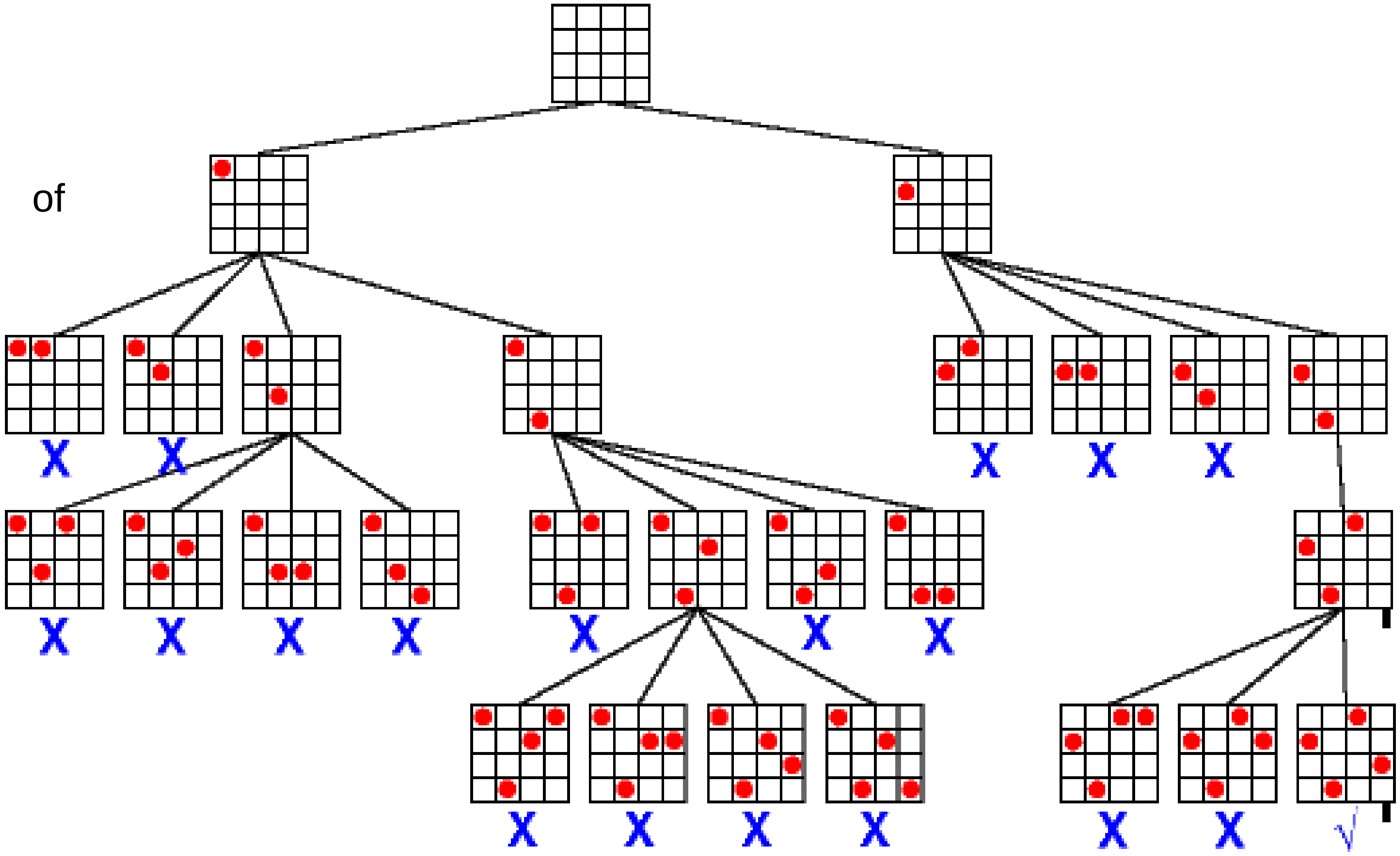
~~2.1. General, definitions & concepts~~

2.2. Solving/Optimizing algorithms

2.3. Filtering algorithms

Systematic research :

- Smart or not,
- Variable ordering,
- with possible propagation of choices,
- Etc



NP-hard problem

Existing Solvers

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Prune variable values which cannot lead to a solution.

1,2,3, 4,5,6, 7,8,9	1,2,3, 4,5,6, 7,8,9	1,2,3, 4,5,6, 7,8,9
1,2,3, 4,5,6, 7,8,9	1,2,3, 4,5,6, 7,8,9	1,2,3, 4,5,6, 7,8,9
1,2,3, 4,5,6, 7,8,9	1,2,3, 4,5,6, 7,8,9	1,2,3, 4,5,6, 7,8,9

9 variables, X1 to X9
9 definition domains = {1..9}
1 constraint : AllDiff (X1..X9)

Search space = 9^9 = 387 420 489

To converge faster : prune inconsistent values during the search

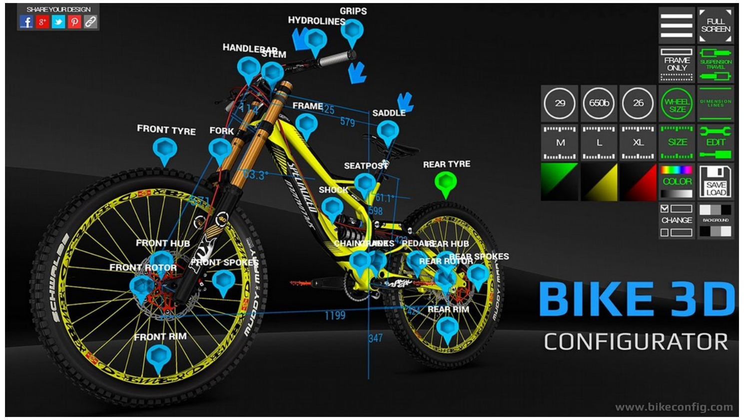
Prune variable values which cannot lead to a solution.

1,~~2,3,~~ ~~4,5,6,~~ ~~7,8,9~~	~~1,~~2,3, 4,5,6, 7,8,9	~~1,~~2,3, 4,5,6, 7,8,9
~~1,~~2,3, 4,5,6, 7,8,9	~~1,~~2,3, 4,5,6, 7,8,9	~~1,~~2,3, 4,5,6, 7,8,9
~~1,~~2,3, 4,5,6, 7,8,9	~~1,~~2,3, 4,5,6, 7,8,9	~~1,~~2,3, 4,5,6, 7,8,9

1,~~2,3,~~ ~~4,5,6,~~ ~~7,8,9~~	~~1,2,3,~~ ~~4,5,6,~~ ~~7,8,~~**9**	~~1,~~2,3, 4,5,6, 7,8,~~9~~
~~1,~~2,3, 4,5,6, 7,8,~~9~~	~~1,~~2,3, 4,5,6, 7,8,~~9~~	~~1,~~2,3, 4,5,6, 7,8,~~9~~
~~1,~~2,3, 4,5,6, 7,8,~~9~~	~~1,~~2,3, 4,5,6, 7,8,~~9~~	~~1,~~2,3, 4,5,6, 7,8,~~9~~

2.3. Filtering algorithms

Filtering methods (AC, K-C, Interval arithmetics, Box-consistency, etc) can be used on their own :



Or combined to solving/optimizing methods to decrease the computational time…

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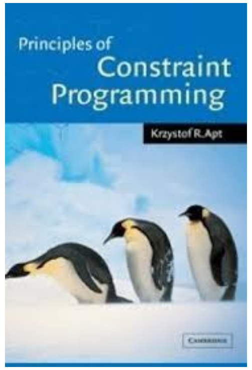
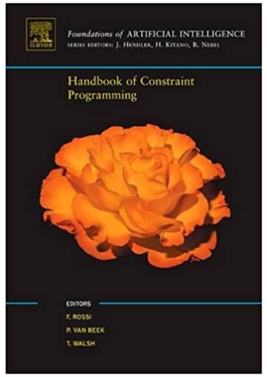
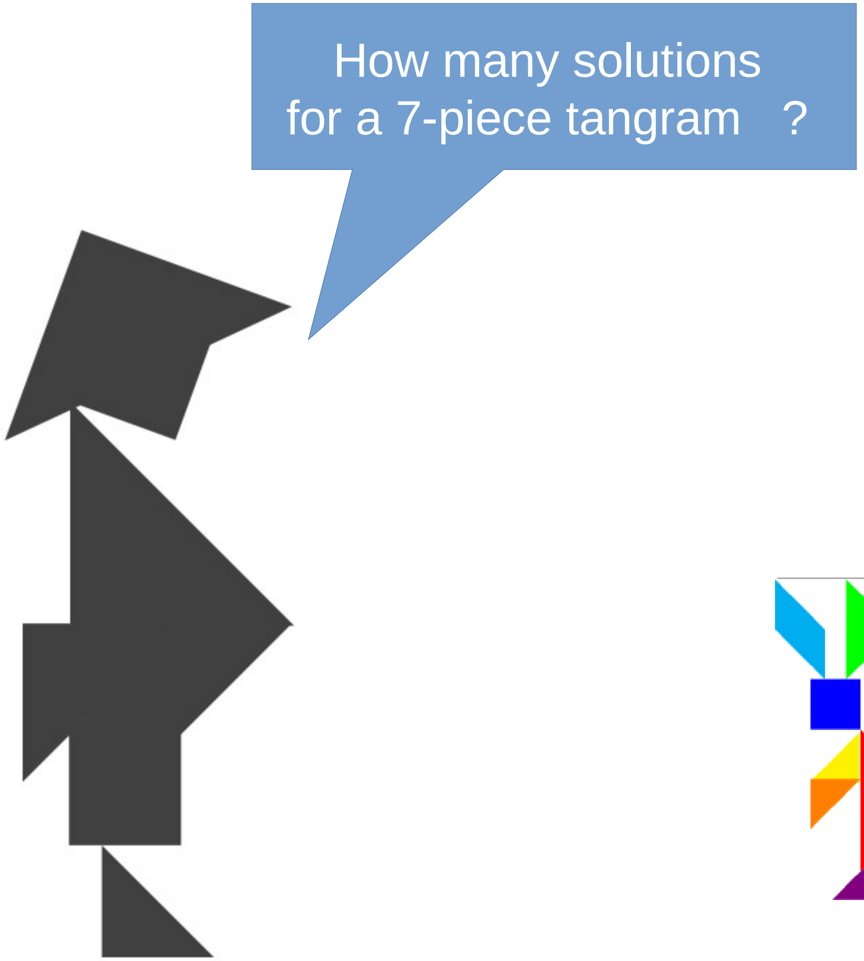
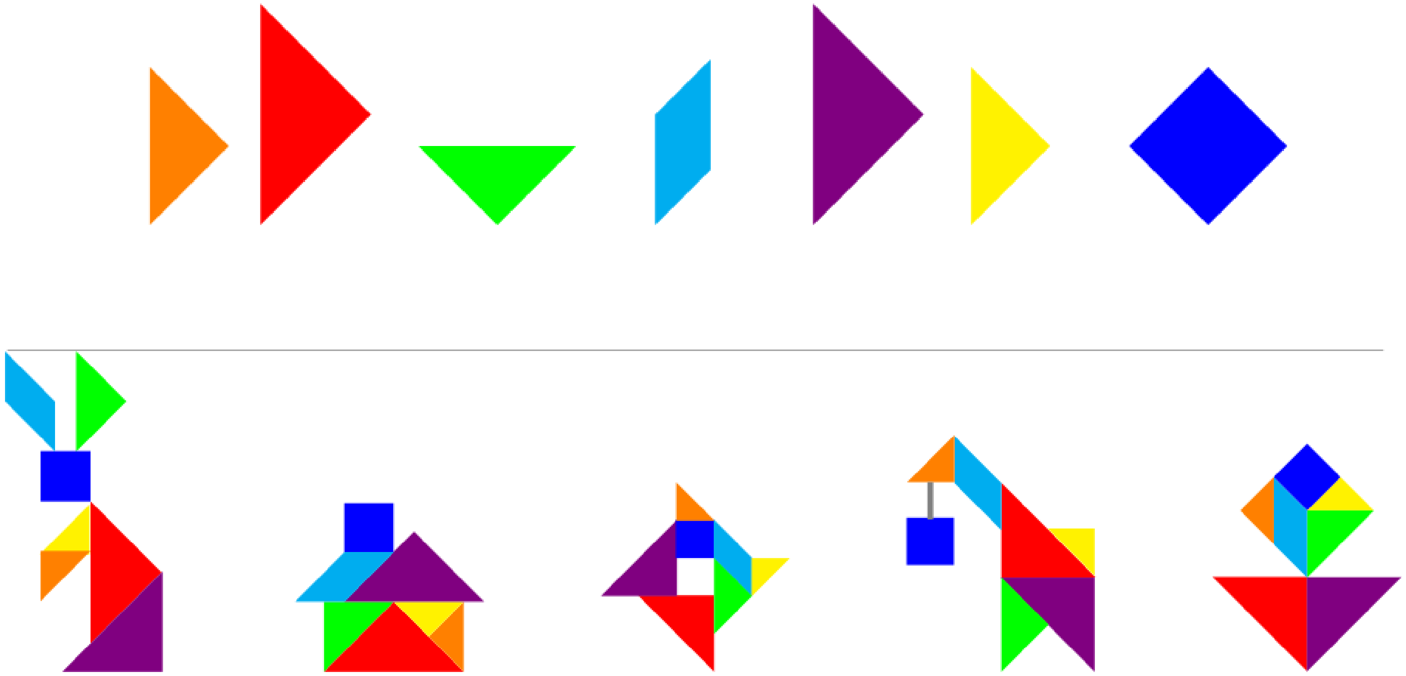
To read on Constraint Programming :

- Accreditation to conduct research, E. Vareilles, 2015, https://perso.imt-mines-albi.fr/~vareille/Vareilles_HdR.pdf
- Handbook of Constraint Programming, K. N. Brown, I. Miguel, in Foundations of Artificial Intelligence, 2006
- http://www.univ-montp3.fr/miap/~jq/OptionIA/cours/cspGoualard.pdf
- https://www.labri.fr/perso/eyraud/pmwiki/uploads/Main/Cours1-Papier.pdf






How many solutions
for a 7-piece tangram ?



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