System identification and Signal Filtering

Laboratory work - Session 2

2 Identification of a mechanical system

The aim of this project is to identify the continuous-time transfer function of a real system and to link the identified model parameters to some physical parameters of the system. A discrete-time model is firstly identified and then transformed to continuous-time domain.

2.1 Description of the process

The electro-mechanical process, illustrated by Figure 1, is composed by a DC motor, an amplifier allowing to control the input voltage of the motor and two sensors for measuring its angular position and velocity. By considering the angular position as output, the process is instable in open-loop. Therefore, a simple proportionnel controller is added to the system in order to perform the identification from closed-loop data.

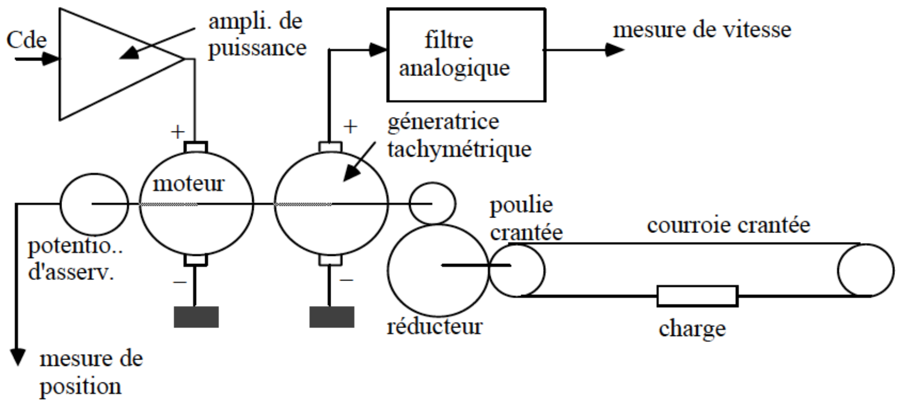

Figure 1. Illustration of the mechanical process.

The following notations will be used
- u the control signal corresponding to the input voltage of the motor,
- y and z the measurement signals of the angular position and the velocity of the motor axis,
- y_c the target angular position,
- P the gain of the proportional corrector.

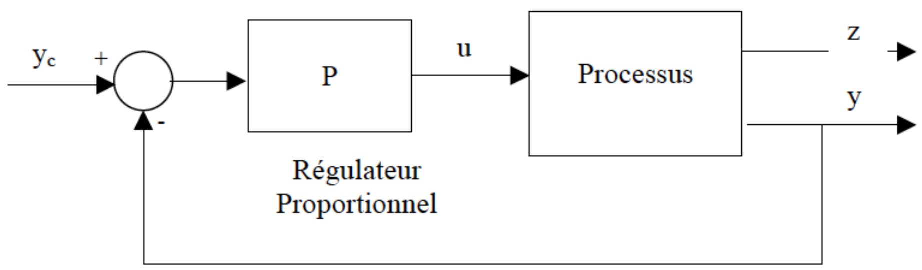

Figure 2. Closed-loop configuration of the controlled system

2.2 Modeling of the process

The physical modeling of the system leads to following block diagram

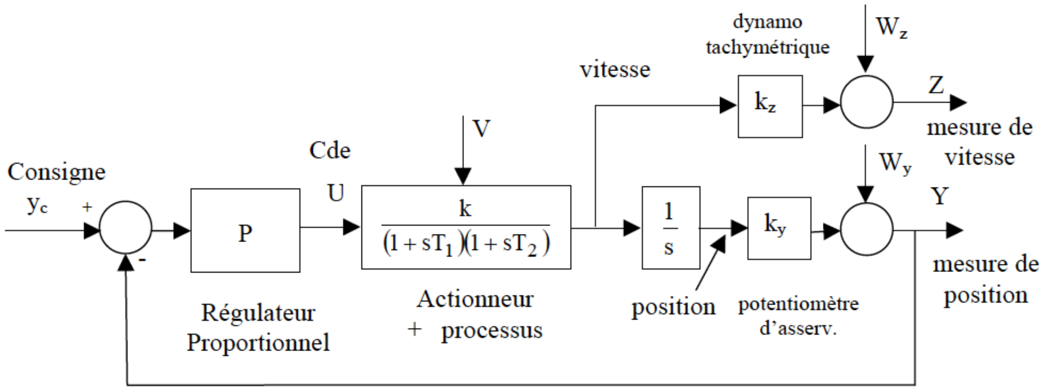

Figure 3. Process modeling.

where $v(t)$ represents the amplifier modeling errors, $w_y(t)$ and $w_z(t)$ are measurement errors of angular position and velocity. All these errors are assumed to be Gaussian and zero mean.

The transfert functions of the process are given in the form

$$\frac{Y(s)}{U(s)} = \frac{K_y}{s(1+T_1 s)(1+T_2 s)} \quad \text{and} \quad \frac{Z(s)}{U(s)} = \frac{K_z}{(1+T_1 s)(1+T_2 s)}$$

where $K_y = (k \cdot k_y)$ and $K_z = (k \cdot k_z)$.

1. Show that the transfer function of the system when considering the velocity as output variable is given as
$$\frac{Z(s)}{Y_c(s)} = \frac{b_1\, s + b_0}{a_3\, s^3 + a_2\, s^2 + a_1 s + a_0}$$
and express the values of the coefficients $(b_0, b_1, a_0, a_1, a_2, a_3)$ with respect to the physical parameters of the process (P, T_1, T_2, K_y, K_z).
2. What is the order of the equivalent discrete-time transfer function if a numerical differentiation method is used for the conversion from continuous-time to discrete-time.

2.3 Identification of the processus

In order to identify the four parameters of the processus T_1, T_2, K_y, K_z from the signals available in the file `Tergane20.mat`. These data correspond to a record of the target position (vector `yc`), measured velocity (vector `z`) and measured position (vector `y`). These data have been obtained when applying a proportional controller with a gain $P = 2.91$ and a sampling frequency $f_s = 1$ kHz.

1. Express the values of the physical parameters of the process (P, T_1, T_2, K_y, K_z) with respect to the coefficients $(b_0, b_1, a_0, a_1, a_2, a_3)$
2. Write a Matlab code allowing to identify the transfer function of the process using an output error model in discrete-time domain $\dfrac{Z(z)}{Y_c(z)}$ (use OE or ARMAX models)
3. Convert this discrete-time to the continuous-time domain under the hypothesis of a zero order hold
4. Deduce the values of the physical parameters.