Econometrics II
Assignment
Paris-Dauphine University
Master 203

MAGERAND Alban

January 2025

Contents

Chapter 1

Timing the Core Equity Factor

1.1 Core Equity Factor & Replication Portfolio

We consider monthly returns data of the Euro Stoxx50 and of its constituents from June 2001 until November 2024.

Core Equity Factor
The Core Equity Factor (CEF) is defined as the first principal component of the PCA on our dataset, such that:

- there is a positive correlation between the ECF and the returns of individual stocks
- the ECF factor has the same volatility as the benchmark.

Mathematically, it can be defined as:

$$CEF = sgn(\rho_{mkt,ECF}) \times PC_0 \times \frac{\sigma_{market}}{\sigma_{PC_0}} \tag{1.1}$$

We standardize the returns and extract the first principal component that we compare to the index returns to get a sense of the appropriate sign in (1.1):

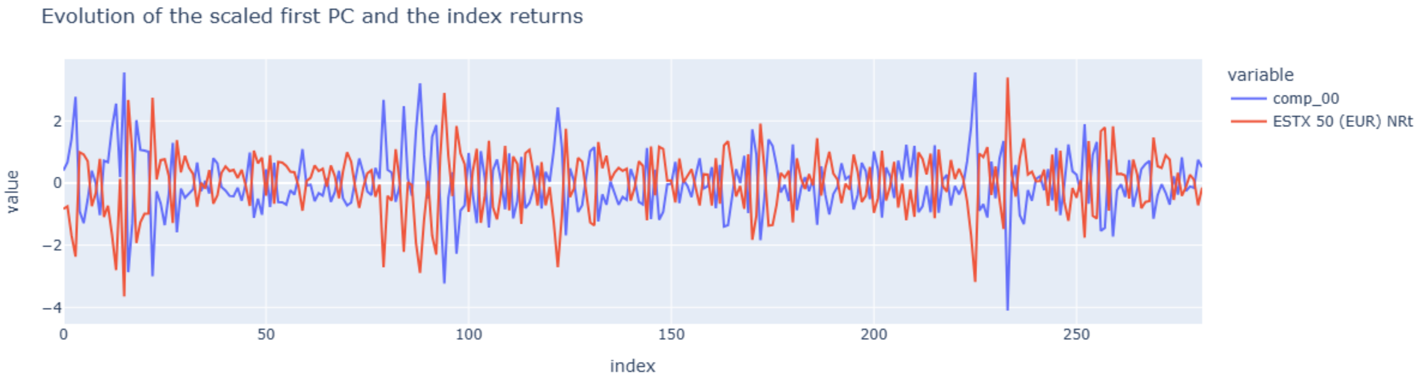

Figure 1.1: Scaled first PC and Euro Stoxx 50

From the opposite evolution (and the negative correlation $\rho_{mkt,ECF} = -0.98$), we deduce that the sign is negative to construct the ECF factor.

Exposure to the CEF factor
We now estimate the sensitivity of each stock to this factor by running the following regression for each stock i:

$$r_{i,t} = \alpha_i + b_{1,i} \times F_{1,t} + \epsilon_t \tag{1.2}$$

We obtain the following histogram of coefficients:

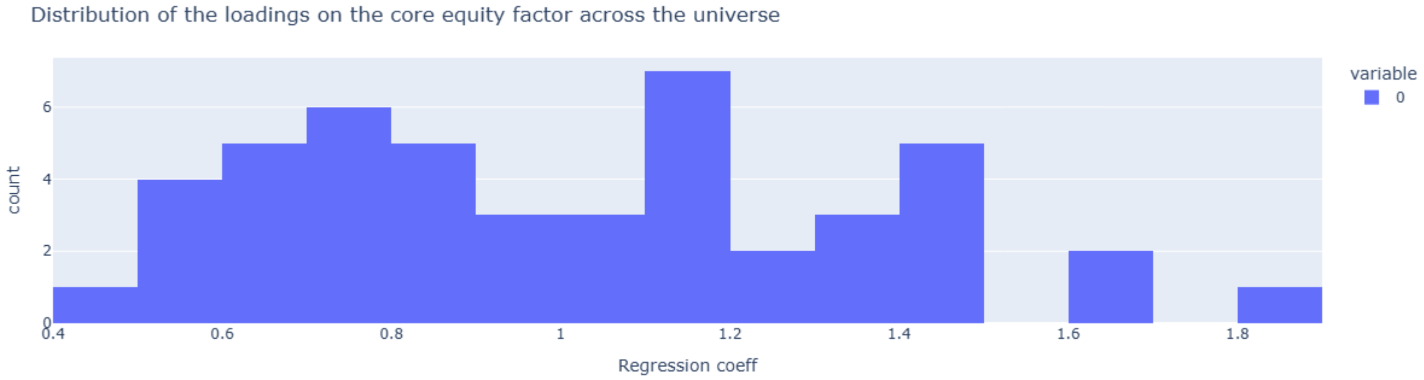

Figure 1.2: Distribution of the $b_{1,i}$

We notice that every stock in our universe has a positive loading on the CEF by design, which is approximately the market. Had we considered a broader universe, the results could have been different (e.g., if more defensive stocks had been included).

Replication Portfolio
To compute the weights enabling us to replicate the CEF factor (which can be seen as a portfolio), we solve the following optimization program:

$$\begin{aligned} & \text{Argmin}_{\mathbf{w}_1} \left(\mathbf{w}_1' \hat{\Omega} \mathbf{w}_1 \right) \\ \text{subject to:} \quad & \sum_{i=1}^{N} w_{1,i} = 1, \quad w_{1,i} \geq 0, \quad \sum_{i=1}^{N} w_{1,i} \hat{b}_{i,1} = 1 \end{aligned} \tag{1.3}$$

After doing so, we obtain the following histogram of weights:

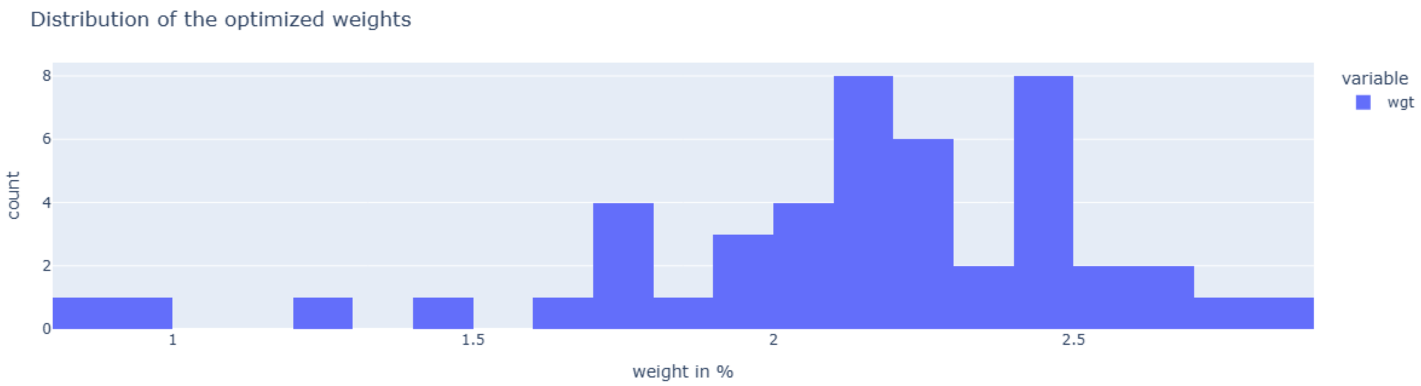

Figure 1.3: Distribution of the weights of the replicating portfolio

Most stocks seem to be represented in the replicating portfolio, and there does not seem to be a huge disparity in the weights across the universe. The evolution of the distribution of the weights depending on how we estimate the covariance matrix will be studied in the below section.

1.2 Alpha of the portfolio and impact of the estimation error

To estimate the alpha of this portfolio against the benchmark, we run the following regression:

$$Rtn_t^{ECF} = \alpha + \beta \times Rtn_t^{mkt} + \epsilon_t \tag{1.4}$$

and look at the significance of the α. We obtain the following output:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     ESTX 50 (EUR) NRt   R-squared:                       0.959
Model:                           OLS   Adj. R-squared:                  0.959
Method:                Least Squares   F-statistic:                     6549.
Date:               Fri, 27 Dec 2024   Prob (F-statistic):          2.99e-196
Time:                       08:50:21   Log-Likelihood:                 883.38
No. Observations:                282   AIC:                            -1763.
Df Residuals:                    280   BIC:                            -1755.
Df Model:                          1
Covariance Type:           nonrobust
================================================================================
                  coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
rtns_ecf_ptf    0.9792      0.012     80.929      0.000       0.955       1.003
const          -0.0045      0.001     -7.090      0.000      -0.006      -0.003
==============================================================================
Omnibus:                       6.946   Durbin-Watson:                   1.825
Prob(Omnibus):                 0.031   Jarque-Bera (JB):                8.814
Skew:                         -0.199   Prob(JB):                       0.0122
Kurtosis:                      3.769   Cond. No.                        19.2
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 1.4: Regression of the CEF portfolio returns against the benchmark

The alpha seems highly significant, with a t-stat of -7.09, potentially indicating a strong source of outperformance with respect to the benchmark.
Aware of the significant influence that covariance matrix estimation can have on the weights of optimized portfolios (e.g., Jorion, 1992), we employ a bootstrapping technique on the covariance matrix used for portfolio optimization. This approach allows us to derive the following distribution of t-statistics:

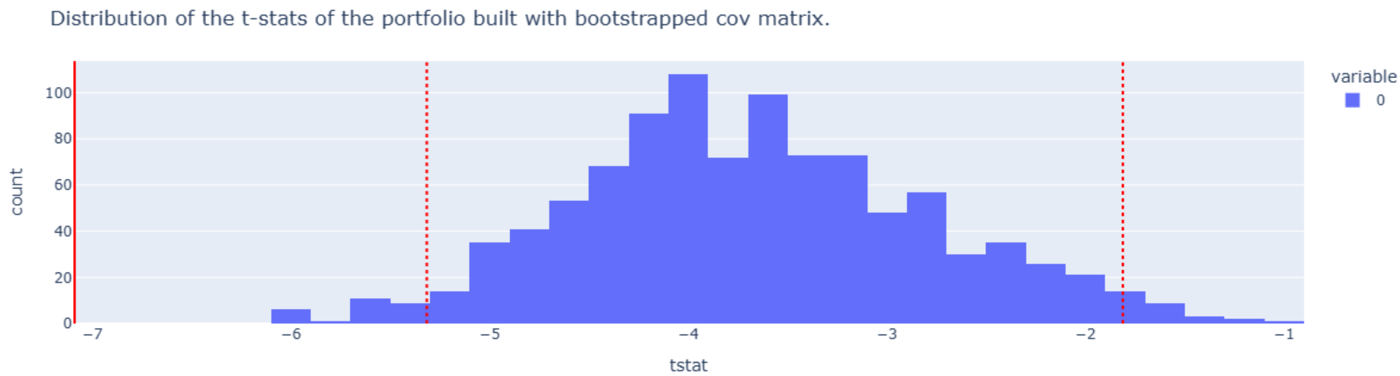

Figure 1.5: Distribution of the t-stats of the alpha of eq(1.4)

With the following 5% confidence interval:

$$t^{\alpha}_{5\%} = [-5.41, -1.82]$$

Since we have 282 observations, we can approximate the t-stats distribution by a normal one, with a 5% threshold of 1.96, making the alpha significant in most simulations.

1.3 Trend model and Investment Strategy

Using the weights obtained previously for the replicating portfolio (which we assume constant), we construct the estimated core portfolio $I_{1,t}$ that we modelize using a local linear trend model:

$$\begin{aligned} I_{1,t} &= T_{1,t} + \epsilon_t, \\ T_{1,t} &= T_{1,t-1} + S_{1,t-1} + u_t, \\ S_{1,t} &= S_{1,t-1} + v_t. \end{aligned}$$

where $T_{1,t}$ is the trend component of the price index, $S_{1,t}$ is the slope of the trend, ϵ_t is the measurement error, u_t and v_t the state innovation errors associated with the trend and the slope respectively, and $\sigma_u^2 = \frac{\sigma_\epsilon^2}{2}$, $\sigma_v^2 = \frac{\sigma_u^2}{2}$.

We estimate this model using a Kalman filter, and construct the following investment strategy:

$$R_{s,t} = \begin{cases} R_{c,t}, \ \text{if } S_{1,t-1} > 0 \\ \\ \frac{3}{12} \ \text{else} \end{cases}$$

We obtain the following summary of investment, which yields a Sharpe ratio of 0.502:

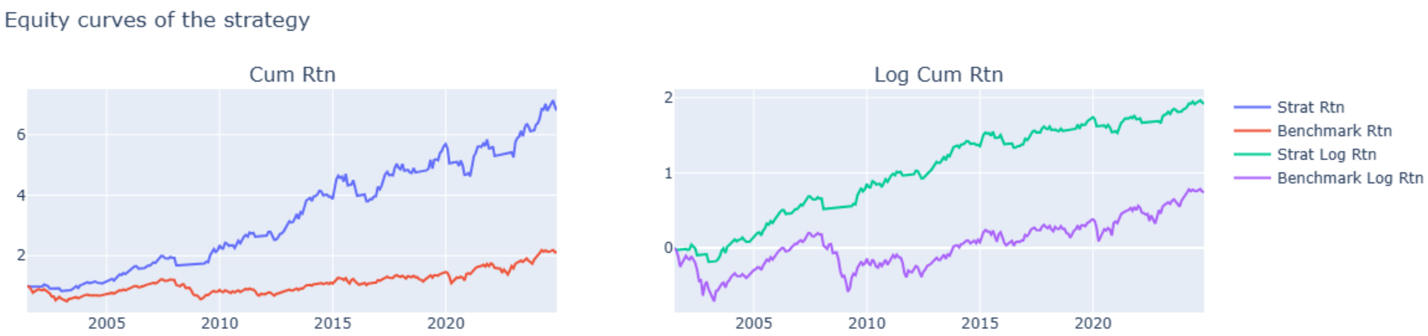

Figure 1.6: Performance of the trend strategy

The strategy yields a solid performance compared to the index, and does not seem to decay recently. On top of that, being long the equity factor when the slope is positive (can be seen as the speed of the returns), and neutral when the price action turns less bullish intuitively makes sense.

To test the legitimacy of this performance, we apply a bootstrapping technique where we randomly assign the weights w_1 N times each month and compute the Sharpe ratio of the resulting strategy. We then derive the following distribution of SRs:

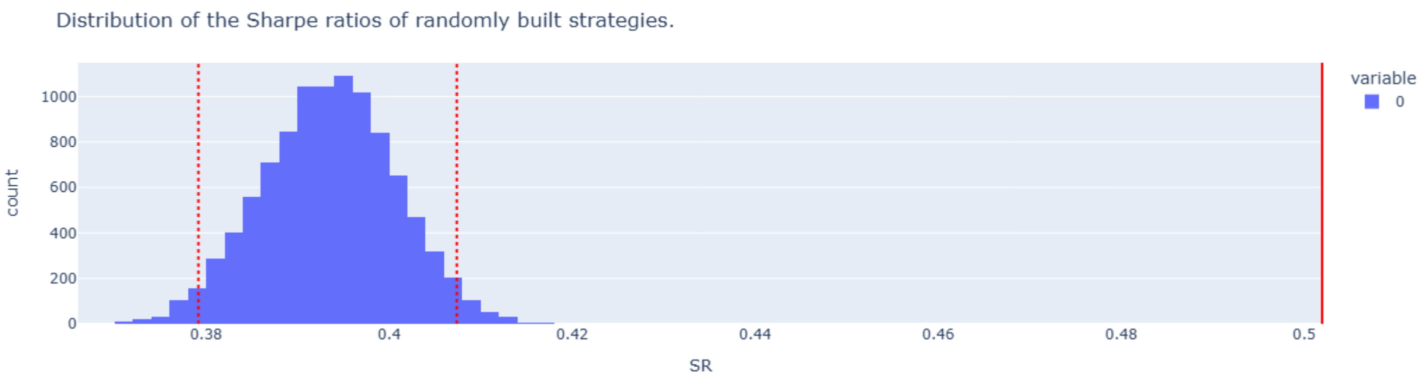

Figure 1.7: Distribution of Sharpe ratios of randomly built strategies

With the following 5% cutoff values:

$$SR_{5\%} = [0.38, 0.41]$$

The initial strategy yields a 0.50 SR, which lies outside this interval and therefore indicates that the observed performance is unlikely due to luck.
Overall, this research yields promising results but could be extended by considering the following aspects:

- more realistic backtest, incorporating other moments of the returns distributions (e.g. drawdowns) and cost.
- impact of the re-balancing frequency on the performance combined with time-varying weights.