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## Software

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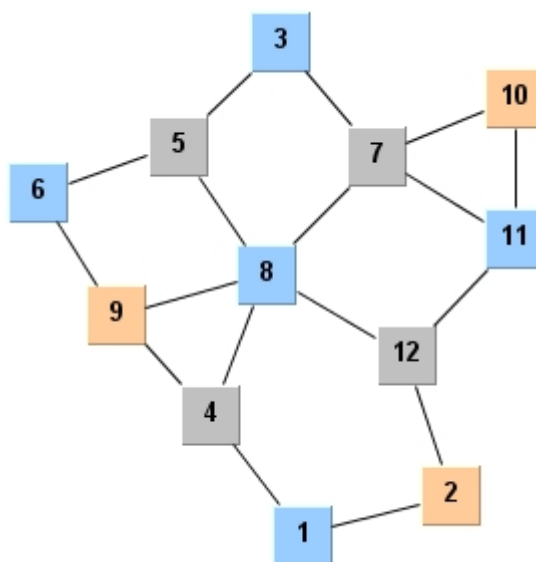
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## Summary



**Figure 1: Three Coloring.**

The graph coloring problem aims at assigning colors to the nodes of a graph such that no two connected nodes have the same color. The graph coloring problem is NP-complete and one of the harder problems to solve. We present a heuristic to solve it using three cascaded algorithms. The graph coloring problem was one of Karp's 21 NP-complete problems, and is also known as the problem of finding the chromatic number of a graph. Several other problems reduce to graph coloring including solving generalized Sudoku puzzles.

The problem of graph coloring can be solved using exact approaches like branch and cut and using heuristics like this work. Exact algorithms are not suitable for very large instances like more than 500 vertices. This paper (Malaguti & Toth, 2010) surveys some of the algorithms that are exact and heuristic and find that the %gap for exact algorithms, even when allowed to run for 7200 seconds is quite large. This algorithm is able to solve several large instances, and finds the optimum chromatic number or at least a very close to optimum solution in a short amount of computational duration. When an exact algorithm is available and suitable, it is of course the preferred way as the solution can be provably optimal. But for very large graphs, this is often unsuitable because of the exponential complexity of NP-hard problems. Our heuristic can help in these cases. Results of our algorithm can be found on our git page (<https://github.com/shah314/graphcoloring>).

I ran the ColPack solver (Gebremedhin, Nguyen, Patwary, & Pothén, 2010) on some of the benchmark instances. The results are at the bottom of the git README and in the below table. To summarize, our method is as good or better on all the randomly chosen instances. On the queen9\_9 and le450\_5d instances, our method does better than ColPack. Also, our method is randomized and iterative. So it can be left running for a few hours to see if the coloring improves further (for large graphs with unknown chromatic numbers). The following table shows the results of the coloring. The run time for JCOL for 100 iterations is shown in the last column (milliseconds). This could be improved by decreasing the number of iterations and by disabling local search.

DataSet	LARGEST FIRST	SMALLEST LAST	INCIDENCE DEGREE	This Algorithm	ColPack(ms)	JCOL(ms)
fpsol2.i.3	30	30	30	30	32	430
inithx.i.3	31	31	32	31	36	457
le450_5d	14	12	14	7	33	361
multsol.i.5	31	31	31	31	38	342
zeroin.i.3	30	30	30	30	30	332
games120	9	9	9	9	30	229
miles750	32	31	32	31	31	276
queen9_9	15	15	15	11	30	218
myciel7	8	8	9	8	30	569

This Java code uses the DSATUR (Brélaz, 1979) heuristics along with iterated greedy heuristics (Culberson, 1992) to color a graph. The DSATUR heuristic orders the nodes of a graph in non-increasing order of the degree of saturation. The degree of saturation is the number of colors found in the adjacency list of a node. The iterated greedy heuristics perform randomized ordering of the vertices to color them in that order. By randomizing this process, improvement can be found in successive iterations. The algorithm then uses min-conflicts local search to improve the coloring. The method is quite successful in finding good colorings of the majority of the publicly available data sets. Results can be found on the git page.

The method uses the following steps:

- 1) Compute a clique (maximum is good)
- 2) Color the clique
- 3) Sort the rest of the vertices in non-increasing order of the degree of saturation
- 4) Color the vertices in the order given by 3. Also, when a vertex is colored, change the degree of saturation of the neighboring vertices so that the order of coloring changes
- 5) Improve the coloring using Iterated Greedy techniques
- 6) Improve the coloring using min-conflicts local search
- 7) Report the coloring

The Java code and a description are available here: <https://github.com/shah314/graphcoloring>.

The problem instances are available here: <https://mat.tepper.cmu.edu/COLOR/instances.html> and <http://sites.nlsde.buaa.edu.cn/~kexu/benchmarks/graph-benchmarks.htm>.

## References

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