Computing the Skyscraper Invariant

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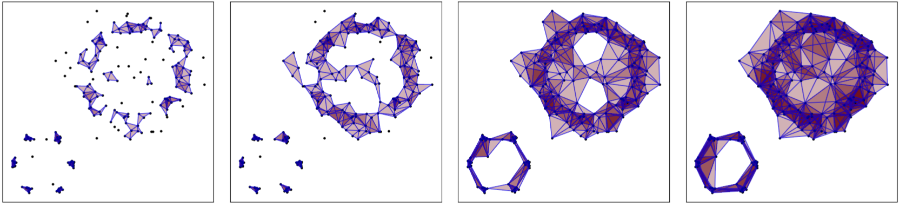
University of Oxford

Persistence under noise

Goal

Estimate the homology groups of noisy samples.

Alpha Complex



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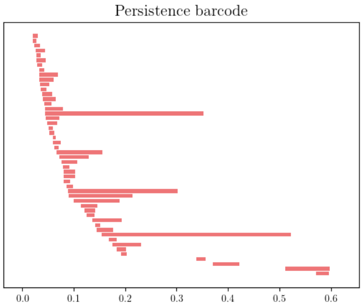

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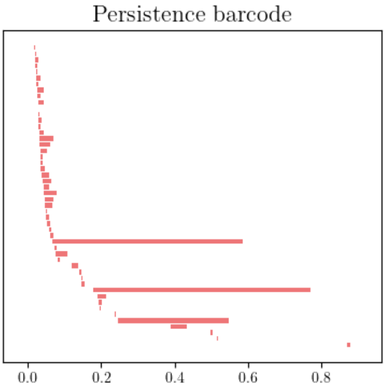

Multiparameter Persistence

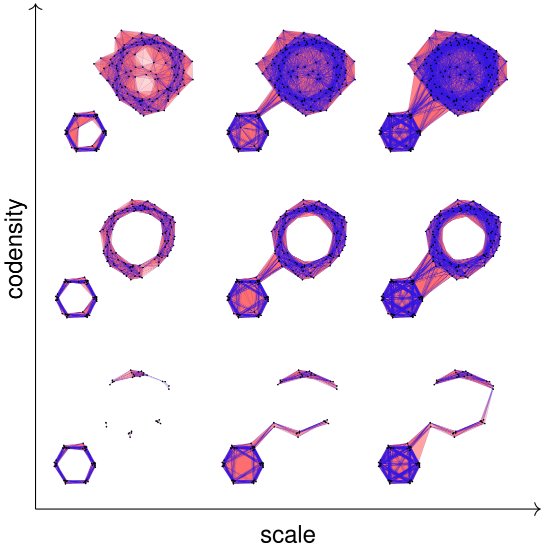

Figure: Density-Rips Bi-filtration

Multiparameter Persistence

$$H_1(-;\mathbb{F}_2) \Longrightarrow \quad \begin{tikzcd}
\mathbb{F}_2 \arrow[r] & 0 \arrow[r] & 0 \\
\mathbb{F}_2^2 \arrow[u, "{\begin{bmatrix} 1 & 0 \end{bmatrix}}"] \arrow[r, "{\begin{bmatrix} 0 & 1 \end{bmatrix}}"] & \mathbb{F}_2 \arrow[u] \arrow[r] & \mathbb{F}_2 \arrow[u] \\
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\end{tikzcd}$$

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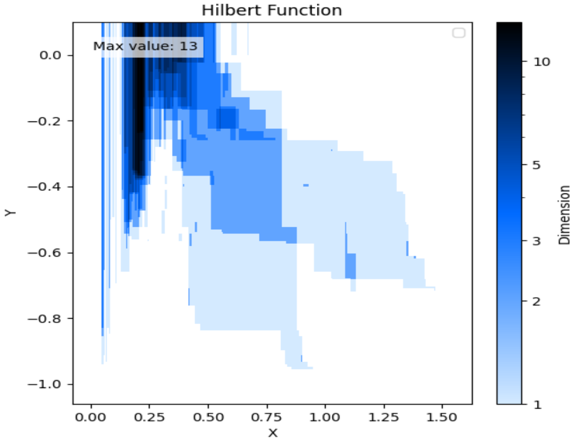

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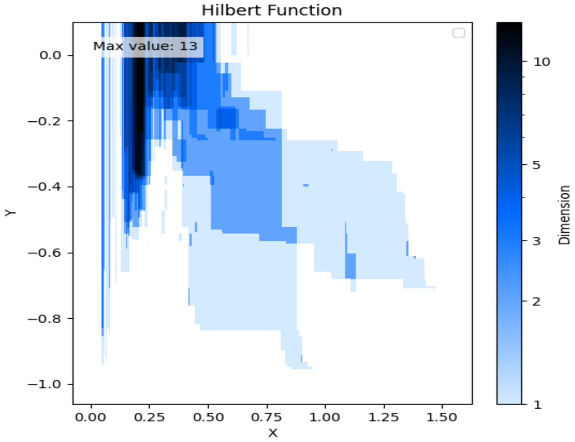

Goal

Find an invariant which describes the module in terms of interval-modules.

You could have invented Mumford Stability

for Persistence Modules

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Definition 1

Let $X\colon \mathbb{R}^2 \to \mathbf{Vect}_{\mathbb{F}_2}$ be a persistence module and $\alpha \in \mathbb{R}^2$.
The *lifetime* of $v \in X_\alpha$ is

$$L_X(v) := \{\beta \in \mathbb{R}^2 \mid \beta \geq \alpha \text{ and } X_{\alpha\to\beta}(v) \neq 0\}$$

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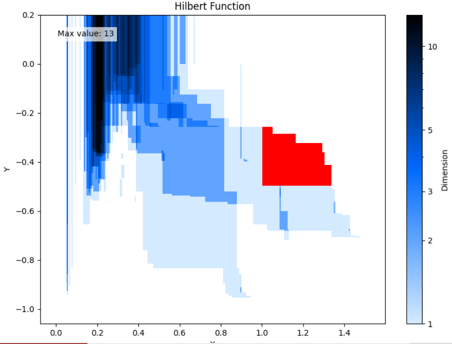

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Definition 2

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Lifetime Filtration (Miller, Zhang)

Find $v_1 \in X$ with the longest lifetime. $\langle v_1 \rangle \subset X$ is an *interval-module*.
Compute $X_1 := X / \langle v \rangle$ and repeat.

$$\Rightarrow \langle v_1 \rangle \hookrightarrow X^1 \hookrightarrow \ldots \hookrightarrow X \qquad X^k / X^{k-1} = \langle v_k \rangle.$$

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\begin{tikzcd}
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$$

Question

Which vectors in the above module have the longest lifetime?

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$$v_1 := \begin{pmatrix} 1 \\ 1 \end{pmatrix} \in \begin{tikzcd} \mathbb{F} \arrow[r] & 0 \arrow[r] & 0 \\ \mathbb{F}_2^2 \arrow[u, "{\begin{bmatrix} 1 & 0 \end{bmatrix}}"] \arrow[r, "{\begin{bmatrix} 0 & 1 \end{bmatrix}}"] & \mathbb{F}_2 \arrow[u] \arrow[r] & \mathbb{F}_2 \arrow[u] \\ \mathbb{F}_2 \arrow[u, "{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}"] \arrow[r] & 0 \arrow[u] \arrow[r] & 0 \arrow[u] \end{tikzcd}$$

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$$X/\langle v_1 \rangle \simeq \begin{tikzcd} 0 \arrow[r] & 0 \arrow[r] & 0 \\ \mathbb{F}_2 \arrow[r] \arrow[u] & 0 \arrow[r] \arrow[u] & 0 \arrow[u] \\ \mathbb{F}_2 \arrow[r] \arrow[u] & 0 \arrow[r] \arrow[u] & 0 \arrow[u] \end{tikzcd}$$

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Problem

The element of largest lifetime is not *additive*.

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The module above decomposes as

$$\begin{tikzcd} \mathbb{F}_2 \arrow[r] & 0 \arrow[r] & 0 \\ \mathbb{F}_2 \arrow[r] \arrow[u] & 0 \arrow[r] \arrow[u] & 0 \arrow[u] \\ \mathbb{F}_2 \arrow[r] \arrow[u] & 0 \arrow[r] \arrow[u] & 0 \arrow[u] \end{tikzcd} \bigoplus \begin{tikzcd} 0 \arrow[r] & 0 \arrow[r] & 0 \\ \mathbb{F}_2 \arrow[r] \arrow[u] & \mathbb{F}_2 \arrow[r] \arrow[u] & \mathbb{F}_2 \arrow[u] \\ 0 \arrow[r] \arrow[u] & 0 \arrow[r] \arrow[u] & 0 \arrow[u] \end{tikzcd}$$

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Instead find the vector of *shortest* lifetime.

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For $v \in X_\alpha$ define its *slope*

$$\mu_\alpha(v) := 1 \bigg/ \int_{\mathbb{R}^2} \mathbb{1}_{L_X(v)} \, dx.$$

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The element of highest slope is still not *unique*.

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Consider also *subspaces* of X_α.

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Definition 4

Fix $\alpha \in \mathbb{R}^2$. For any vector subspace $V \subset X_\alpha$, define the *slope*

$$\mu_\alpha(V) := \dim_{\mathbb{F}_2} V \bigg/ \int_{\mathbb{R}^2} \dim_{\mathbb{F}_2} \langle V \rangle \ .$$

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$$\mu(\mathbb{F}_2^2) = 2/5.$$

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Definition 5

- Find a maximal subspace $V \subset X_\alpha$ of highest slope $\mu_\alpha(V)$.
- Set $X_1 := X \big/ \langle V \rangle$ and repeat.

The resulting filtration

$$X^0 \hookrightarrow X^1 \hookrightarrow \ldots \hookrightarrow \langle X_\alpha \rangle \hookrightarrow X$$

is called the *Harder-Narasimhan-Filtration* at α.

Definition 6

A sub-space V is called *stable*, if for all $W \subset V$ we have

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Proposition

The filtration factors $\langle V \rangle^i := X^i / X^{i-1}$ in the HN-Filtration are stable and sorted by slope.

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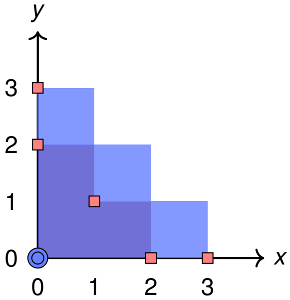

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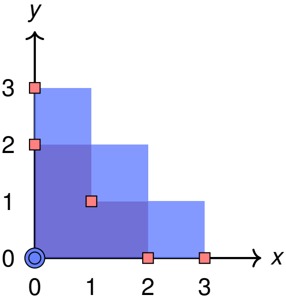
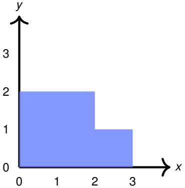
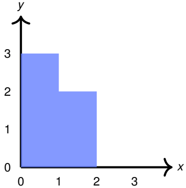
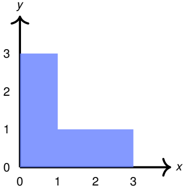


$$\begin{tikzcd} & \mathbb{F}_2 & \mathbb{F}_2 \\ & \mathbb{F}_2^2 \arrow[u, "\begin{bmatrix}1 & 0\end{bmatrix}"] \arrow[ur, "\begin{bmatrix}1 & 1\end{bmatrix}"] \arrow[r, "\begin{bmatrix}0 & 1\end{bmatrix}"'] & \mathbb{F}_2 \end{tikzcd}$$







Mumford Stability for Persistence Modules

Congratulations

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Stability Conditions

Let ν, ξ be adequate measures on $\mathbb{R}^2$. For any submodule $Y \subset X$ they define a slope

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We are using the special case $\nu = \delta_\alpha, \xi = d\lambda$ - a *Skyscraper*.

The Skyscraper Invariant

Definition 7

For $\alpha \leq \beta \in \mathbb{R}^2$ and $\langle V \rangle^i := X^i / X^{i-1}$ the stable filtration factors of the HN-Filtration. We define the *Skyscraper Invariant* as

$$s^{\theta}_{\alpha,\beta}(X) := \bigoplus_{i \text{ st. } \mu(V^i) > \theta} \dim_{\mathbb{F}_2} V^i_\beta.$$

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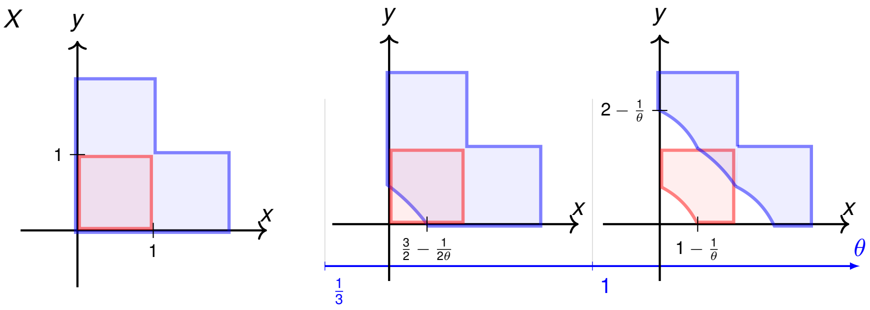

Computing Harder-Narasimhan Filtrations

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Let $\varepsilon > 0$. We denote by $E_\varepsilon(X) := (\varepsilon\mathbb{Z})^2 \cap \mathrm{supp}(X)$ the equidistant grid on X.

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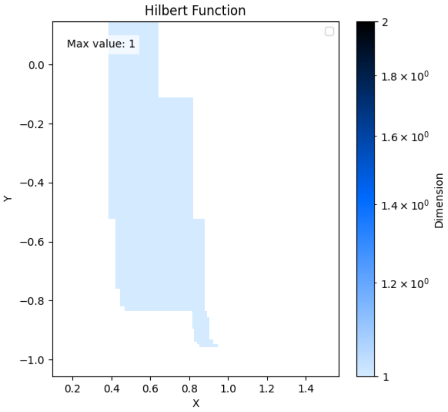

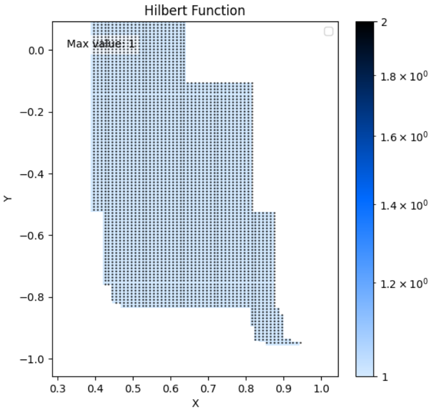

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Cheng's algorithm performs reduction on matrices of size $\mathcal{O}(\theta(X)^2 d^2)$

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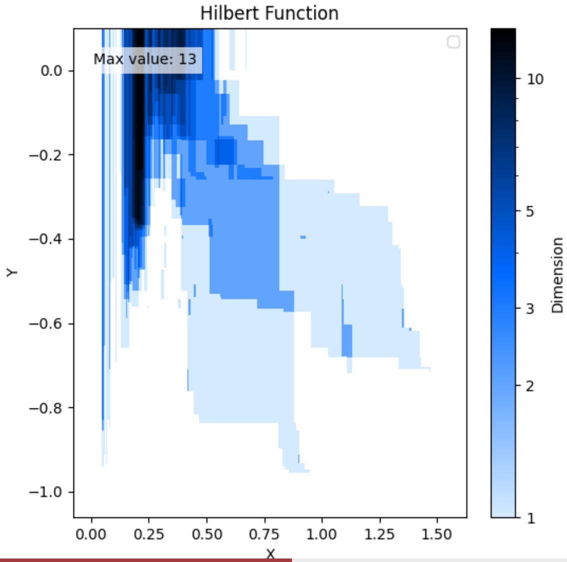

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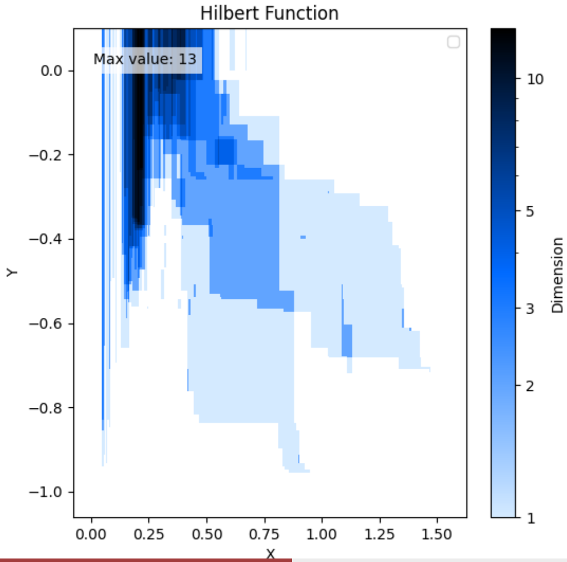
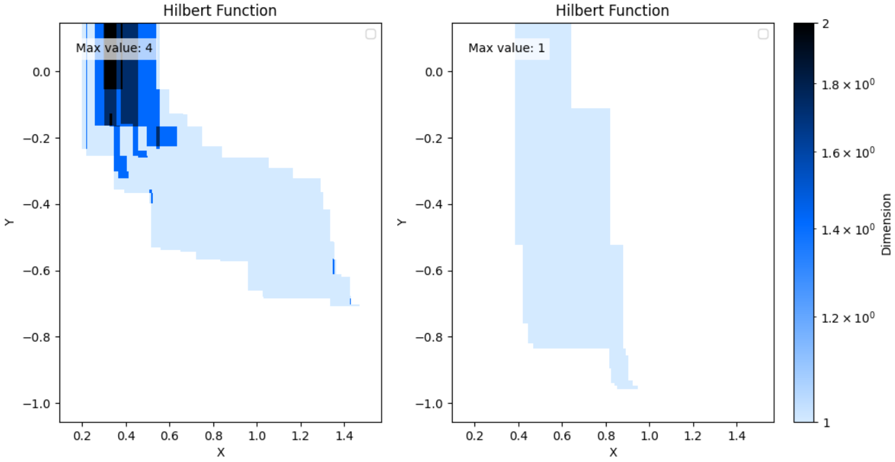


Decompose with AIDA!

The Skyscraper Algorithm



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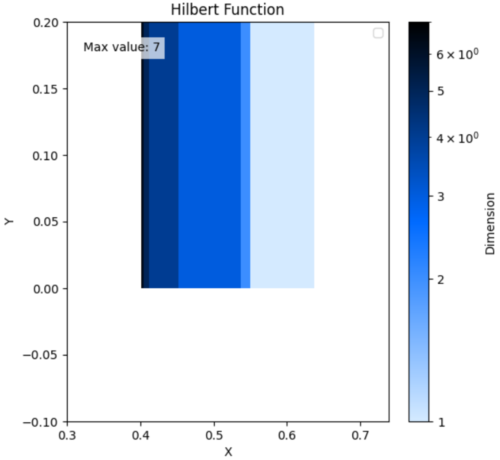

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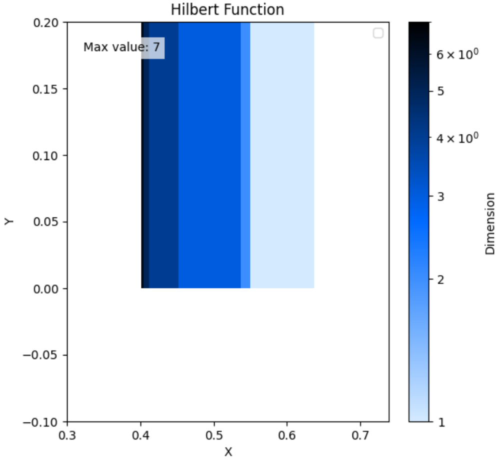

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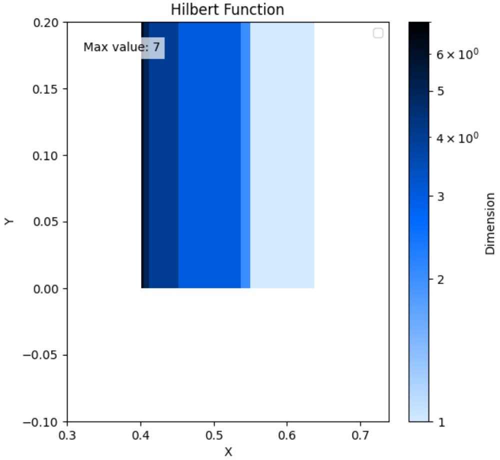

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$\Rightarrow$ Decompose again!

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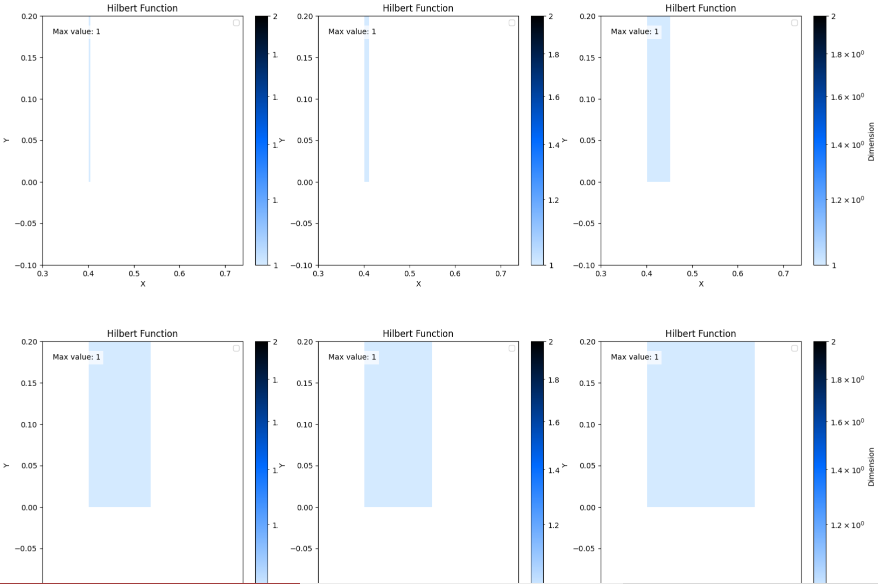

Empirical Results

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The modules $\langle X_\alpha \rangle$ are highly decomposable.

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The pointwise dimension of the indecomposable components of $\langle X_\alpha \rangle$ for typical bifiltrations is always ≤ 3!

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Experimental Observations

The modules $\langle X_\alpha \rangle$ are highly decomposable.
The pointwise dimension of the indecomposable components of $\langle X_\alpha \rangle$ for typical bifiltrations is always ≤ 3!

$\Rightarrow$ Can brute-force the computation of the HN filtration.

The Skyscraper Algorithm

Still need to compute this for every $\alpha \in E_\varepsilon(X)$ for every indecomposable component.

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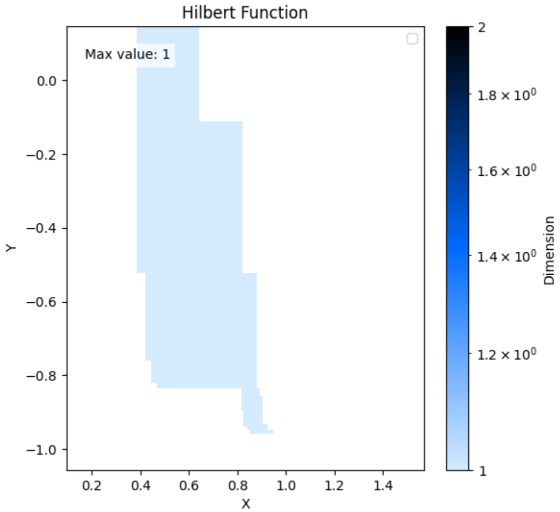

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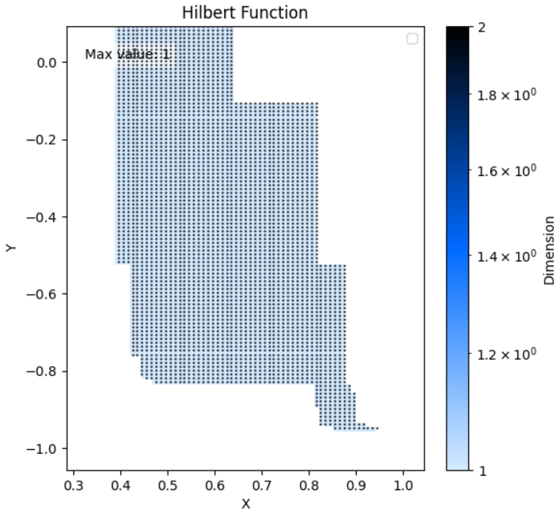

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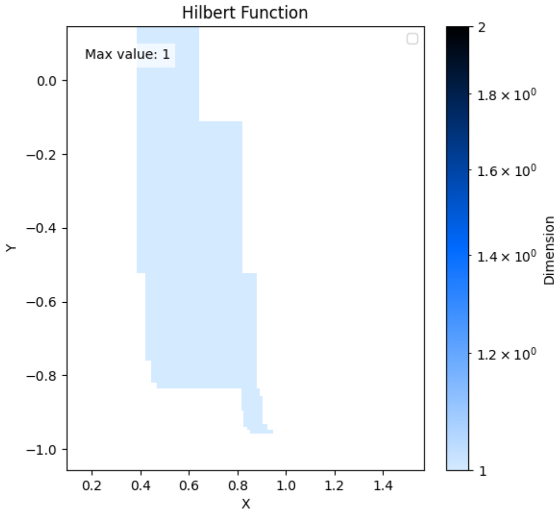

The Induced Grid of a Module

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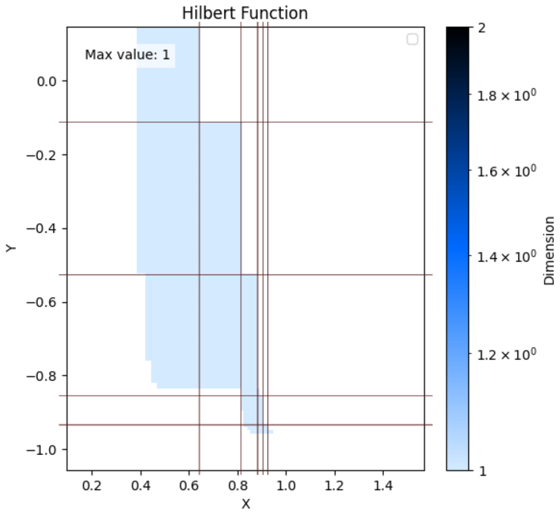

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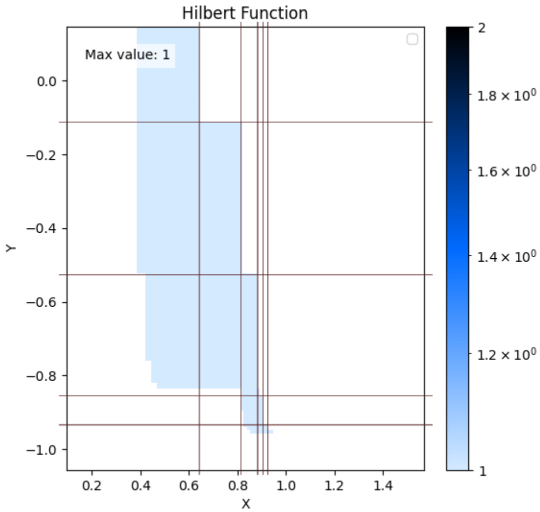
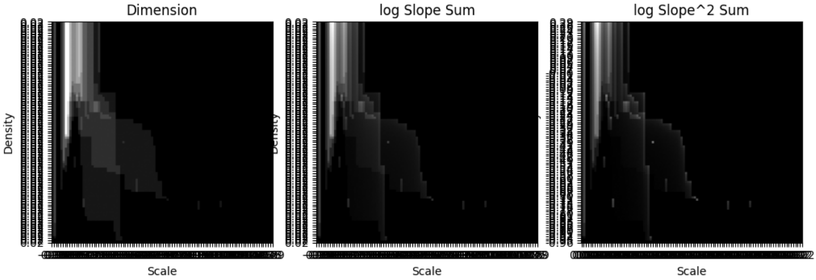


Idea

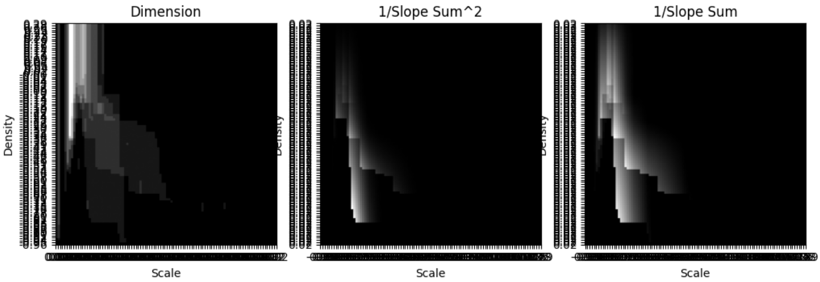
Let $\alpha \leq \beta$. $\langle X_\alpha \rangle$ and $\langle X_\beta \rangle$ are almost the same if there are no generators or relations in $\langle \alpha \rangle \setminus \langle \beta \rangle$.

Results

Results


Dimension
log Slope Sum
log Slope^2 Sum
Density
Scale
Scale
Scale

Results



The size of Stable Submodules

Previous computation, 100×100 grid:
$\sim$ 5.0 s when going over whole grid, $\sim$ 0.5 s using the induced grid.

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Open Questions

- Is there a geometric reason for this?

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Open Questions

- Is there a geometric reason for this?
- How to compute HN filtrations for other central charges
- Compute the resulting filtered landscapes and signed barcodes fast.

The Skyscraper Algorithm

Software

- github.com/jendjan/Skyscraper-Invariant
- github.com/jendjan/AIDA
- github.com/jendjan/Persistence-Algebra

The Skyscraper Algorithm

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Thank you!