

92586 Computational Linguistics

Lesson 7. More Math¹

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¹Lesson 6 was based on notebooks

Previously

- ▶ Pre-processing
- ▶ BoW representation
- ▶ One rule-based sentiment model
- ▶ One statistical model (Naïve Bayes)

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These slides cover roughly chapter 3 of Lane et al. (2019)

From BoW to *tf*

Intuition

1. The frequency of a token t in a document d is an important factor of its **relevance**
2. The relative frequency of a word in a document wrt **all other documents** in the collection provides better information

Binary Bag of Words

We departed from a binary representation.

We were simply interested in the existence (or not) of a word in a document.


$$\begin{array}{rcccccccccccccccc} & & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 & w_9 & w_{10} & w_{11} & w_{12} & w_{13} \\ d_1 = & [& 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 &] \\ d_2 = & [& 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 &] \end{array}$$

“Counting” Bag of Words

A word that appears often contributes more to the “meaning” of document

A document with many occurrences of “good”, “awesome”, “best” is more **positive** than one in which they occur only once.

$$\begin{array}{rcccccccccccccccc} d_1 = & [& 0 & 1 & 0 & 0 & 2 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & 0 &] \\ d_2 = & [& 2 & 3 & 5 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 2 &] \end{array}$$

 Let us see the difference...

Already a useful representation to diverse tasks, such as detecting **spam** and computing “**sentiment**”

tf : Term Frequency

tf represents the number of times a word appears in a document

(In general) the frequency of a word depends on the length of the document

- ▶ Shorter document → lower frequencies
- ▶ Longer document → higher frequencies

Ideally, our counting should be document-length independent.

Normalisation!

tf: Term Frequency (Normalised)

Why normalise?

word dog appears 3 times in d_1

word dog appears 100 times in d_2

Intuition: dog is way more important for d_2 than for d_1

d_1 is an email by a veterinarian (30 words)


d_2 is *War & Peace* (580k words)

If normalised...

$$tf(\text{dog}, d_1) = 3/30 = 0.1$$

$$tf(\text{dog}, d_2) = 100/580000 = 0.00017$$

Remember: normalised frequencies are probabilities

 Let us see

tf: Term Frequency (Normalised)

Playing with a longer text

[https:](https://en.wikipedia.org/wiki/Coronavirus_disease_2019)

[//en.wikipedia.org/wiki/Coronavirus_disease_2019](https://en.wikipedia.org/wiki/Coronavirus_disease_2019)

Coronavirus disease 2019 (COVID-19) is an infectious disease caused by severe acute respiratory syndrome coronavirus 2 (SARS coronavirus 2, or SARS-CoV-2), a virus closely related to the SARS virus. The disease was discovered and named during the 2019–20 coronavirus outbreak. Those affected may develop a fever, dry cough, fatigue, and shortness of breath. A sore throat, runny nose or sneezing is less common. While the majority of cases result in mild symptoms, some can progress to pneumonia and multi-organ failure. [...]

Note. The examples use NLTK. Nowadays, there are better tools. For instance, parsing with **spaCy** is faster and more accurate

tf: Term Frequency (Normalised)

Playing with a longer text

- ▶ Loading frequencies into a dictionary
- ▶ Vectorising frequencies
- ▶ Normalising frequencies

tf: Term Frequency


From a single to multiple documents

- ▶ The vectors have to be comparable across documents → **normalisation**
- ▶ Each value in the vectors must represent **the same word**

This is when representations become sparse: many values become 0

Sparse vector most of the elements are **zero**

Dense vector most of the elements are **nonzero**

 Let us see

See https://en.wikipedia.org/wiki/Sparse_matrix

Vectors of Term Frequency

Vectors

- ▶ Primary building blocks of linear algebra
- ▶ Ordered list of numbers, or coordinates, in a vector space
- ▶ They describe a location in that space. . .
- ▶ or identify a direction/magnitude/distance in that space

Vector space Collection of all possible vectors that could appear

$[1, 4] \rightarrow$ 2D vector space

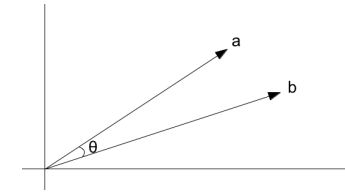
$[1, 4, 9] \rightarrow$ 3D vector space

We have a 18D vectors space (we have seen 1kD and bigger ones!)

Comparing Vectors

Cosine similarity

The cosine of the angle between two vectors (θ theta)




$$\cos \theta = \frac{A \cdot B}{|A||B|} \quad (1)$$

where

$A \cdot B$ is the **dot product** (we know it!)

$|A|$ is the **magnitude** of vector A

 Let us see an implementation (but there are efficient libraries to do it)

Comparing Vectors

Cosine similarity

Properties of the cosine similarity

- ▶ It is ranged in $[-1, 1]$
- ▶ This is a very convenient range for ML
- ▶ $\cos = 1$ represents identical normalized vectors that point in exactly the same direction
- ▶ $\cos = 0$ represents two vectors that share no components (they are perpendicular in all dimensions)
- ▶ In *tf*-like representations, cosine is ranged in $[0, 1]$ (frequencies are not negative)

Zipf's Law


Zipf's Law

Given some corpus of natural language utterances, the frequency of any word is inversely proportional to its rank in the frequency table.²

$pos(w)$	$freq(w)$
1st	k
2nd	$k/2$
3rd	$k/3$
...	...

The system behaves **“roughly” exponentially**

Examples population dynamics, economic output and **COVID-19**

 Let's see it for text

²George K. Zipf; 1930s

Zipf's Law

Frequencies of the Brown corpus: expected vs actual

w	$f_{exp}(w)$	$f_{act}(w)$
the	—	69,971
of	34,985	36,412
and	23,323	28,853
to	17,492	26,158
a	13,994	23,195
in	11,661	21,337
that	9,995	10,594
is	8,746	10,109
was	7,774	9,815
he	6,997	9,548
for	6,361	9,489
it	5,830	8,760
with	5,382	7,289
as	4,997	7,253
his	4,664	6,996



Zipf's Law

Stats

- ▶ This distribution only holds with large volumes of data (not in a sentence, not in a couple of texts)
- ▶ By computing this distribution, we can obtain an a priori likelihood that a word w will appear in a document of the corpus

Inverse Document Frequency

idf–Inverse Document Frequency

There are two ways to count tokens

tf per document

idf across a full corpus

📖 Let's see...

IDF How strange is it that this token is in this document?

If w appears in d a lot, but rarely in any other $d' \in D \mid d' \neq d$
 w is quite important for d !

📖 Let's see

IDF and Zipf

Let us assume a corpus D , such that $|D| = 1M$

- ▶ 1 document $d \in D$ contains “cat”
 $idf(cat) = 1,000,000/1 = 1,000,000$
- ▶ 10 documents $\{d_1, d_2, \dots, d_{10}\} \in D$ contain “dog”
 $idf(dog) = 1,000,000/10 = 100,000$

According to Zipf's Law, when comparing w_1 and w_2 , even if $f(w_1) \sim f(w_2)$, one will be **exponentially higher** than the other one!

We need the inverse of $exp()$ to mild the effect: $log()$

$$\begin{aligned} idf(cat) &= \log(1,000,000/1) = \log(1,000,000) = 6 \\ idf(dog) &= \log(1,000,000/10) = \log(100,000) = 5 \end{aligned}$$

tf-idf

$$tf(t, d) = \frac{count(t, d)}{\sum_t count(t, d)} \quad (2)$$

$$idf(t, D) = \log \frac{\text{number of documents}}{\text{number of documents containing } t} \quad (3)$$

$$tfidf(t, d, D) = tf(t, d) * idf(t, D) \quad (4)$$

- ▶ The more often t appears in d , the higher the TF (and hence the TF-IDF)
- ▶ The higher the number of documents containing t , the lower the IDF (and hence the TF-IDF)

tf-idf

Outcome The importance of a token in a specific document given its usage across the entire corpus.

“TF-IDF, is the humble foundation of a simple search engine”
(Lane et al., 2019, p. 90)

📖 Let's see

Coming Next

- Towards “semantics”

References

Lane, H., C. Howard, and H. Hapkem
2019. *Natural Language Processing in Action*. Shelter Island,
NY: Manning Publication Co.