92586 Computational Linguistics

Lesson 5. Naïve Bayes

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Previously

- Pre-processing (e.g., tokenisation, stemming, stopwording)
- BoW representation
- One rule-based sentiment analyser

Table of Contents

Naïve Bayes

Naïve Bayes

1. Introduced in the IR community by Maron (1961)
2. First machine learning approach
3. It is a **supervised** model
4. It applies Bayes' theorem with strong (naïve) independence assumptions between the features
   - they are independent
   - they contribute "the same"

Let us take it easy

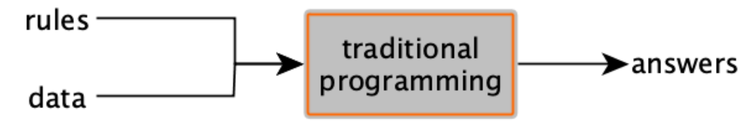
Machine Learning

"the scientific study of algorithms and statistical models that computer systems use to perform a specific task **without using explicit instructions**, relying on patterns and inference instead"

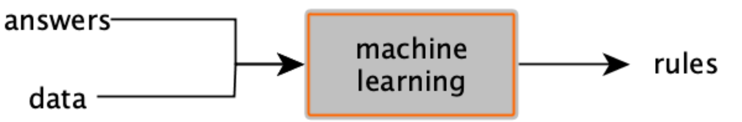
Machine Learning

A change of paradigm

From hand-crafted rules.



To training



Diagrams borrowed from L. Moroney's Introduction to TensorFlow for Artificial Intelligence, Machine Learning, and Deep Learning

Supervised vs Unsupervised

Supervised: The algorithms build a mathematical model of a set of data including...
- **the inputs**
- **desired outputs**

Unsupervised: The algorithms take a set of data that contains...
- **only inputs**

...and find structure in the data

https://en.wikipedia.org/wiki/Machine_learning

Naïve Bayes

A conditional probability model

Given an instance represented by a vector

$$\mathbf{x} = (x_1, \dots, x_n) \tag{1}$$

representing n **independent** features $x_1, x_2, x_3, \dots, x_{n-2}, x_{n-1}, x_n$ n could be $|V|$ (the size of the vocabulary)

The model assigns the instance the probability

$$p(C_k \mid \mathbf{x}) = p(C_k \mid x_1, \dots, x_n) \tag{2}$$

for each of the k possible outcomes C_k

where $C_k = \{c_1, \dots, c_k\}$

From
`https://en.wikipedia.org/wiki/Naive_Bayes_classifier`

Naïve Bayes'

Using Bayes' Theorem

The conditional probability $p(C_k \mid x_1, \dots, x_n)$ can be decomposed as

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\ p(\mathbf{x} \mid C_k)}{p(\mathbf{x})} \tag{3}$$

How to read this

$$\text{posterior} = \frac{\text{prior} \times \text{likelihood}}{\text{evidence}}$$

But $p(x)$ does not depend on the class (it's constant!):

$$p(C_k \mid \mathbf{x}) = p(C_k)\ p(\mathbf{x} \mid C_k) \tag{4}$$

From
`https://en.wikipedia.org/wiki/Naive_Bayes_classifier`

Naïve Bayes

Going deeper (assuming a binary classifier)

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \tag{5}$$

$$\text{posterior probability} = \frac{\text{class prior probability} \times \text{likelihood}}{\text{predictor prior probability}}$$

$p(C \mid \mathbf{x})$ Posterior probability of the class given the input[1]

```
if p > 0.5:
    class = positive
else:
    class = negative
```

[1] Symbol | means "given": the probability of the class given the representation vector

Naïve Bayes

Going deeper (assuming a binary classifier)

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \tag{6}$$

$$\text{posterior probability} = \frac{\text{class prior probability} \times \text{likelihood}}{\text{predictor prior probability}}$$

$p(C)$ Class **prior** probability
How many **positive** instances I have seen (during training)?

Naïve Bayes

Going deeper (assuming a binary classifier)

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \tag{7}$$

$$\text{posterior probability} = \frac{\text{class prior probability} \times \text{likelihood}}{\text{predictor prior probability}}$$

$p(\mathbf{x} \mid C)$ Likelihood
The probability of the document given the class

Rough Idea

- The value of a particular feature is **independent** of the value of any other feature, given the class variable
- All features contribute the same to the classification

- It tries to find keywords in a set of documents that are predictive of the target (output) variable
- The internal coefficients will try to map tokens to scores
- Same as VADER, but without manually-created rules
  the machine will estimate them!

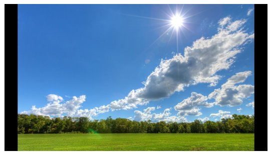
From (Lane et al., 2019, p. 65–68)

Naïve Bayes

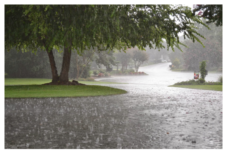
A toy example: Should I grab a beer from the bar today?

One single factor: *zona*


giallo arancione rosso


sunny


overcast


rainy

(get ready for some of the densest slides I have ever made!)

Naïve Bayes

A toy example: Should I grab a beer from the bar today?

Dataset

Zona	
	yes
	yes
	no
	yes
	yes
	yes
	yes
	yes
	yes
	no
	no
	yes
	no
	no

Computing **all** the probabilities by "counting"

Frequency table

Zona	**yes**	**no**
	3	2
	4	0
	2	3

Likelihood table

Zona	**yes**	**no**
	3/9	2/5
	4/9	0/5
	2/9	3/5

Adapted from `http://www.saedsayad.com/naive_bayesian.htm`

Naïve Bayes

A toy example: Should I grab a beer from the bar today?

Likelihood table

Zona	yes	no
	3/9 [1]	2/5
	4/9	0/5
	2/9	3/5
	9/14 [2]	5/14

1

$$p(x \mid c) = p(\mid \text{yes}) = 3/9 = 0.33$$

2

$$p(c) = p(\text{yes}) = 9/14 = 0.64$$

$$p(x) = p() = 5/14 = 0.36$$

What is the Naïve Bayes' probability of **yes** if ?

$$\begin{aligned} p(c \mid x) &= p(c)p(x \mid c)/p(x) \\ p(\text{yes} \mid) &= p(\text{yes})p(\mid \text{yes})/p(v) \\ p(\text{yes} \mid) &= 0.64 * 0.33/0.36 \\ p(\text{yes} \mid) &= 0.59 \end{aligned}$$

Adapted from `http://www.saedsayad.com/naive_bayesian.htm`

Naïve Bayes

A toy example: Should I grab a beer from the bar today?

If. . . let's do it !

Naïve Bayes

A toy example: Should I grab a beer from the bar today?

Considering more data

Zona	Temp	Humidity	Windy	
	hot	high	false	no
	hot	high	true	no
	hot	high	false	yes
	mild	high	false	yes
	cool	normal	false	yes
	cool	normal	true	no
	cool	normal	true	yes
	mild	high	false	no
	cool	normal	false	yes
	mild	normal	false	yes
	mild	normal	true	yes
	mild	high	true	yes
	hot	normal	false	yes
	mild	high	true	no

Adapted from `http://www.saedsayad.com/naive_bayesian.htm`

Naïve Bayes

A toy example: Should I grab a beer from the bar today?

Frequency tables

Zona	yes	no
	3	2
	4	0
	2	3

Humid	yes	no
high	3	4
normal	6	1

Temp	yes	no
hot	2	2
mild	4	2
cool	3	1

Windy	yes	no
false	6	2
true	3	3

Likelihood tables

Zona	yes	no
	3/9	2/5
	4/9	0/5
	2/9	3/5

Humid	yes	no
high	3/9	4/5
normal	6/9	1/5

Temp	yes	no
hot	2/9	2/5
mild	4/9	2/5
cool	3/9	1/5

Windy	yes	no
false	6/9	2/5
true	3/9	3/5

Adapted from `http://www.saedsayad.com/naive_bayesian.htm`

Naïve Bayes

Likelihood tables

Zona	**yes**	**no**
	3/9	2/5
	4/9	0/5
	2/9	3/5

Temp	**yes**	**no**
hot	2/9	2/5
mild	4/9	2/5
cool	3/9	1/5

Humid	**yes**	**no**
high	3/9	4/5
normal	6/9	1/5

Windy	**yes**	**no**
false	6/9	2/5
true	3/9	3/5

zona	**temp**	**humidity**	**windy**	play
	cool	high	true	?

$$
\begin{aligned}
p(\text{yes} \mid x) &= \frac{p(\text{yes})p(\ \mid \text{yes})p(\text{cool} \mid \text{yes})p(\text{high} \mid \text{yes})p(\text{true} \mid \text{yes})}{p(\ )p(\text{cool})p(\text{high})p(\text{true})} \\
&= \frac{9/14 \times 2/9 \times 3/9 \times 3/9 \times 3/9}{5/14 \times 4/14 \times 7/14 \times 6/14} \\
&= 0.00529/0.02811 = 0.188 \sim 0.2 \ \textbf{no}
\end{aligned}
$$

Adapted from `http://www.saedsayad.com/naive_bayesian.htm`

Naïve Bayes

Back to the math...

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{p(\mathbf{x})} \tag{8}$$

The probability $p(\mathbf{x})$ is constant for any given input!

$$p(C \mid \mathbf{x}) = \frac{p(C)\ p(\mathbf{x} \mid C)}{\cancel{p(\mathbf{x})}} \tag{9}$$

$$p(c \mid \mathbf{x}) \propto p(c)p(\mathbf{x} \mid c) \tag{10}$$

Naïve Bayes

Back to the math...

$$p(c \mid \mathbf{x}) \propto p(c)p(\mathbf{x} \mid c) \tag{11}$$

But $\mathbf{x}$ is a vector

$$p(c \mid x_1 \ldots x_n) \propto p(c)p(x_1 \mid c) \times p(x_2 \mid c) \times \ldots \times p(x_n \mid c) \tag{12}$$

Eq.(12) can be rewritten as

$$p(c \mid x_1 \ldots x_n) \propto p(c) \prod_{i=1}^{n} p(x_i \mid c) \tag{13}$$

Naïve Bayes

The classification process

Back to the toy example

$$
\begin{aligned}
p(\text{yes} \mid x) &\propto p(\text{yes})p(\ \mid \text{yes})p(\text{cool} \mid \text{yes})p(\text{high} \mid \text{yes})p(\text{true} \mid \text{yes}) \\
&\propto 9/14 \times 2/9 \times 3/9 \times 3/9 \times 3/9 \\
&\propto 0.00529 \ \textbf{not a probability!}
\end{aligned}
$$

Classification: the maximum for all the classes

$$c \propto \arg\max_{c} p(c) \prod_{i=1}^{n} p(x_i \mid c) \tag{14}$$

```
compute p(yes|x)
compute p(no|x)
if p(yes|x) > p(no|x):
    yes
else:
    no
```

Training a Machine Learning Model

The dataset

We need a bunch of documents with their associated **class**

kind	**examples**
binary	{positive, negative}
	{0, 1}
	{-1, 1}
multiclass	{positive, neutral, negative}
	{0,1,2}

In our case, we need the positivity sentiment:

d_1	pos	d_5	neg	d_9	neu
d_2	neu	d_6	neg	d_{10}	pos
d_3	pos	d_7	neg	d_{11}	neu
d_4	pos	d_8	pos	d_{12}	neg

The dataset

Option 1 **You use a corpus created by somebody else**

Option 2 You build your own corpus

(a) You have/hire experts to do it
(b) You engage non-experts through gamification
(c) You hire non-experts through explicit crowdsourcing
(d) There are many other ways to get annotated data

Let us go and build a classifier with a corpus built by Hutto and Gilbert (2014)[2]

For this, you have to download and install the software companion of NLP in Action:

`https://github.com/totalgood/nlpia`

[2] http://comp.social.gatech.edu/papers/icwsm14.vader.hutto.pdf

What I did on OsX

I use pipenv[3]

```
$ pipenv install --skip-lock nlpia
```

On Github they explain how to install it with conda or pip if you plan to contribute to the project

[3] https://pipenv.readthedocs.io/en/latest/

References

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