# 92586 Computational Linguistics

## Lesson 4. More Math

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# Previously

- Pre-processing
- BoW representation

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- Pre-processing
- BoW representation
- One rule-based sentiment model
- One statistical model (Naïve Bayes)

# Table of Contents



These slides cover roughly chapter 3 of Lane et al. (2019)

## From BoW to tf

# Intuition

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2. The relative frequency of a word in a document wrt **all other documents** in the collection provide better information

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Let us see the difference...

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Already a useful representation to diverse tasks, such as detecting **spam** and computing **"sentiment"**

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**Normalisation!**

# *tf*: Term Frequency (Normalised)

Why should we normalise?

$d_1$ contains word `dog` 3 times
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**Remember:** normalised frequencies are indeed probabilities

# *tf*: Term Frequency (Normalised)

Playing with a longer text

https://en.wikipedia.org/wiki/Coronavirus_disease_2019

```
Coronavirus disease 2019 (COVID-19) is an infectious
disease caused by severe acute respiratory syndrome
coronavirus 2 (SARS coronavirus 2, or SARS-CoV-2), a
virus closely related to the SARS virus. The disease was
discovered and named during the 2019{20 coronavirus
outbreak. Those affected may develop a fever, dry cough,
fatigue, and shortness of breath. A sore throat, runny
nose or sneezing is less common. While the majority of
cases result in mild symptoms, some can progress to
pneumonia and multi-organ failure.
[...]
```

**Note.** The examples use NLTK. Nowadays, there are better tools. For instance, parsing with **spaCy** is faster and more accurate

# *tf*: Term Frequency (Normalised)

Playing with a longer text

- Loading frequencies into a dictionary
- Vectorising frequencies
- Normalising frequencies

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From a single to multiple documents

- The vectors have to be comparable across documents → **normalisation**

See https://en.wikipedia.org/wiki/Sparse_matrix

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This is when representations become sparse: many values become 0

Sparse vector most of the elements are **zero**

Dense vector most of the elements are **nonzero**

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Let us see

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We have a 18D vectors space (we have seen 1kD and bigger ones!)

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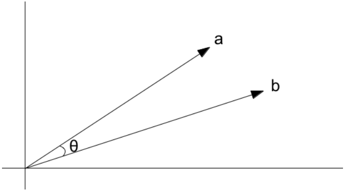

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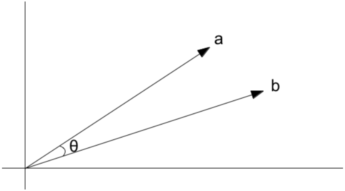


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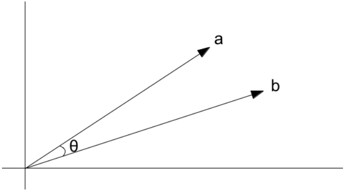


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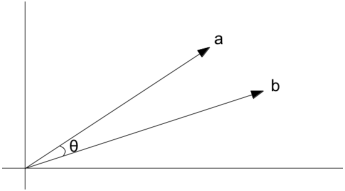


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$|A|$ is the **magnitude** of vector $A$

Let us see an implementation (but there are efficient libraries to do it)

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Properties of the cosine similarity

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Let's see this in words

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Frequencies of the Brown corpus

| $w$ | $f_{exp}(w)$ | $f_{act}(w)$ |
|---|---|---|
| the | – | 69,971 |
| of | 34,985 | 36,412 |
| and | 23,323 | 28,853 |
| to | 17,492 | 26,158 |
| a | 13,994 | 23,195 |
| in | 11,661 | 21,337 |
| that | 9,995 | 10,594 |
| is | 8,746 | 10,109 |
| was | 7,774 | 9,815 |
| he | 6,997 | 9,548 |
| for | 6,361 | 9,489 |
| it | 5,830 | 8,760 |
| with | 5,382 | 7,289 |
| as | 4,997 | 7,253 |
| his | 4,664 | 6,996 |

- This distribution only holds with large volumes of data (not in a sentence, not in a couple of texts)

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- This distribution only holds with large volumes of data (not in a sentence, not in a couple of texts)
- By computing this distribution, we can obtain an a priori likelihood that a word *w* will appear in a document of the corpus

## Inverse Document Frequency

# *idf*–Inverse Document Frequency

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$$\begin{aligned} idf &= log(1,000,000/1) = 6 \\ idf &= log(1,000,000/10) = 5 \end{aligned}$$

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- The more often $t$ appears in $d$, the higher the TF (and hence the TF-IDF)
- The higher the number of documents containing $t$, the lower the IDF (and hence the TF-IDF)

# *tf-idf*

**Outcome** The importance of a token in a specific document given its usage across the entire corpus.

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"TF-IDF, is the humble foundation of a simple search engine" (Lane et al., 2019, p. 90)

Let's see

# Coming Next

- Towards "semantics"

# References

Lane, H., C. Howard, and H. Hapkem
2019. *Natural Language Processing in Action*. Shelter Island, NY: Manning Publication Co.