ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA
CAMPUS DI FORLÌ

91258 / B0385
Natural Language Processing

Lesson 6. Term Frequency–Inverse Document Frequency

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Table of Contents

1. From BoW to term frequency

2. Zipf's Law

3. Inverse Document Frequency

Previously

- Pre-processing
- BoW representation
- One rule-based sentiment model

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- One statistical model (Naïve Bayes)

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These slides cover roughly chapter 3 of Lane et al. (2019)

From BoW to term frequency

Intuition

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2. The relative frequency of a word in a document wrt all other documents in the collection provides even better information

Binary Bag of Words

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$$\begin{array}{ccc} & & \begin{array}{ccccccccccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 & w_9 & w_{10} & w_{11} & w_{12} & w_{13} \end{array} \\ d_1 & = & \left[\begin{array}{ccccccccccccc} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \\ d_2 & = & \left[\begin{array}{ccccccccccccc} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{array} \right] \end{array}$$

“Counting” Bag of Words

A word that appears often contributes more to the “meaning” of the document

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Let us see...

Already a useful representation for diverse tasks, such as detecting spam and computing "sentiment"

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Normalisation!

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word `dog` appears 3 times in d_1
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Intuition: `dog` is way more important for d_2 than for d_1

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$$tf(\text{dog}, d_1) = 3/300 = 0.01$$
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Let us see

tf: Term Frequency (Normalised)

Playing with a longer text

- Loading frequencies into a dictionary
- Vectorising frequencies
- Normalising frequencies

tf: Term Frequency

From a single to multiple documents

- The vectors have to be comparable across documents $\rightarrow$ normalisation

See `https://en.wikipedia.org/wiki/Sparse_matrix`

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Sparse vector: most of the elements are zero

Dense vector: most of the elements are non-zero

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We have an 18D vector space (we have seen $20k$+D ones!)

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Cosine similarity

The cosine of the angle between two vectors (θ theta)

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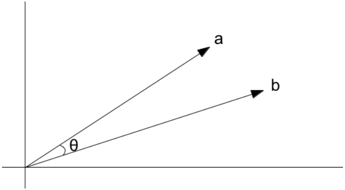

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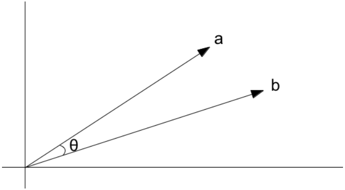

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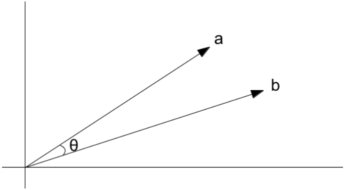

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$A \cdot B$ is the dot product (we know it!)

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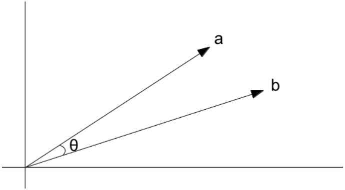

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where

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$|A|$ is the magnitude of vector A

Let us see an implementation (but there are efficient libraries to do it)

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- In *tf*-like representations, cosine is ranged in $[0, 1]$
  (no negative frequencies)

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Let's see it for text

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Zipf's Law

Frequencies of the Brown corpus: *exp*ected vs *act*ual

w	$f_{exp}(w)$	$f_{act}(w)$
the	–	69,971
of	34,985	36,412
and	23,323	28,853
to	17,492	26,158
a	13,994	23,195
in	11,661	21,337
that	9,995	10,594
is	8,746	10,109
was	7,774	9,815
he	6,997	9,548
for	6,361	9,489
it	5,830	8,760
with	5,382	7,289
as	4,997	7,253
his	4,664	6,996

- This distribution only holds with large volumes of data (not in a sentence, not in a couple of texts)

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- By computing this distribution, we can obtain an *a priori* likelihood that a word w will appear in a document of the corpus

Inverse Document Frequency

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Two ways (among many others) to count tokens

tf per document

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According to Zipf's Law, when comparing w_1 and w_2, even if $f(w_1) \sim f(w_2)$, one will be exponentially higher than the other one!

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$$\begin{aligned} idf(cat) &= log(1,000,000/1) &&= log(1,000,000) &&= 6 \\ idf(dog) &= log(1,000,000/10) &&= log(100,000) &&= 5 \end{aligned}$$

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- The more often t appears in d, the higher the TF (and hence the TF-IDF)
- The higher the number of documents containing t, the lower the IDF (and hence the TF-IDF)

tf-idf

Outcome The importance of a token in a specific document given its usage across the entire corpus.

tf-idf

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> "TF-IDF, is the humble foundation of a simple search engine"
> (Lane et al., 2019, p. 90)

Let's see

tf-idf Implementation

- We "hand-coded" the *tf-idf* implementation

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BM	best matching
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- Cosine similarity is a top choice metric for many text vector representations.
- Nothing prevents you from weighting n-grams, for $n = [1, 2, \ldots]$

Coming Next

- Towards "semantics"

References

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