Digital modulation

Lecture 4

Digital modulation

- **Digital modulation** is the **process** of **changing** one of the **characteristics** of an analog signal based on the information in digital data.
  - ✓ This is also known as **"Digital-to-Analog Conversion"**

- Topics discussed in this section:
  - **Aspects of Digital-to-Analog Conversion**
  - **Amplitude Shift Keying**
  - **Frequency Shift Keying**
  - **Phase Shift Keying**
  - **Quadrature Amplitude Modulation**

Digital modulation

- **Digital modulation** is the **process** of **changing** one of the **characteristics** of an analog signal based on the information in digital data.

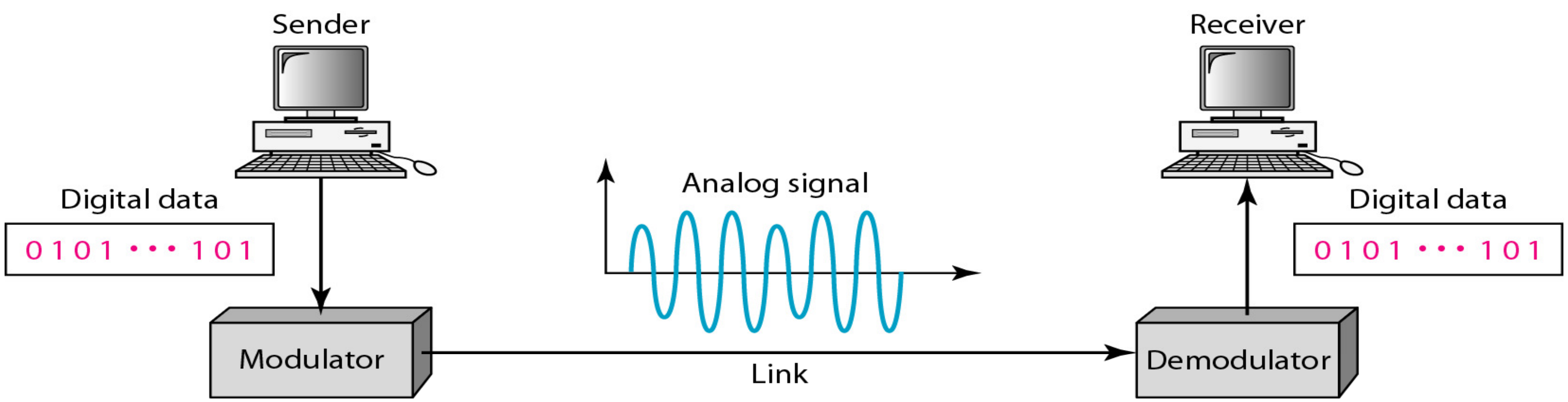
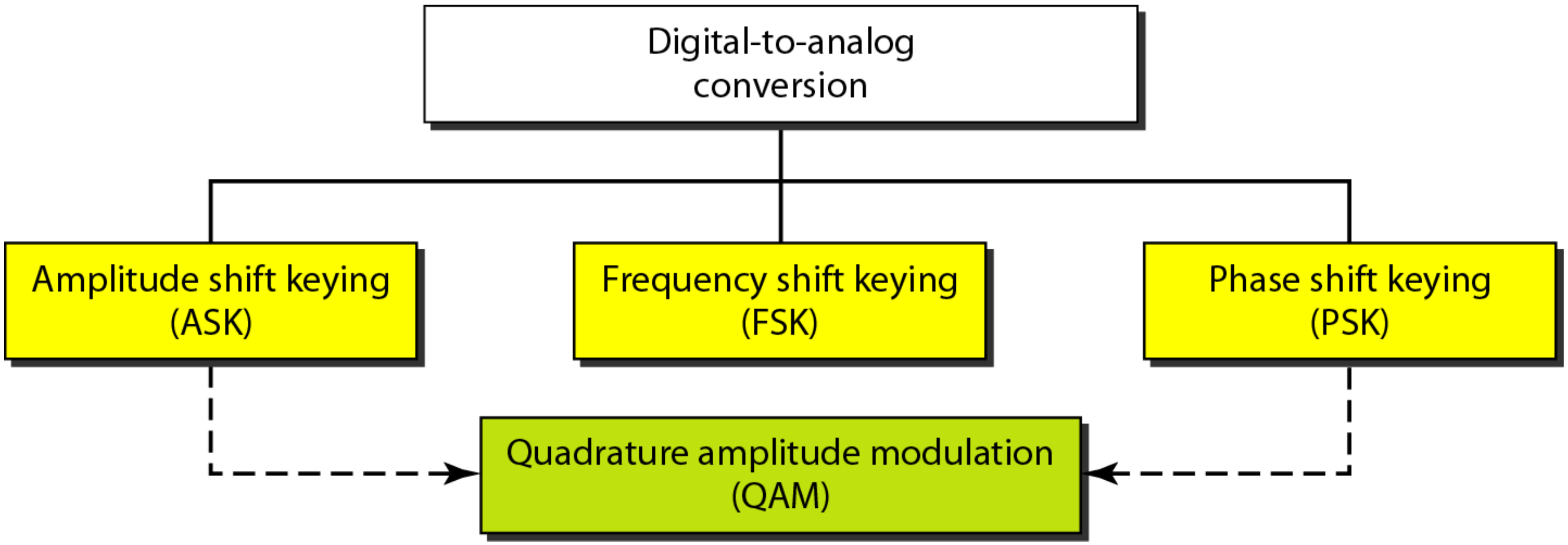


Fig. 1: Digital modulation

- **Type of various Digital modulation techniques are:**
  - Amplitude Shift Keying
  - Frequency Shift Keying
  - Phase Shift Keying
  - Quadrature Amplitude Modulation

Digital-to-Analog Conversion

- **Types of Digital-to-Analog conversion:**



Digital-to-Analog Conversion

- **Some Aspects of Digital-to-Analog Conversion:**

  - ***Bit Rate:*** **Bit rate** is the **number of bits** per second.

  - ***Baud Rate:*** **Baud rate** is the **number of signal elements** per second.

- In the analog transmission of digital data, the **baud rate** is **less than or equal** to the **bit rate.**

Digital-to-Analog Conversion

- ***Example 1:***

An analog signal carries ***4 bits*** per **signal element.** If ***1000*** **signal elements** are sent per second, find the ***bit rate***.

- ***Solution:***

In this case***, r = 4, S = 1000,*** and ***N*** is unknown. We can find the value of ***N*** from

$$S = N \times \frac{1}{r} \quad \text{or} \quad N = S \times r = 1000 \times 4 = 4000 \text{ bps}$$

- ***Example 2:***

An analog signal has a **bit rate** of ***8000 bps*** and a **baud rate** of ***1000 baud.*** How many ***data elements*** are carried by each signal element? How many ***signal elements*** do we need?

- ***Solution:***

In this example, ***S = 1000, N = 8000,*** and ***r*** and ***L*** are unknown. We find first the value of ***r*** and then the value of ***L***.

$$S = N \times \frac{1}{r} \rightarrow r = \frac{N}{S} = \frac{8000}{1000} = 8 \text{ bits/baud}$$

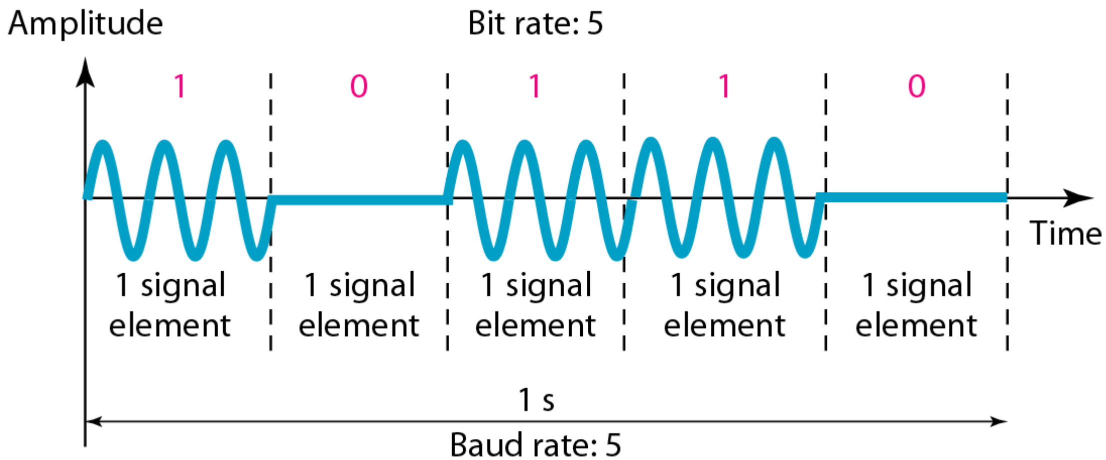
$$r = \log_2 L \rightarrow L = 2^r = 2^8 = 256$$

Amplitude Shift Keying (ASK)

- **Amplitude Shift Keying (ASK):**

In Amplitude Shift Keying, the **amplitude** of the carrier signal is **varied** to create signal elements. Both **frequency** and **phase** remain **constant** while the **amplitude changes**.

- **Binary amplitude shift keying:**



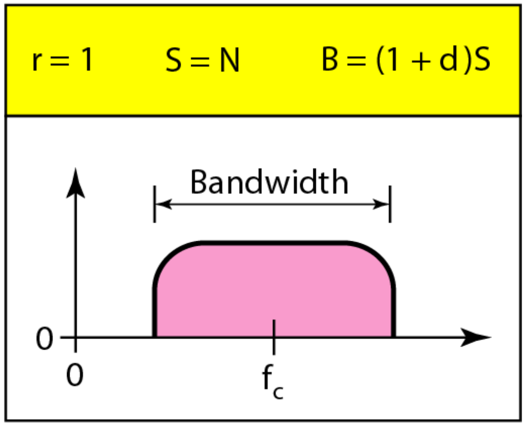


Fig. 3: Binary amplitude shift keying

Amplitude Shift Keying (ASK)

- ASK is implemented by **changing** the amplitude of a carrier signal to reflect amplitude levels in the digital signal.

- For example: a digital ***"1"*** could not affect the signal, whereas a digital ***"0"*** would, by making it zero.

- The **line encoding** will determine the values of the analog waveform to reflect the digital data being carried.

- The ***bandwidth B*** of ASK is proportional to the signal rate ***S.***

$$\boxed{B = (1+d)*S}$$

- ***"d"*** is due to **modulation and filtering**, lies between ***0 and 1.***

Implementation of Binary ASK

- **Implementation of binary ASK:**

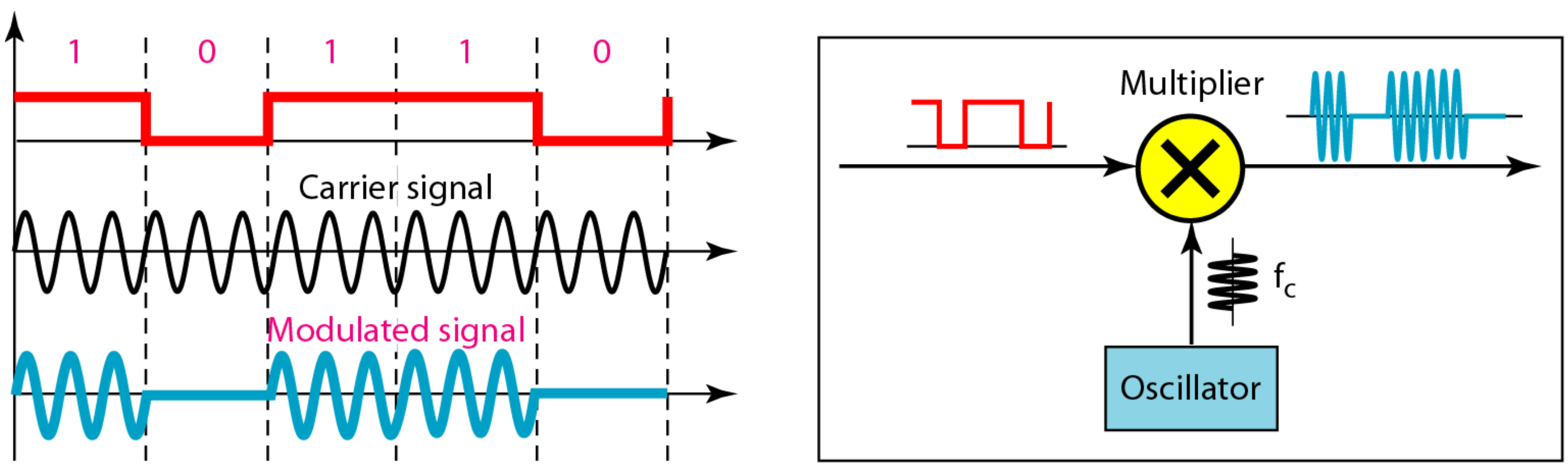


Fig. 4: Implementation of Binary ASK

ASK

- demodulation: only the presence or absence of a sinusoid in a given time interval needs to be determined
- advantage: simplicity
- disadvantage: ASK is very susceptible to noise interference – noise usually (only) affects the amplitude, therefore ASK is the modulation technique most affected by noise
- application: ASK is used to transmit digital data over optical fiber

ASK

- ***Example 3:***

We have an available bandwidth of ***100 kHz*** which spans from ***200 to 300 kHz.*** What are the ***carrier frequency*** and the ***bit rate*** if we modulated our data by using **ASK** with ***d = 1?***

- ***Solution:***

The middle of the bandwidth is located at ***250 kHz.*** This means that our carrier frequency can be at ***f_c = 250 kHz.*** We can use the formula for bandwidth to find the bit rate ***(with d = 1 and r = 1).***

$$B = (1 + d) \times S = 2 \times N \times \frac{1}{r} = 2 \times N = 100 \text{ kHz} \quad \longrightarrow \quad N = 50 \text{ kbps}$$

ASK

- ***Example 4:***

In data communications, we normally use **full-duplex** links with communication in both directions. We need to **divide** the bandwidth into two with **two carrier frequencies,** as shown in ***Figure 5.*** The figure shows the **positions** of **two carrier frequencies** and the **bandwidths.** The available bandwidth for each direction is now ***50 kHz,*** which **leaves** us with a data rate of ***25 kbps*** in each direction.

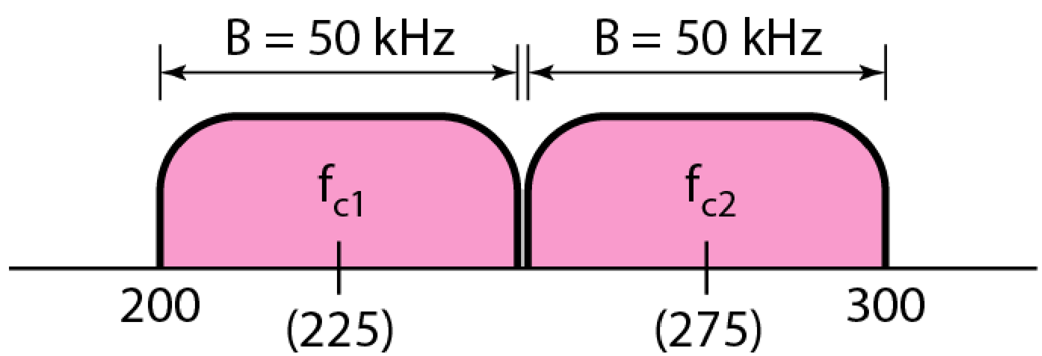


Fig. 5: Bandwidth of full-duplex ASK used in Example 4.

ASK

- **Multilevel ASK:**

The above discussion uses only **two amplitude levels.** We can have **multilevel ASK** in which there are more than two levels.

We can use **4, 8, 16, or more different amplitudes** for the signal and modulate the data using **2, 3, 4, or more bits at a time.**

In these cases, **r=2, r=3, r=4, and so on.** It is implemented with **QAM.**

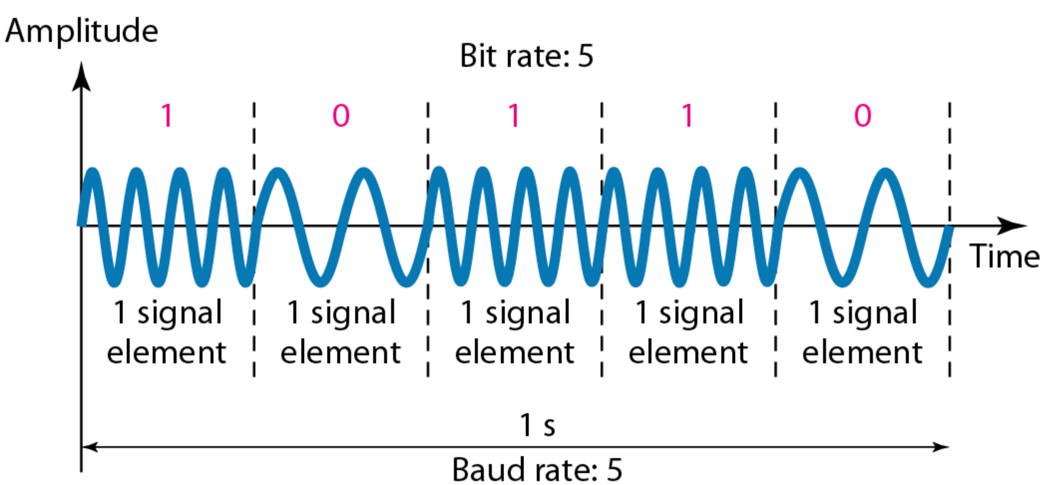
Frequency Shift Keying (FSK)

- **Frequency Shift Keying (FSK):**

In Frequency Shift Keying (FSK), the **frequency** of the carrier signal is **varied** to **represent** digital data.

The **frequency** of the modulated signal is **constant** for the duration of **one signal element,** but **changes** for the next element if the **data element changes.**

Both **peak amplitude** and **phase** remain **constant** for all signal elements.



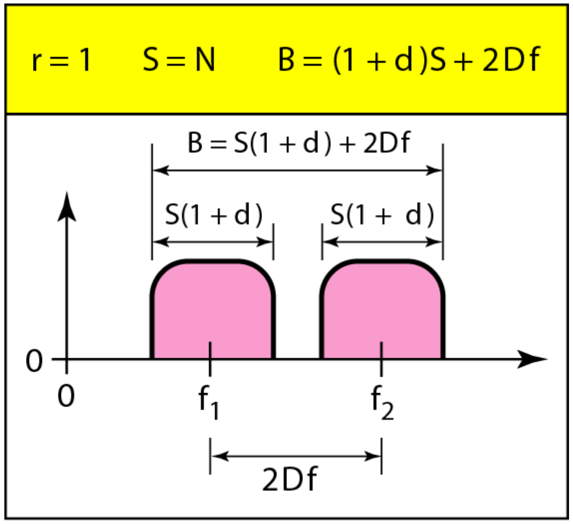
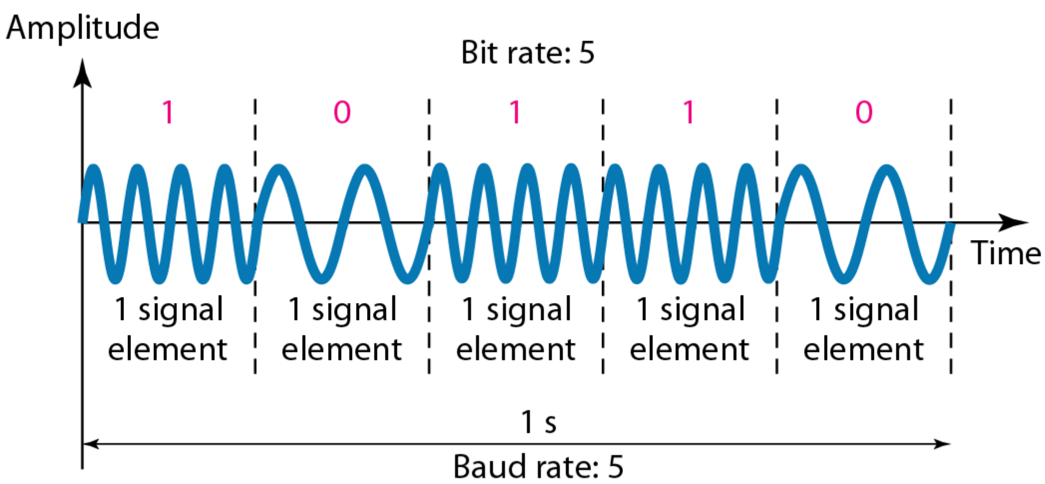


Fig. 6: Binary frequency shift keying

Frequency Shift Keying (FSK)

- The digital data stream changes the frequency of the carrier signal, f_c.
- For example, a *"1"* could be represented ***by*** $f_1=f_c+\Delta f$, and a *"0"* could be represented by $f_2=f_c-\Delta f$.
- If the difference between the two frequencies (f_1 ***and*** f_2) is $2\Delta f$, then the required ***BW B*** will be:

$$\boxed{B = (1+d) \times S + 2\Delta f}$$



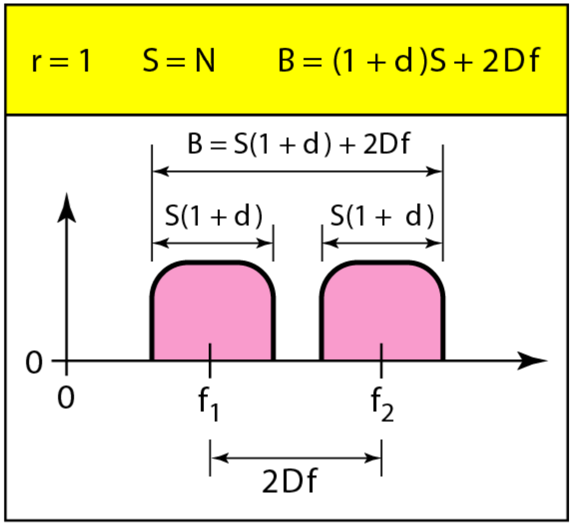


Fig. 6: Binary frequency shift keying

Implementation of binary FSK

- **Implementation of binary FSK:**

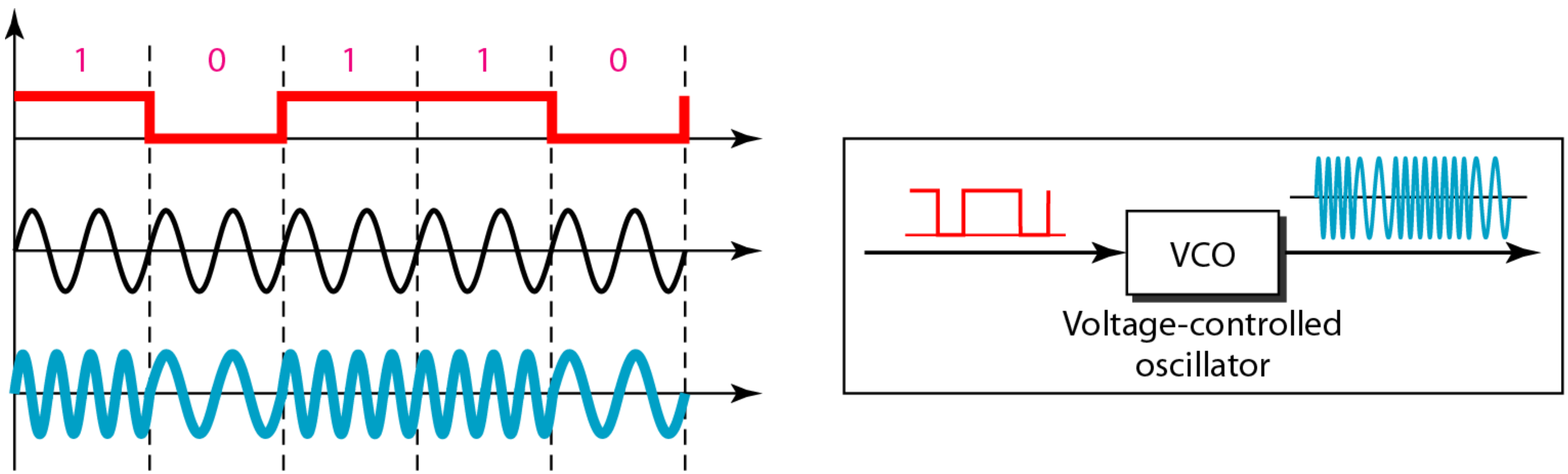


Fig. 7: Implementation of Binary FSK

Frequency Shift Keying (FSK)

- demodulation: demodulator must be able to determine which of two possible frequencies is present at a given time
- advantage: FSK is less susceptible to errors than ASK – receiver looks for specific frequency changes over a number of intervals, so voltage (noise) spikes can be ignored
- disadvantage: <u>FSK spectrum is 2 x ASK spectrum</u>
- application: over voice lines, in high-freq. radio transmission, etc.

Frequency Shift Keying (FSK)

- ***Example 5:***

We have an available bandwidth of ***100 kHz*** which spans from ***200 to 300 kHz.*** What are the ***carrier frequency*** and the ***bit rate*** if we modulated our data by using **FSK** with ***d* = *1?***

- ***Solution:***

This problem is similar to ***Example 3,*** but we are modulating by using **FSK**. The **midpoint** of the band is at ***250 kHz.*** We choose ***2Δf*** to be ***50 kHz;*** this means

$$B = (1 + d) \times S + 2\Delta f = 100 \longrightarrow 2S = 50 \text{ kHz} \quad S = 25 \text{ kbaud} \quad N = 25 \text{ kbps}$$

FSK

- **Multilevel FSK:**

In Multilevel FSK modulation (MFSK), we can use **more than two frequencies.**

For example, we can use **four** different frequencies ***f1, f2, f3 and f4*** to send ***2 bits*** at a time.

To send ***3 bits*** at a time, we can use ***eight frequencies*** and so on.

For the proper operation of the modulator and demodulator, it can be shown that the minimum value of ***2Δf* need to be equal to *S.*** The bandwidth with ***d=0*** is,

$$B = (1+d) \times S + (L-1)2\Delta f = L \times S$$

FSK

- **Example 6:**

We need to send data ***3 bits*** at a time at **a bit rate of *3 Mbps.*** The **carrier frequency** is ***10 MHz.*** Calculate the ***number of levels (different frequencies), the baud rate, and the bandwidth.***

- **Solution:**

We can have ***$L = 2^3 = 8$.*** The baud rate is ***S = 3 Mbps/3 = 1 Mbaud.*** This means that the carrier frequencies must be ***1 MHz apart ($2\Delta f$ = 1 MHz).*** The bandwidth is ***$B = 8 \times 1$ MHz = 8 MHz.*** **Figure 8** shows the **allocation** of **frequencies** and **bandwidth**.

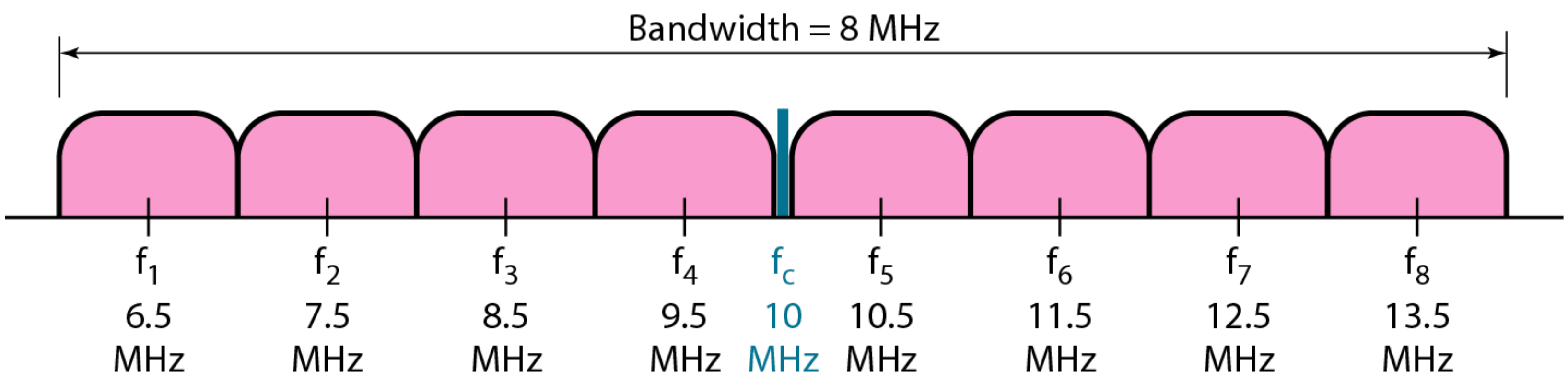


Fig. 8: Bandwidth of MFSK used in example 6.

Phase Shift Keying (PSK)

PSK – phase of carrier signal is varied to represent binary 1 or 0

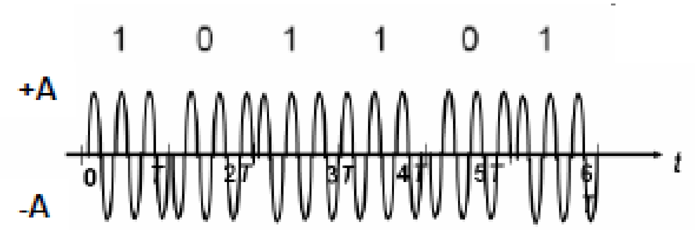
- peak amplitude & freq. remain constant during each bit interval
- example: binary 1 = 0° phase, binary 0 = 180° (πrad) phase

⇒ PSK is equivalent to multiplying carrier signal by +1 when the information is 1, and by -1 when the information is 0

2-PSK, or **Binary PSK**, since only 2 different phases are used.

$$s(t)=\begin{cases}A\cos(2\pi f_c t), & \text{binary 1}\\ A\cos(2\pi f_c t+\pi), & \text{binary 0}\end{cases}$$

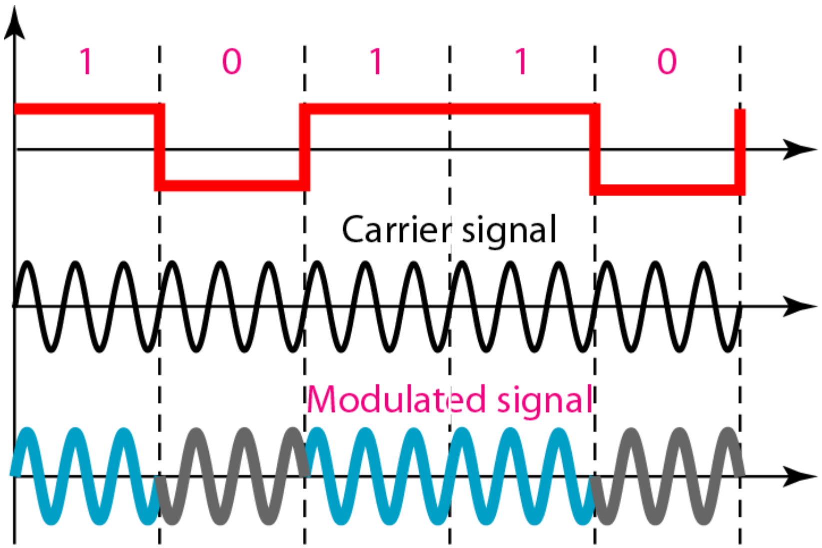
$$s(t)=\begin{cases}A\cos(2\pi f_c t), & \text{binary 1}\\ -A\cos(2\pi f_c t), & \text{binary 0}\end{cases}$$



- demodulation: demodulator must determine the phase of received sinusoid with respect to some reference phase
- advantage:
  - PSK is less susceptible to errors than ASK, while it requires/occupies the same bandwidth as ASK
  - more efficient use of bandwidth (higher data-rate) are possible, compared to FSK !!!
- disadvantage: more complex signal detection / recovery process, than in ASK and FSK

Phase Shift Keying (PSK)

- **Implementation of BPSK:**



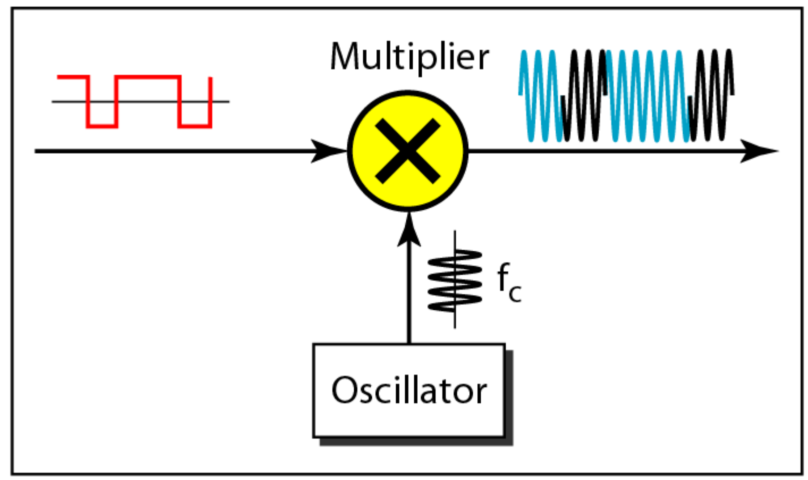
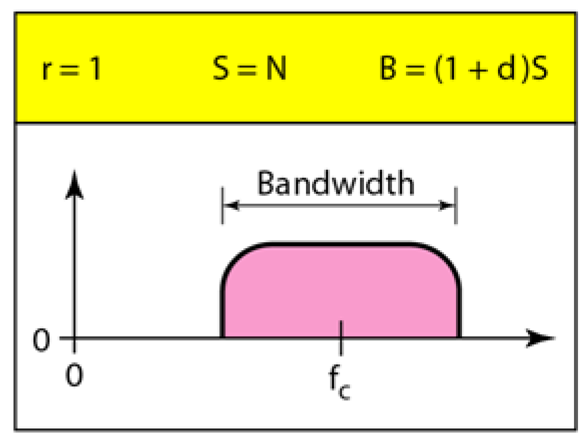


Fig. 10: Implementation of BPSK



The **bandwidth** requirement: $\boxed{B = (1+d) \times S}$

Phase Shift Keying (PSK)

PSK Detection

$$\cos^2 A = \frac{1}{2}(1+\cos 2A)$$

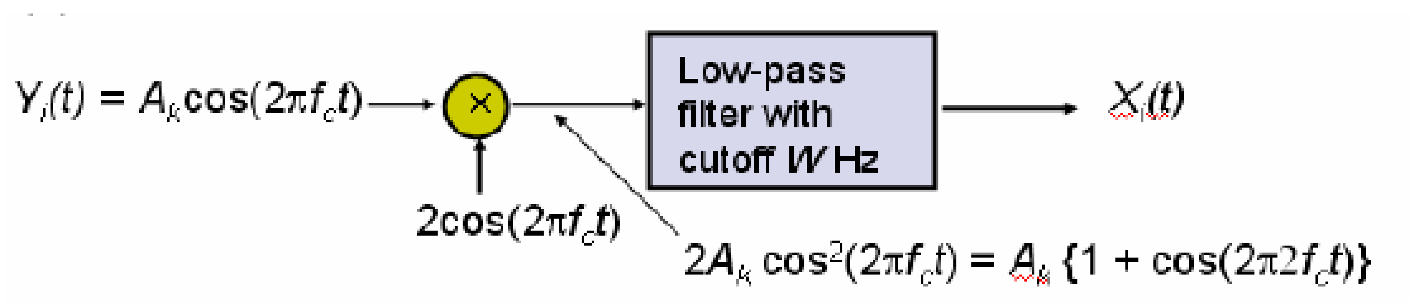
– **multiply the received / modulated signal $\pm A\cos(2\pi f_c t)$ by <u>$2*\cos(2\pi f_c t)$</u>**

- **resulting signal**

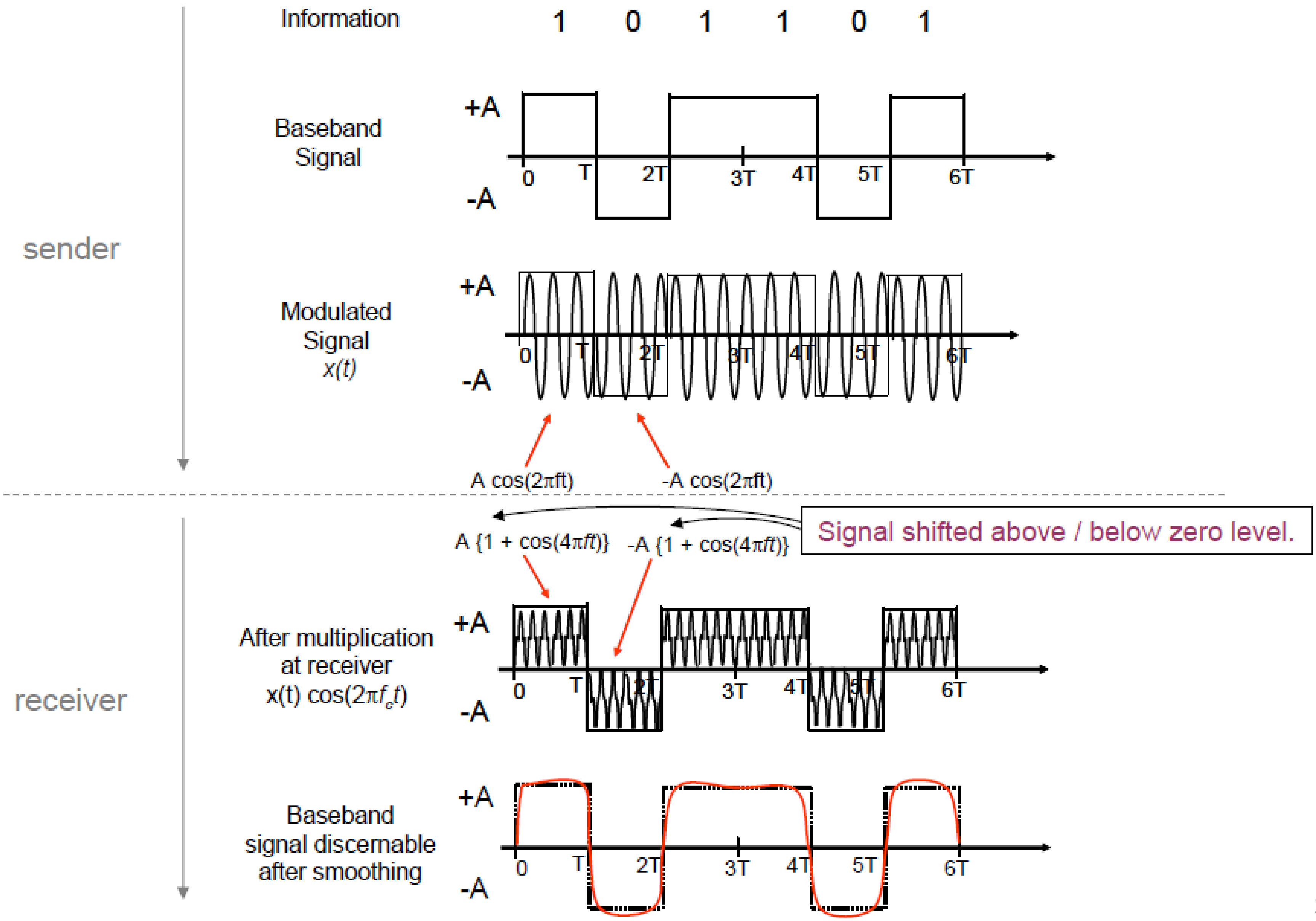
$$2A\cos^2(2\pi f_c t) = A[1+\cos(4\pi f_c t)], \quad \text{binary 1}$$

$$-2A\cos^2(2\pi f_c t) = -A[1+\cos(4\pi f_c t)], \quad \text{binary 0}$$

- **by removing the oscillatory part with a low-pass filter, the original baseband signal (i.e. the original binary sequence) can be easily determined**



Phase Shift Keying (PSK)



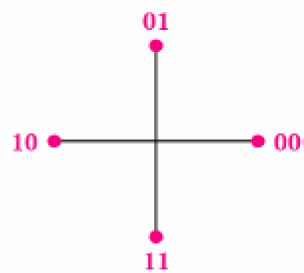
Quadrature PSK(QPSK)

QPSK = 4-PSK – **PSK that uses phase shifts of 90°=π/2 rad ⇒ 4 different signals generated, each representing 2 bits**

$$s(t)=\begin{cases} A\cos(2\pi f_c t), & \text{binary } 00 \\ A\cos(2\pi f_c t+\frac{\pi}{2}), & \text{binary } 01 \\ A\cos(2\pi f_c t+\pi), & \text{binary } 10 \\ A\cos(2\pi f_c t+\frac{3\pi}{2}), & \text{binary } 11 \end{cases}$$

Dibit	Phase
00	0
01	90
10	180
11	270

Dibit
(2 bits)



Constellation diagram

- **advantage**: **higher data rate than in PSK (2 bits per bit interval), while bandwidth occupancy remains the same**
- **4-PSK can easily be extended to 8-PSK, i.e. n-PSK**
- **however, higher rate PSK schemes are limited by the ability of equipment to distinguish small differences in phase**

QPSK and its implementation

- **QPSK and its implementation:**

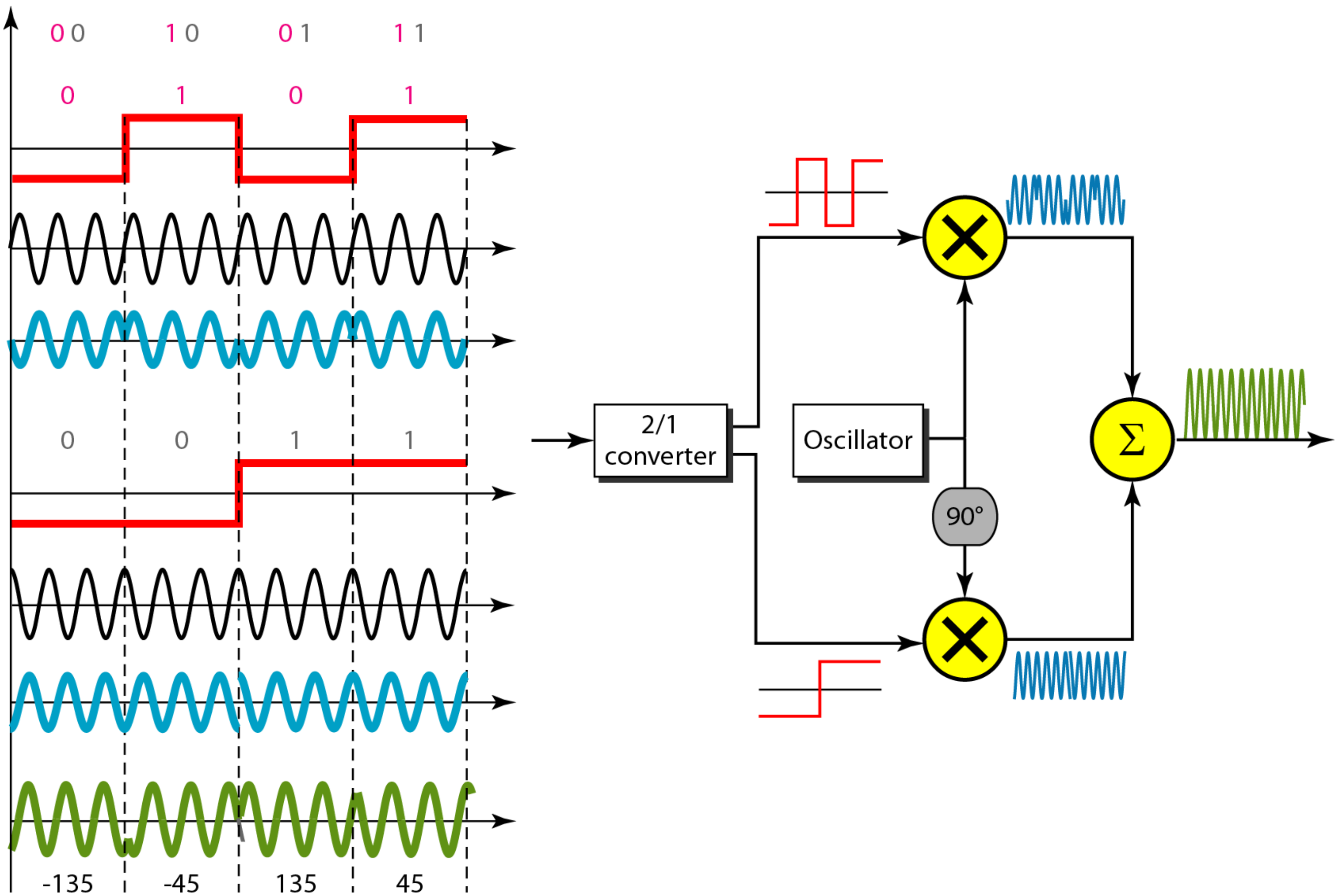


Fig. 11: Implementation of QPSK

Quadrature PSK(QPSK)

- **Example 7:**

Find the bandwidth for a signal transmitting at ***12 Mbps*** for **QPSK**. The value of $d = 0$.

- **Solution:**

For QPSK, ***2 bits*** is carried by one signal element. This means that $r = 2$. So the signal rate (baud rate) is $S = N \times (1/r) = 6$ ***Mbaud.*** With a value of $d = 0$, we have $B = S = 6$ ***MHz.***

Quadrature Amplitude Modulation (QAM)

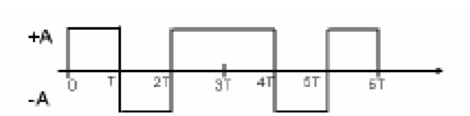
Quadrature Amplitude Modulation (QAM) – uses "two-dimensional" signalling

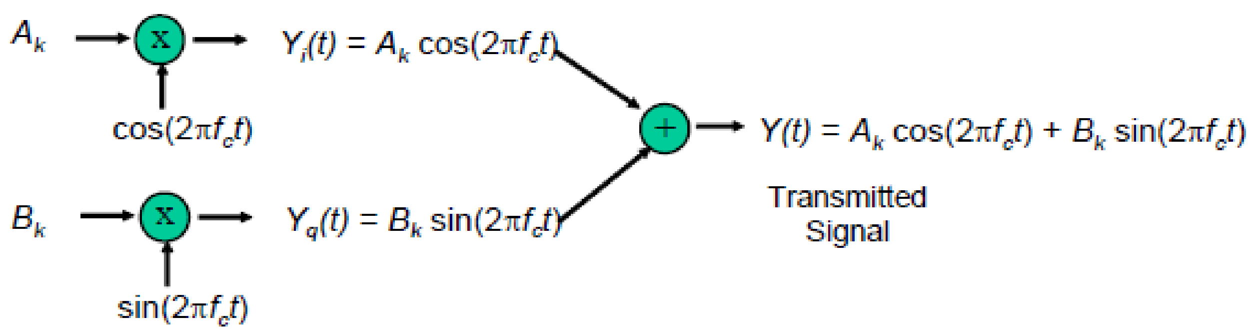
- original information stream is <u>split into two sequences</u> that consist of odd and even symbols, e.g. B_k and A_k

1	0	1	1	0	1	…
1	-1	1	1	-1	1	…
B_1	A_1	B_2	A_2	B_3	A_3	…

- A_k sequence (in-phase comp.) is modulated by $\cos(2\pi f_c t)$
  B_k sequence (quadrature-phase comp.) is modulated by $\sin(2\pi f_c t)$

- composite signal $A_k \cos(2\pi f_c t) + B_k \sin(2\pi f_c t)$ is sent through the channel



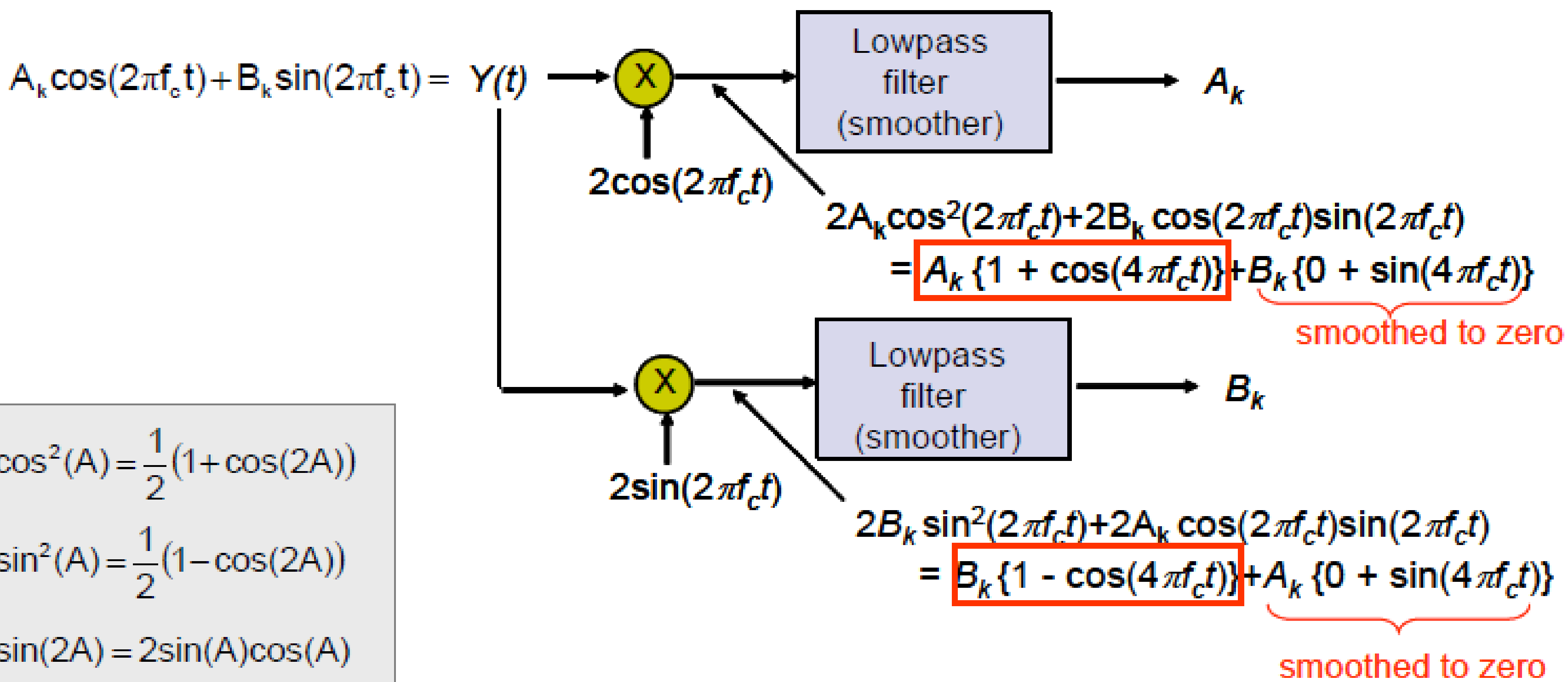


- advantage: data rate = 2 bits per bit-interval!

Quadrature Amplitude Modulation (QAM)

QAM Demodulation

- **by multiplying Y(t) by $2 \cdot \cos(2\pi f_c t)$ and then low-pass filtering the resultant signal, sequence A_k is obtained**
- **by multiplying Y(t) by $2 \cdot \sin(2\pi f_c t)$ and then low-pass filtering the resultant signal, sequence B_k is obtained**



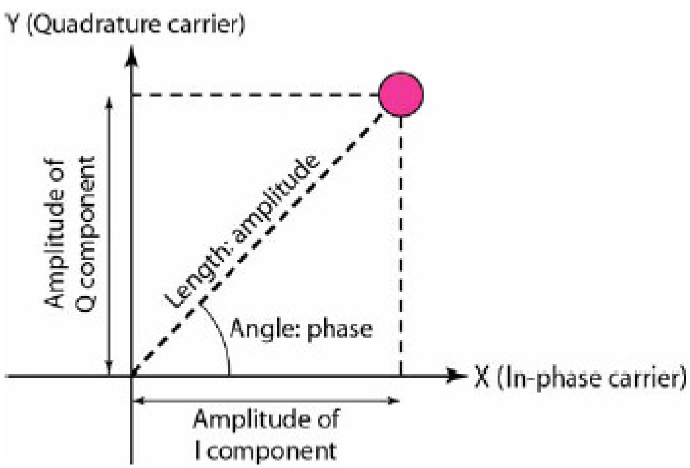
$$\cos^2(A) = \frac{1}{2}(1+\cos(2A))$$

$$\sin^2(A) = \frac{1}{2}(1-\cos(2A))$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

Signal Constellation

Constellation Diagram – used to represents possible symbols that may be selected by a given modulation scheme as <u>points</u> in 2-D plane



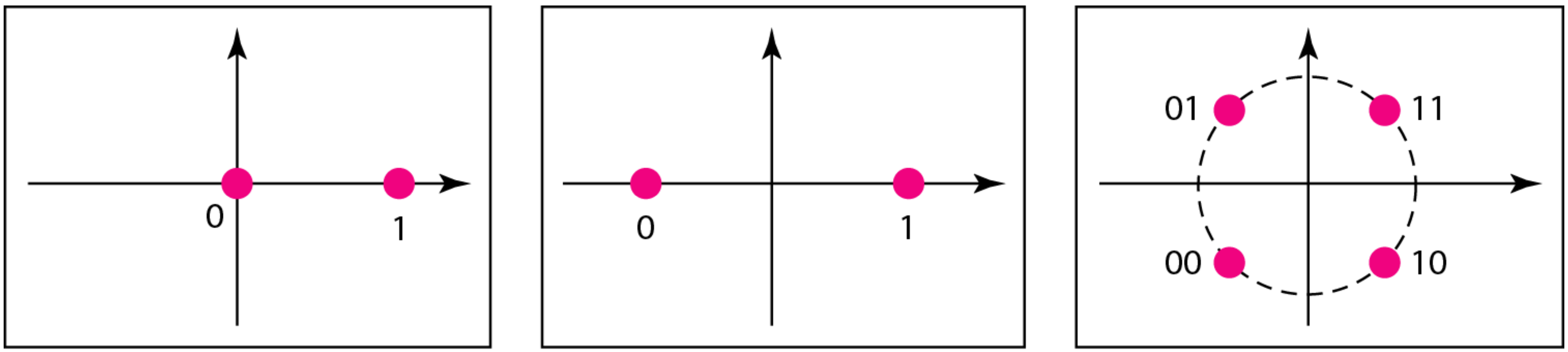
- X-axis is related to in-phase carrier: $\cos(\omega_c t)$
  - the projection of the point on the X-axis defines the peak amplitude of the in-phase component
- Y-axis is related to quadrature carrier: $\sin(\omega_c t)$
  - the projection of the point on the Y-axis defines the peak amplitude of the quadrature component
- the length of line that connects the point to the origin is the peak amplitude of the signal element (combination of X & Y components)
- the angle the line makes with the X-axis is the phase of the signal element

Constellation Diagrams

- **Example 8:**

Show the **constellation** diagrams for an **ASK (OOK), BPSK, and QPSK signals.**

- **Solution:**



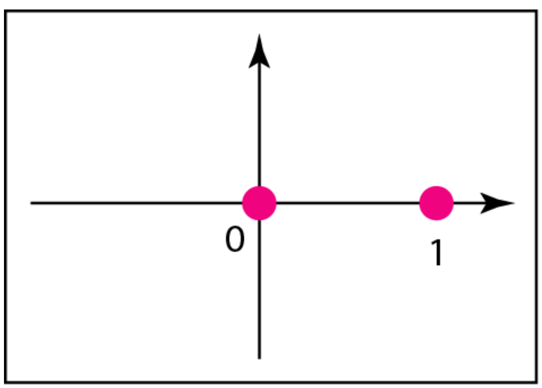
a. ASK (OOK) b. BPSK c. QPSK

- For ASK, we are using only an in-phase carrier. Therefore, the **two points** should be on the **X axis.** ***Binary 0*** has an amplitude of ***0V; binary 1*** has an amplitude of ***1 V***(for example). The points are located at the origin and at ***1 unit.***

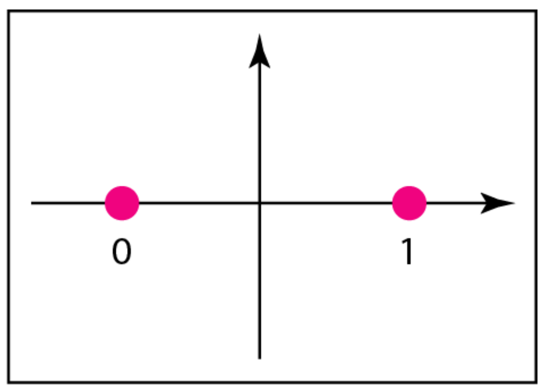
Constellation Diagrams

- **BPSK** creates two different signal elements, **one** with amplitude ***1 V and in phase*** and the **other** with amplitude ***-1 V and 180^0 out of phase.***

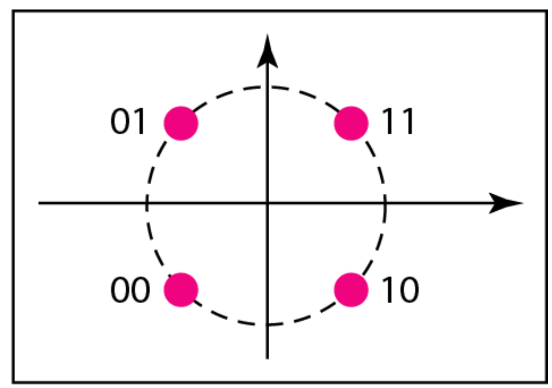
- **QPSK** uses **two carriers,** one **in-phase** and the **other quadrature.** The point representing ***11*** is made of **two combined signal elements, both** with an amplitude of **1V. One element** is **represented** by an **in-phase** carrier, the other element by a **quadrature** carrier*.*



a. ASK (OOK)



b. BPSK

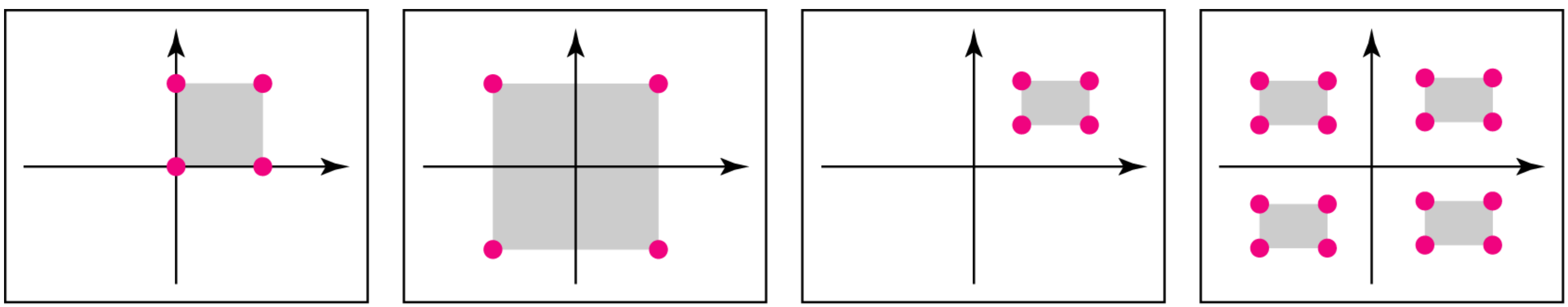


c. QPSK

Constellation Diagrams

- **Quadrature Amplitude Modulation (QAM):**

The **idea** of using two carriers, one in-phase and the other quadrature, **with different amplitude levels** for **each** carrier is the concept behind **quadrature amplitude modulation(QAM).**



a. 4-QAM b. 4-QAM c. 4-QAM d. 16-QAM

Fig. 13: Constellation diagrams for some QAMs

Continuous-phase frequency-shift keying (CPFSK)

- **Continuous-phase frequency-shift keying** (CPFSK) is a commonly used variation of frequency-shift keying (FSK), which is itself a special case of analog frequency modulation.
  - FSK is a method of modulating digital data onto a sinusoidal carrier wave, encoding the information present in the data to variations in the carrier's instantaneous frequency **between one of two frequencies** (referred to as the space frequency and mark frequency).
  - In general, a standard **FSK signal does not have continuous phase**, as the modulated waveform switches instantaneously between two sinusoids with different frequencies.
- As the name suggests, the phase of a CPFSK is in fact continuous;
  - this attribute is desirable for signals that are to be transmitted over **bandlimited channel**, as **discontinuities in a signal introduce wideband frequency components.**
  - In addition, some classes of amplifiers exhibit **nonlinear behavior** when driven **with nearly discontinuous signals**; this could have undesired effects on the shape of the transmitted signal.

Minimum Shift Keying (MSK)

- **Minimum Shift Keying (MSK):**
  - **Minimum shift keying (MSK)** is a special type of **continuous phase-frequency shift keying (CPFSK)** with a **modulation index** of ***0.5.***
  - A modulation index of **0.5 corresponds** to the **minimum frequency spacing** that **allows** two FSK signals to be **coherently orthogonal,** and the name minimum shift keying **implies** the **minimum frequency separation** (i.e. bandwidth) that **allows orthogonal detection.**
- **Gaussian Minimum Shift keying (GMSK):**
  - Gaussian Minimum Shift Keying (GMSK)is a modification of MSK (i.e. CPFSK with m = 1/2).
  - A **filter** used to reduce the **bandwidth** of a **baseband pulse train** prior to modulation is called a **pre-modulation filter.**
  - The Gaussian pre-modulation filter smoothens the phase trajectory of the MSK signal thus limiting the instantaneous frequency variations. The result is an **FM modulated signal** with a much **narrower bandwidth.** This bandwidth reduction does not come for free since the pre-modulation filter smears the individual pulses in pulse train. As a consequence of this smearing in time, adjacent pulses interfere with each other generating what is commonly called **inter-symbol interference or ISI.** In the applications where GMSK is used, the trade-off between power efficiency and bandwidth efficiency is well worth the cost.

Shannon's Theorem

- **Shannon's Theorem:**

Shannon's Theorem gives an upper bound to the capacity of a link, in bits per second (bps), as a function of the available bandwidth and the **signal-to-noise ratio** of the link.

The Theorem can be stated as:

$$\boxed{C = B * \log2(1+ S/N)}$$

where ***C*** is the **achievable channel capacity,** ***B*** is the **bandwidth** of the line, ***S*** is the **average signal power** and **N** is the **average noise power.**

The **signal-to-noise ratio (S/N)** is usually expressed in ***decibels (dB)*** given by the formula:

$$\boxed{10 * \log_{10}(S/N)}$$

so for example a signal-to-noise ratio of ***1000*** is commonly expressed as

.

$$\boxed{10 * \log_{10}(1000) = 30\ dB}$$

END