

1 Program 7, Phase 1: Tallying the Text

We will start with a very short message in order to keep this process simple enough to diagram:

Morals rule everything! (Or is it money?)

The first step is to count the letter frequencies in this text. We will use a simple algorithm:

- Create an array of 256 integers initialize to 0.
- Read the file one keystroke at a time and use the ASCII code of each char to index the array.
- Increment the counter at that array location.

The non-zero results of tallying our text are shown below in ASCII sequence order:

!	()	?	'	'	M	O	a	e	g	h	i	l	m	n	o	r	s	t	u	v	y
1	1	1	1	6	1	1	1	1	4	1	1	3	2	1	2	2	4	2	2	1	1	2

2 Program 7, Phase 2: Building the Heap

For each non-zero-count character, make a new Huffman Node using the character and its frequency. Put all of the nodes into an array, starting with slot 1. The result for our text will be:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
										0	1	2	3	4	5	6	7	8	9	0	1	2			
	!	()	?	'	M	O	a	e	g	h	i	l	m	n	o	r	s	t	u	v	y			
0	1	1	1	1	6	1	1	1	4	1	1	3	2	1	2	2	4	2	2	1	1	2			

Figure 1: The heap array before heapify().

Next, we must *heapify()* this array. Since there are 22 data items in the array, position 11 is the rightmost heap node that is a parent. Everything after that is a leaf node (colored green in the diagram.) So we set $k = 11$ and call *downHeap*(11).

During the *downHeap*(11) execution, we set $s = 22$ and $r = 23$. We do not try to compare the priorities of the sons, since there is only one son. Now we compare $k.priority$ to $s.priority$ and find that they are in the right order already. So we are done with *downheap*(11).

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
		!	()	?	'	'	M	O	a	e	g													
0	1	1	1	1	6	1	1	1	4	1															
												h													
												1													

1	1	1	1	1	1	1	1	2	2	2
2	3	4	5	6	7	8	9	0	1	2
i	l	m	n	o	r	s	t	u	v	y
3	2	1	2	2	4	2	2	1	1	2

\wedge
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 s

Figure 2: The first step of heapify().

Now we decrement k and execute *downHeap*(10). We set $s = 20$ and $r = 21$ and compare $s.priority$ to $r.priority$. Since they are the same, we do not change s . Now we compare $k.priority == 1$ to $s.priority == 1$ and find that they are in the right order already. So we are done with *downheap*(10).

Now we decrement k and execute *downHeap*(9). We set $s = 18$ and $r = 19$ and compare $s.priority$ to $r.priority$. Since they are the same, we do not change s . Now we compare $k.priority == 4$

Since they are the same, we do not change s . Now we compare $k.\text{priority}==6$ to $s.\text{priority}==1$ and find that they are in the wrong order. (See Figure 5) We swap the (space,6) with the (u,1). Since position 20 is a leaf, we are done with this pass. (See Figure 6)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040	1041	1042	1043	1044	1045	1046	1047	1048	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079	1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117	1118	1119	1120	1121	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147	1148	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168	1169	1170	1171	1172	1173	1174	1175	1176	1177	1178	1179	1180	1181	1182	1183	1184	1185	1186	1187	1188	1189	1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213	1214	1215	1216	1217	1218	1219	1220	1221	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the array $(y, 2)$ is moved to slot 1. The diagrams show the progress of the *downHeap()* operation that moves the $(y, 2)$ to its proper place in the tree. This job required 4 passes – the maximum possible with a tree of this size.

- In the top row, we see that positions 2 and 3 are the sons of position 1, and s is set to position 2 because position 3 does not have greater priority. So we compare the priorities in positions 1 and 2 and see that a swap is necessary.
 - The second row shows that $(y, 2)$ has moved to position 2 and is ready to compare with position 4. Again, a swap is necessary.
 - The third row shows the $(y, 2)$ in position 4, ready to compare to its sons. We compare y 's priority to position 9's priority and see that a swap is necessary.
 - Row 4 shows the $(y, 2)$ in position 8 ready to compare to its better son in position 16. No swap is necessary this time, and we are done because position 16 is a leaf. Figure 8 shows the array after this swap takes place.
- Remove a second Node from the heap. The second node is a parenthesis with frequency 1. We replace it by the $(v, 1)$ at the end of the heap and do a *downHeap(1)* operation. This terminates right away because the priorities of both sons are no greater than the priority of $(v, 1)$. The result is shown in Figure 9.

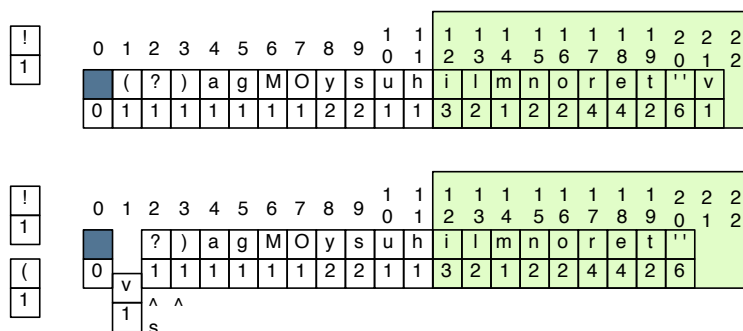


Figure 8: Deleting the second item.

- Create a new Node whose left and right sons are the two Nodes you removed and add the new Node to the heap. The priority of this node equals the sum of the priorities of the two sons. The diagram shows the heap immediately after the new node has been stored there,

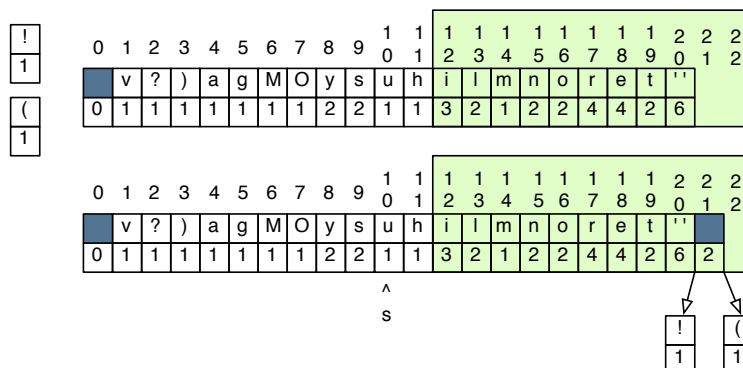


Figure 9: Putting the first combined Node into the heap.

and before the *upHeap*(21) operation. During *upHeap*(), the priority of the parent node, position 10, is compared to the priority of the new node in position 21. They are in the right order so no swap takes place. Figure 9 shows the final position of the data after finishing the *upHeap*(21).

- Repeat this process (remove, remove, add) until there is only one node left on the tree. Each time we do the process, the heap is shortened by 1 position. We diagram the heap during the second iteration, after deleting the first node, deleting the second, and finally inserting the combined node:

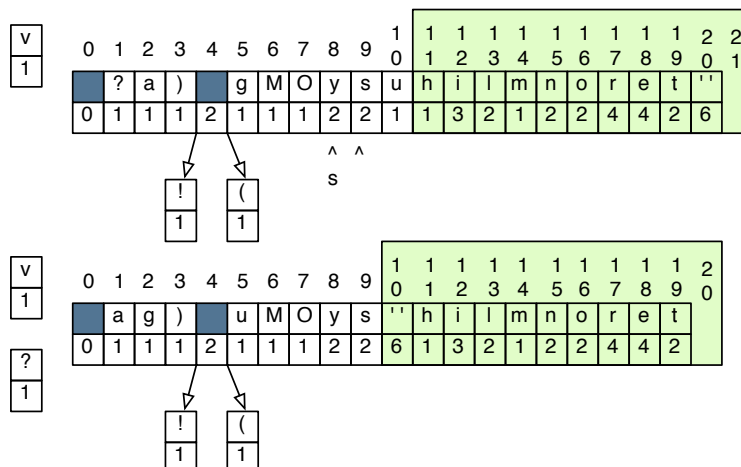


Figure 10: Removing two more nodes.

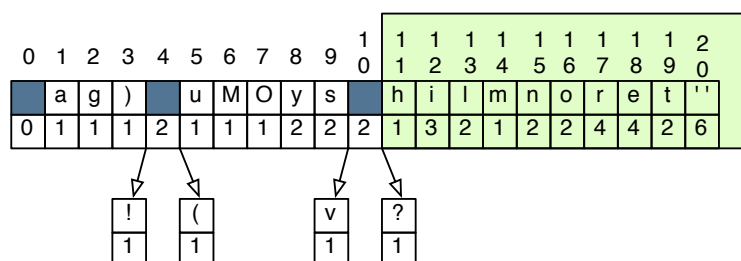


Figure 11: After adding the second combined Node to the heap.

- The final node left in the heap is a Huffman code tree. Do a recursive tree-walk to print the tree, then return the pointer to the tree for use in the next phase of the program.

4 Extra Credit: Phase 4: Creating the Huffman Code

We can create a code by traversing the tree. Every left-branch corresponds to a 0 in the code, and every right-branch corresponds to a 1. We traverse the tree in depth-first order, writing down the 0's and 1's as we go. When we reach a leaf node (which contains a letter) the current string of 0's and 1's becomes the code for that letter. The table below shows the codes that would be generated from the tree above, in order of generation.

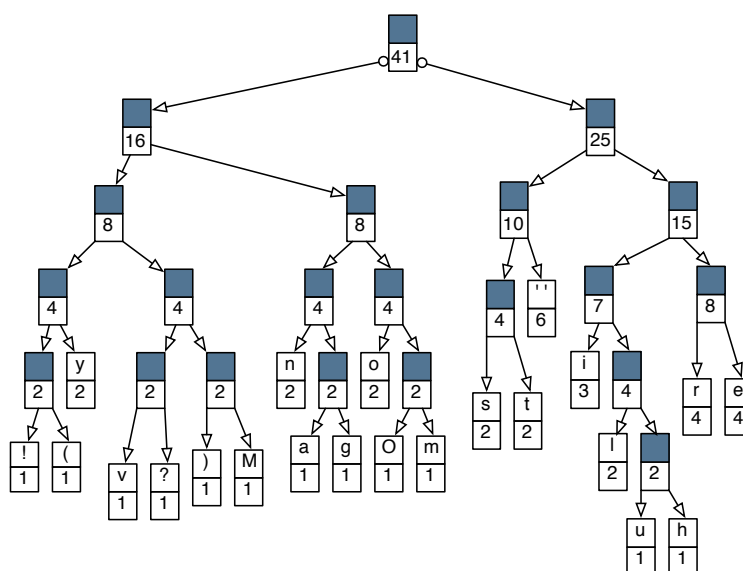


Figure 12: After adding the second combined Node to the heap.

!	5	0	0	0	0	0	O	5	0	1	1	1	0	
(5	0	0	0	0	1	m	5	0	1	1	1	1	
y	4	0	0	0	1		s	4	1	0	0	0		
v	5	0	0	1	0	0	t	4	1	0	0	1		
?	5	0	0	1	0	1	space	3	1	0	1			
)	5	0	0	1	1	0	i	4	1	1	0	0		
M	5	0	0	1	1	1	l	5	1	1	0	1	0	
n	4	0	1	0	0		u	6	1	1	0	1	1	0
a	5	0	1	0	1	0	h	6	1	1	0	1	1	1
g	5	0	1	0	1	1	r	4	1	1	1	0		
o	4	0	1	1	0		e	4	1	1	1	1		

This kind of code has some very special properties:

- It is a variable length code – common letters have shorter representations than uncommon ones.
- No code is a prefix of the code for any other letter. This enables us to decode messages.
- When a message is encoded, the bits are packed into bytes. Every bit counts. The last byte is padded with 0 bits, if necessary.