Compressive Sensing of Frequency-Hopping Spread Spectrum Signals

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ABSTRACT

In this paper, compressive sensing strategies for interception of *Frequency-Hopping Spread Spectrum* (FHSS) signals are introduced. Rapid switching of the carrier among many frequency channels using a pseudorandom sequence (unknown to the eavesdropper) makes FHSS signals difficult to intercept. The conventional approach to intercept FHSS signals necessitates capturing of all frequency channels and, thus, requires the *Analog-to-Digital Converters* (ADCs) to sample at very high rates. Using the fact that the FHSS signals have sparse instantaneous spectra, we propose compressive sensing strategies for their interception. The proposed techniques are validated using *Gaussian Frequency-Shift Keying* (GFSK) modulated FHSS signals as defined by the Bluetooth specification.

Keywords: Compressive Sensing, FHSS, Scatter Matrix, Compressed Classification

1. INTRODUCTION

Frequency-Hopping Spread Spectrum (FHSS) is a common modulation technique for spread spectrum communications. In FHSS, the carrier frequency is repeatedly switched in a pseudorandom manner and such rapid switching of the carrier frequency makes FHSS signals hard to intercept or jam. A cooperative receiver with the knowledge of the pseudorandom switching sequence can operate in synchronization with the transmitter and can stay tuned to the same carrier frequency as the transmitter. For non-cooperative receivers, however, the conventional approach requires sampling at very high rates in order to capture the full range of possible transmission frequencies. Such high sampling rates make interception of FHSS signal difficult and/or expensive.

The recently introduced compressive sensing (CS) theory[1,2] illustrates how certain class of signals can be recovered from significantly fewer samples than suggested by the Nyquist-Shannon sampling theorem. Such reduced sampling requirements are very desirable in many applications since they can lead to significant simplification of sensor hardware. The basic principle of CS is that signals that are sparse or compressible can be recovered from a small number of linear measurements provided that the *Restricted Isometry Property* (RIP) is satisfied.[1,2]

In this paper, we introduce novel CS methods for FHSS signals. Using the fact that the FHSS signals have sparse instantaneous spectra, we formulate a CS framework for interception of FHSS signals without the knowledge of the hopping sequence. We propose a sensing architecture for capturing the FHSS signals. In conventional CS, measurements are made using random linear projections of the signal since such random projections have been shown to satisfy RIP.[1] However in many applications, prior information about the signal (beyond sparsity) exists and it is desirable to incorporate such prior information into the design of the sensing matrices.[3] We propose a method to design compressive measurement kernels using prior knowledge of the FHSS signal structure and illustrate the advantages of such designed measurement kernels over random ones. The proposed techniques are validated using *Gaussian Frequency-Shift Keying* (GFSK) modulated FHSS signals as defined by the Bluetooth specification.[4]

The remainder of this paper is organized as follows: In Section 2, a brief introduction to CS theory is provided and an architecture for CS of sparse bandlimited signals is discussed. CS for FHSS signals is introduced in Section 3 and compressive classification is discussed in Section 4. Section 5 introduces a method for designing the sampling kernels. Experimental results are presented in Section 6 and conclusions are provided in Section 7.

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2. COMPRESSIVE SENSING OF SPARSE BANDLIMITED SIGNALS

Let $\mathbf{s} \in \mathbb{C}^N$ represent an N-dimensional signal. In CS, it is assumed that the signal $\mathbf{s}$ has a sparse representation such that

$$\mathbf{s} = \mathbf{\Psi x} \tag{1}$$

where the vector $\mathbf{x}$ denotes the sparse domain coefficients of the signal and $\mathbf{\Psi}$ denotes the sparse dictionary. In general, the matrix $\mathbf{\Psi}$ has size $N \times L$. In cases where the dictionary is a complete orthogonal basis, $N = L$. Since the vector $\mathbf{x}$ is sparse, l_0-norm of $\mathbf{x}$, $\|\mathbf{x}\|_0 = K$ where $K \ll N$.

In CS, the signal $\mathbf{s}$ is measured using M linear projections. In other words, let $\mathbf{\Phi}$ denote the $M \times N$ sensing matrix and $\mathbf{y} \in \mathbb{C}^M$ denote the measurements. The measurement process can then be expressed as

$$\mathbf{y} = \mathbf{\Phi s} = \mathbf{\Phi \Psi x} \tag{2}$$

momentarily disregarding the noise effects. The goal of CS is to recover $\mathbf{x}$ from $\mathbf{y}$ when $M \ll N$.

An architecture for CS of sparse bandlimited signals was proposed in[5] and is shown in Figure 1. In this block diagram, the signal (which might be corrupted by additive noise $\mathbf{n_r} \in \mathbb{C}^N$) is first mixed with a sampling kernel. The low-pass filter in the figure acts as an accumulator, and the resulting signal is sampled at the reduced sampling rate to obtain the measurements $\mathbf{y}$. Note that the additive noise term $\mathbf{n_s} \in \mathbb{C}^M$ in the figure represents the quantization noise during the ADC.

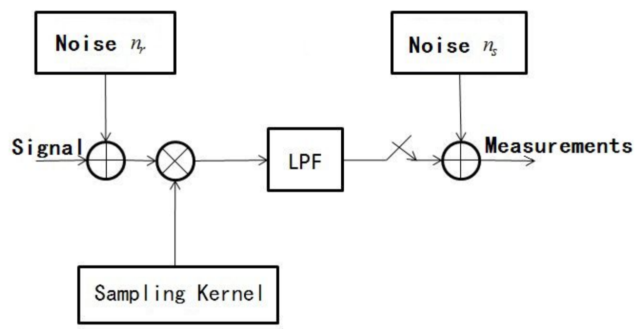

Figure 1. Block Diagram of Compressive Sensing Receiver

The sensing architecture illustrated in Figure 1 can be mathematically represented using sensing matrices $\mathbf{\Phi}$ that are in block-diagonal form. A representative sensing matrix in this form is shown in Figure 2. In the figure, each row represents the sampling kernel used to make one measurement.The bright areas represent the non-zero entries of the sampling kernel, and the dark areas represent the zero values. Additional parallel branches of the architecture in Figure 1 can be employed to obtain more simultaneous measurements (i.e. increasing the block size of the block-diagonal matrix in Figure 2), albeit at increased hardware cost. In conventional CS, the projections are often randomly generated since such random projections have been shown to satisfy RIP[1] . Thus, earlier works[5] using the sensing architecture shown in Figure 1 have suggested that the sampling kernels (i.e. the non-zero entries of the matrix $\mathbf{\Phi}$) can be selected randomly. Our goal in this work is to illustrate that careful design of the sensing matrix $\mathbf{\Phi}$ using prior knowledge about the signal can lead to improved performance.

3. COMPRESSIVE SENSING FOR FHSS SIGNALS

FHSS is a commonly used spread spectrum technique. By repeatedly switching the carrier frequency in a pseudo-random manner, FHSS transmitter/receiver pairs can avoid interception and jamming. Since the pseudorandom hopping sequence is not known to the interceptor, interception of FHSS signals are often very challenging. Our goal in this work is to use the fact that FHSS signals have sparse instantaneous spectra to develop CS methods for interception of FHSS signals without the knowledge of the hopping sequence.

While many different types of FHSS modulation exist, we have chosen to work with the Bluetooth specification[4] in this work. Bluetooth signals are obtained by combining GFSK modulation of binary symbols with FHSS.

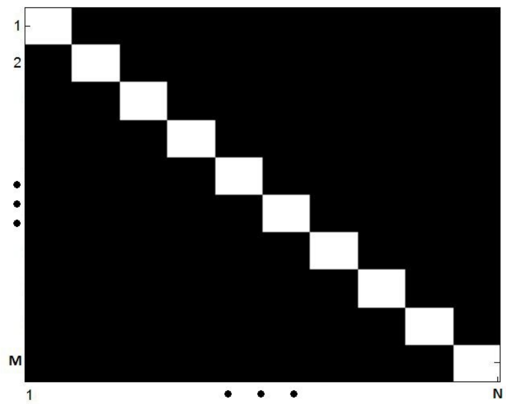

Figure 2. Illustration of a Representative Sensing Matrix Corresponding the Block Diagram in Figure 1

The standard specifies 79 RF channels displaced by 1 MHz starting at 2.402 GHz and stopping at 2.480 GHz. This 2.4 GHz RF band is globally designed as unlicensed Industrial, Scientific and Medical (ISM) short-range RF band. The symbol period is specified as $1\mu s$ and the signal hops between the 79 RF channels every 625 symbols.

Disregarding the 2.4 GHz carrier, we model the complex envelope of the Bluetooth signal as

$$s(t) = Ae^{i(2\pi f_k(t)+\theta)} \tag{3}$$

where A is a positive constant that represents the magnitude of the signal and θ is an unknown phase. The index $k \in \{1, \ldots, 79\}$ denotes the subcarrier used to transmit the current symbol. The function $f_k(t)$ describes the GFSK modulation of the transmitted binary symbols and is defined as

$$f_k(t) = f_c^k t + f_{shift} \int_0^t g_{Gaus}(m(b(\tau))d\tau \tag{4}$$

where f_c^k denotes the k^{th} subcarrier frequency and f_{shift} denotes the frequency shift from the subcarrier to transmit each symbol with FSK modulation. In Equation (4), $b(t)$ denotes the transmitted binary symbol (i.e. 0 or 1) as a function of time and $m(b(t))$ is a modulation function for the transmitted symbol defined as

$$m(b(t)) = \begin{cases} -1 & \text{when } b(t) = 0 \\ 1 & \text{when } b(t) = 1 \end{cases} \tag{5}$$

$g_{Gaus}(\cdot)$ is a Gaussian filter used to make the pulse smoother so as to limit its spectral width. This Gaussian filtering introduces *Inter-Symbol Interference* (ISI).

In the proposed CS model, we assume that the 2.4 GHz carrier frequency has been demodulated and that the Bluetooth signal given by Equation (3) is applied as input to the CS architecture in Figure 1. We assume that the analog front-end of the architecture in Figure 1 operates at the full Nyquist bandwidth (80 MHz). Note that since the hopping sequence is unknown, the signal would have to be sampled at this rate to capture the entire bandwidth, if CS techniques were not being used. Thus, we refer to the sampling rate required to capture this entire bandwidth as the Nyquist sampling rate. The Bluetooth signals are then represented as

$$\mathbf{s} = \mathbf{\Psi}\mathbf{x} \tag{6}$$

where $\mathbf{\Psi}$ is the $N \times L$ dimensional sparsity dictionary. In Equation (6), $\mathbf{x} = e^{i\theta}\mathbf{u}$ is an L-dimensional vector that is 1-sparse, where θ is the random phase defined in Equation (3) and $\mathbf{u}$ is an L-dimensional indicator vector. If we consider the binary symbols transmitted at 79 possible subcarrier frequencies, $L = 2 \times 79 = 158$. Alternatively, it is possible to consider a larger signal dictionary to account for ISI. For example, if the previous and the next binary symbol are also considered in addition to the current symbol, $L = 2^3 \times 79 = 632$. Sensing of the Bluetooth signal is then modeled as

$$\mathbf{y} = \mathbf{\Phi}(\mathbf{s} + \mathbf{n}_r) + \mathbf{n}_s = \mathbf{\Phi}(e^{i\theta}\mathbf{\Psi}\mathbf{u} + \mathbf{n}_r) + \mathbf{n}_s \tag{7}$$

where both $\mathbf{n}_r$ and $\mathbf{n}_s$ are i.i.d. Gaussian noise vectors. Based on this assumption and the fact that the rows of the sensing matrix $\mathbf{\Phi}$ are orthogonal due to the non-overlapping block diagonal structure shown in Figure 2, Equation (7) can be rewritten using an equivalent i.i.d. Gaussian noise vector $\mathbf{n}_e$ as

$$\mathbf{y} = e^{i\theta}\mathbf{\Phi}\mathbf{\Psi}\mathbf{u} + \mathbf{n}_e \tag{8}$$

4. COMPRESSIVE CLASSIFICATION

Most of the existing CS literature focuses on reconstruction problems where the goal is to recover the signal of interest as accurately as possible from compressive measurements. In this application, however, our goal is to intercept the communication between the transmitter and the receiver using compressive measurements. Thus, this problem can be formulated as a classification problem. A framework for compressed classification and compressed matched filtering was proposed in.[6] In this work, we follow similar steps to develop a compressed classification technique for our application.

The goal of recovering the hopping sequence together with the transmitted symbol can be stated as determining which one of the L possible signals in our dictionary was transmitted. This is equivalent to selecting between L hypotheses:

$$H_i : \mathbf{y} = e^{i\theta}\mathbf{\Phi}\mathbf{\Psi}\mathbf{u}_i + \mathbf{n}_e \tag{9}$$

for $i = 1, \ldots, L$ where $\mathbf{u}_i$ denotes the indicator vector corresponding to the ith signal. In this work, we select the hypothesis that yields the maximum a posteriori probability given by

$$\max_i Pr(\mathbf{u}_i|\mathbf{y}) \tag{10}$$

While we forgo the detailed derivation here, it can be shown that Equation (10) can be restated to yield

$$\max_i (e^{-(\mathbf{\Phi}\mathbf{\Psi}\mathbf{u}_i)^H \mathbf{E}_e^{-1}(\mathbf{\Phi}\mathbf{\Psi}\mathbf{u}_i)} I_0[2|\mathbf{y}^H\mathbf{E}_e^{-1}\mathbf{\Phi}\mathbf{\Psi}\mathbf{u}_i|]) \tag{11}$$

under the assumption of complex Gaussian i.i.d. noise $\mathbf{n}_e$ and uniformly distributed $\theta \in [0, 2\pi]$. In Equation (11), $\mathbf{E}_e$ denotes the covariance matrix of $\mathbf{n}_e$ and $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind.

5. SENSING MATRIX DESIGN

While most CS literature concentrates on random measurements, recent works illustrate that it may be advantageous to incorporate prior information into sensing matrix design.[3] Our goal in this work is to classify the transmitted signal into one of L classes. For this task, the sensing matrix should be designed to maximize class separation. Class separation for $L > 2$ can be defined in a variety of ways which will lead to different methods. Here, we present a simple method motivated by Linear Discriminant Analysis (LDA).[7] Let us start by defining an L-class scatter matrix as

$$\mathbf{R} = \sum_{k=1}^{L-1} \sum_{l=k+1}^{L} w_{k,l}(\mathbf{s}_k - \mathbf{s}_l)(\mathbf{s}_k - \mathbf{s}_l)^H \tag{12}$$

where $\mathbf{s}_k$ and $\mathbf{s}_l$ are the signals from the k^{th} and l^{th} class, respectively; and $w_{k,l}$ is a weight parameter that accounts for the relative importance of discriminating between classes k and l. The weights $w_{k,l}$ can be selected in a variety of ways. One possible approach is to set $w_{k,l} = p_k p_l$ where p_i denotes the *a priori* probability of class i.

We propose to use the principal eigenvector of the scatter matrix $\mathbf{R}$ in the design of the measurement matrix $\mathbf{\Phi}$. The principal eigenvector of the scatter matrix $\mathbf{R}$ is used as the non-zero entries of the block diagonal sensing matrix $\mathbf{\Phi}$ as illustrated in Figure 2. Our experimental results presented in the next section illustrate the advantages of this approach compared to random projections that are common in CS literature.

6. EXPERIMENTS

In this section, we present results of experiments that were conducted to evaluate the performances of the proposed techniques. Signals were generated according to the Bluetooth model described in Section 3. An unknown random phase uniformly distributed between 0 to 2π was added to the Bluetooth signals. Additive white Gaussian noise was then added independently to the real and imaginary parts of the signal prior to the measurement process. The standard deviation of the noise was adjusted to simulate varying channel Signal-to-Noise Ratios (SNRs). The effects of quantization noise $\mathbf{n}_s$ was ignored since this noise is very small compared to the channel noise $\mathbf{n}_r$ at the channel SNR levels used in these experiments. The noisy signal was sensed using different methods. First, traditional Nyquist sampling at the full Nyquist rate (i.e. no compression) was used. The transmitted symbol was then detected using conventional matched filtering of the Nyquist sampled signal. The performance of this method is used as a benchmark for the compressive techniques. The probability of symbol detection error was selected as the performance metric. Two CS methods were tested and compared. The first compressive method uses conventional CS. In this method, the non-zero entries of the sensing matrix were generated using complex Gaussian i.i.d. random numbers with zero mean and unit variance. Each row of the sensing matrix was then normalized to unit 2-norm. This method is referred to as the random CS method. The second compressive method uses sensing matrices designed by the approach described in Section 5. During the design, the scatter matrix was created with each class being equally likely (i.e. $w_{k,l}$ constant). This method is referred to as the designed CS method. The compressive measurements from both methods were used with the compressive classification method (as discussed in Section 4) to detect the transmitted symbols. The CS techniques were compared at various compression ratios (CRs) where CR is defined as the ratio of the number of measurements in the Nyquist-sampled signal to the number of compressive measurements. Compression ratios of 5, 10, and 15 were used in the experiments. Monte Carlo simulations using 400 random hops with 625 symbols per hop were used to test each case.

Plots of symbol detection error rate vs SNR at $CR = 5, 10$, and 15 are presented in Figure 6. These plots were obtained using a dictionary of size $L = 2^3 \times 79$ to account for the ISI. Several observations can be made based on these results: First the trade-off between sensor complexity and performance as defined by symbol detection error rate is apparent in these plots. Sensor complexity can be reduced by increasing the CR since fewer measurements are needed at higher CRs. However, this reduced complexity comes at the expense of increased symbol detection error rate. This observation holds for both CS methods. The results in Figure 6 also indicate that the designed CS can significantly outperform the random CS in this application. In fact, for CR=15, the symbol detection error rate can be lowered by more than two orders of magnitude at high SNRs using designed CS. Equivalently, the designed CS can achieve the same symbol detection error rate as random CS while operating at more than 5 dB lower SNR.

7. CONCLUSIONS

In this paper, a CS framework for interception of FHSS signals without the knowledge of the hopping sequence is presented. The traditional approach for such interception necessitates ADCs operating at very high rates. Using the fact that the FHSS signals have sparse instantaneous spectra, we propose CS strategies for this problem. Methods for compressive classification and sensing matrix design are proposed. These techniques are validated using GFSK modulated FHSS signals as defined by the Bluetooth specification. Experimental results suggest that the proposed designed sensing matrices can yield greater than two orders of magnitude decrease in probability of detection error under certain conditions.

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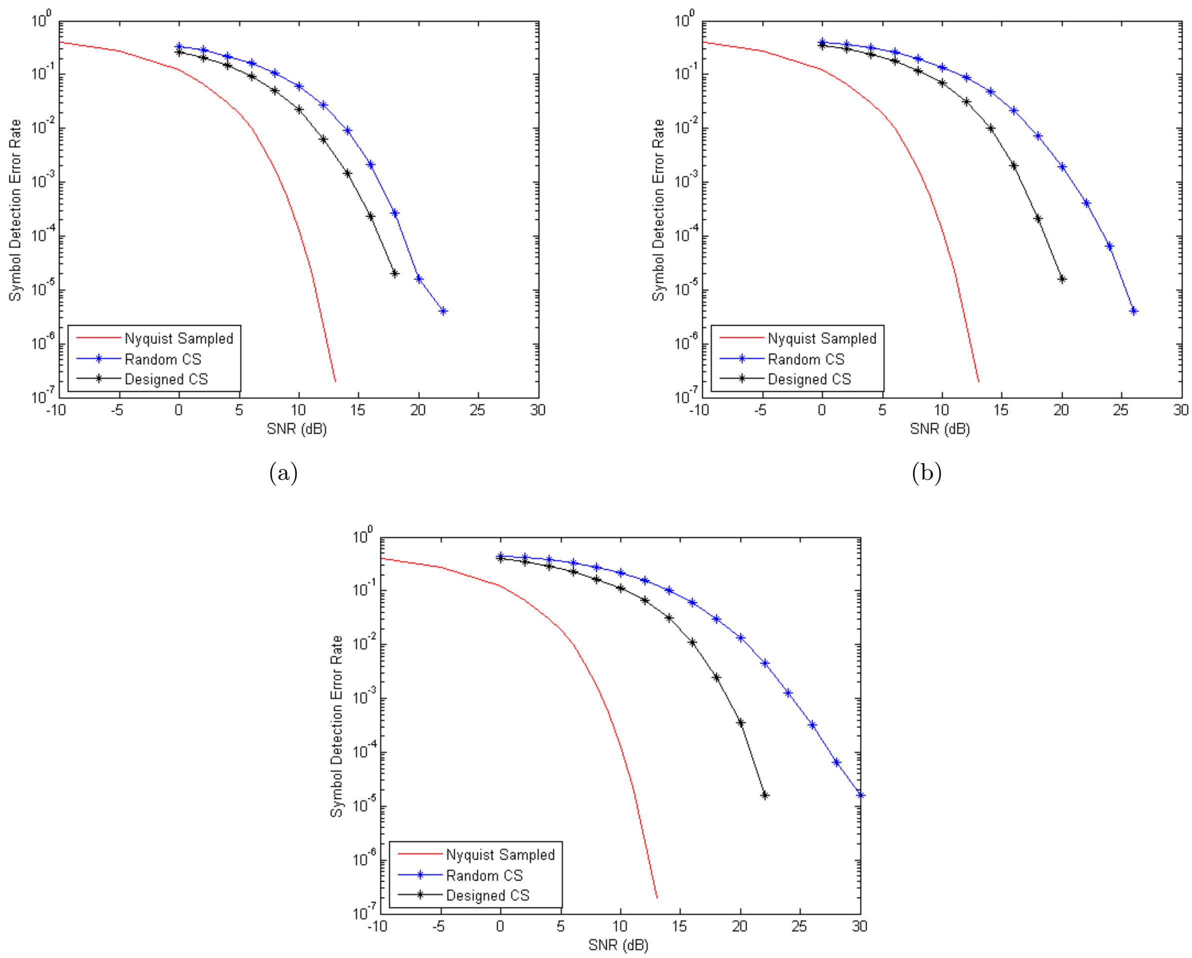

(a) (b)

(c)

Figure 3. Symbol Detection Error Rate vs. SNR for Different Methods. (a) CR=5 (b) CR=10 (c) CR=15

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