Capacity Approaching Analog Fountain Codes

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***Abstract*—In this paper, we propose an analog fountain code (AFC) to approach the capacity of the Gaussian channel in a wide range of signal to noise ratios (SNRs). The proposed code is rateless as the number of the generated coded symbols is potentially limitless; thus, enabling the transmitter to automatically adapt to the channel condition and sending as many coded symbols as required by the destination. Each coded symbol in AFC is directly generated from information symbols, by linearly combining them with real weighting coefficients. Weight coefficients and the degree of each coded symbol are chosen from predetermined weight set and degree distribution function, respectively. We further formulate an optimization problem to find the optimum weight set in order to maximize the efficiency of the proposed code. Simulation results show that the proposed code can approach the capacity of the Gaussian channel across a wide range of SNR values.**

***Index Terms*—Analog fountain code, rateless codes, Gaussian channels, belief propagation.**

I. Introduction

One of the fundamental problems in wireless communication is to effectively increase the throughput of wireless transmission in time-varying channels. Generally, transmission of signals over wireless links faces severe channel impairment, including noise, fading, pathloss and interference, which are generally varying during the time. Therefore, to achieve a high throughput, the communication system needs to adapt well to channel variations and performing well in all channel conditions.

To enable such a transmission strategy, several schemes have been recently proposed in the literature. In Adaptive Modulation and Coding (AMC) scheme, a large number of physical layer configurations, such as channel codes with different code rates, signal constellations, and bit-to-symbols mapping strategies, are used in both transmitter and receiver sides. The transmitter then selects the best configuration according to the feedback of real-time channel conditions obtained from the receiver. For instance, the high-throughput mode of IEEE 802.11n uses an AMC scheme based on LDPC codes, where 16 modulation and coding schemes (MCS) with 4 code rates and 4 different modulation types are used [1]. However, in this approach, the transmitter needs to select between these large number of configurations and also a real-time feedback from the receiver is required, which dramatically increase the overall system complexity. Furthermore, in cases of rapid or unpredictable channel variations, the transmitter cannot precisely follow the channel variation, leading to a significant performance loss in the wireless network.

To overcome these problems, several adaptive systems based on rateless codes have been proposed [1–4], where the transmitter can effectively adapt to the channel condition without knowledge of channel statistics. Due to the random nature of rateless codes, an infinite number of coded symbols can be potentially generated; thus, the transmitter can send as many coded symbols as required by the destination. Although rateless codes were originally designed and optimized for erasure channels [5, 6], they have been recently extended to approach the capacity of the wireless channels. In [1], rateless spinal codes have been proposed, which use an approximate maximum-likelihood (ML) decoding algorithm to achieve the Shannon capacity of both BSC and Gaussian channels. However, the complexity of this decoding algorithm is polynomial in the size of message bits, and is still exponential in block-size [7], which makes it very complex in large block sizes. In [3], a layered approach was proposed to use fixed-rate channel codes, like LDPC, and generate transmission symbols in a rateless fashion. It is shown that by increasing the number of layers, the channel capacity can be achieved by the proposed rateless code. However, the practical deployment of this method requires further research. Despite the good performance, the encoding and decoding complexity of such codes are prohibitively high and thus impractical for real applications.

In this paper, we propose a new type of fountain codes, referred to as the analog fountain code (AFC) based on the original work in [8]. AFC is mainly characterized by a message length, a degree distribution, and a weight set. To generate each coded symbol, information symbols are selected uniformly and randomly according to the degree distribution function, and then linearly combined in real domain to generate a coded symbol, where the weight coefficients are chosen from the weight set. To minimize the error floor, which is defined as the minimum achievable bit error rate for a given number of coded symbols, we modify the AFC encoding process in a way that the minimum variable node degree is maximized and further we precode the entire data before AFC encoding. We analyse the decoding error probability for the proposed AFC code, and further formulate an optimization problem to minimize the error probability and keep the output signal distribution as close as possible to the Gaussian distribution. Simulation results show that the AFC code can approach the capacity of the Gaussian channel in a wide range of SNR values with linear encoding and decoding complexity in terms of the number of information symbols.

978-1-4799-3354-9/14/$31.00

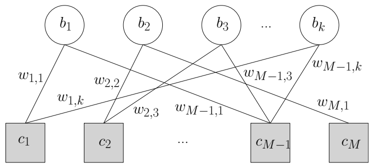


Fig. 1. Weighted bipartite graph of the analog fountain code. Circles and squares show information and coded symbols, respectively. Each edge is assigned with a weight coefficient which is randomly chosen from a weight set.

The rest of the paper is organized as follows. In Section II, we present analog fountain codes. The belief propagation decoding algorithm is then presented in Section III. In Section IV, the error probability analysis of the AFC code is presented. We formulate an optimization problem to find the optimum weight set for the AFC code in Section V. Simulation results are shown in Section VI, followed by concluding remarks in Section VII.

II. Analog Fountain Codes

Let us assume that the transmitter has k information bits to be delivered at the destination. The entire information sequence is first BPSK modulated, where 0 and 1 are mapped to -1 and $+1$, respectively, to obtain k information symbols, $b_i \in \{-1, 1\}$, for $i = 1, ..., k$. Like binary rateless codes [5, 6], to generate an AFC coded symbol, an integer d, called degree, is first obtained based on a predefined probability distribution function, called *degree distribution*. In the next step, d different information symbols are selected uniformly at random and linearly combined in real domain with real weighting coefficients chosen from a predefined *weight set*, to generate a coded symbol. By considering information and coded symbols as variable and check nodes, respectively, the encoding process of AFC can be described by a weighted bipartite graph as in Fig. 1.

Let Ω_d be the probability that the degree of a coded symbol is d, then the degree distribution can be described by its generator polynomial $\Omega(x) = \sum_{d=1}^{D} \Omega_d x^d$, where D is its maximum degree. We further assume the weight set $\mathcal{W}_s = \{a_1, a_2, ..., a_D\}$, where $a_i \in \mathbb{R}^+$ and $\mathbb{R}^+$ is the set of positive real numbers. A coded symbol c_j of degree d can then be calculated as follows:

$$c_j = \sum_{i=1}^{k} g_{j,i} b_i, \tag{1}$$

where only d of $g_{j,i}$'s are non-zero and each nonzero $g_{j,i}$ is chosen from $\mathcal{W}_s$. Let N be the number of AFC coded symbols transmitted by the source and $\mathbf{G}$ is a N by k matrix where $g_{j,i}$ is the i^{th} entry in the j^{th} row of $\mathbf{G}$, then (1) can be further written in a matrix form as follows,

$$\mathbf{C} = \mathbf{G}\mathbf{b}^t, \tag{2}$$

where $\mathbf{b}^t$ is the transpose of $\mathbf{b}$. The number of non-zero entries of each row in matrix $\mathbf{G}$ is determined by the degree distribution $\Omega(x)$, and each non-zero entry of matrix $\mathbf{G}$ is independently selected from the set $\mathcal{W}_s$.

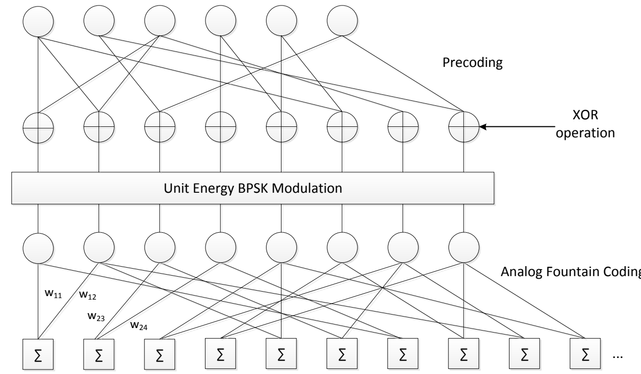


Fig. 2. The AFC encoding process by using a precoder.

Since information symbols are selected uniformly at random, the variable-node degree distribution becomes asymptotically Poisson $v(x) = \exp(\alpha(x-1))$ [6], where $\alpha = N\mu/k$ and k and m are the number of information and coded symbols, respectively, that are both very large values and μ is the average code degree. Accordingly, the probability that a given variable node is not connected to any coded symbol is $e^{-\alpha}$; thus, this code cannot achieve a decoder error probability lower than $e^{-\alpha}$ (error floor). To ensure that all information symbols are connected and also to maximize the average variable node degree, similar to [9], we modify the encoder structure of AFC as follows. To generate a coded symbol of degree d, d information symbols are randomly selected among those with the smallest degrees. As a result, each variable node has either degree d_v or $d_v - 1$, where d_v is the smallest integer larger than or equal to α. In [6], it has been proposed to precode the data before applying the AFC code in order to minimize the error floor. We also use this approach to minimize the error floor by precoding the data using a high rate LDPC code. The overall code structure, including the precoder and AFC code is shown in Fig. 2.

III. Belief Propagation Decoding of AFC

The decoding process of the proposed analog fountain code contains two steps. In the first step, the AFC part of the code will be decoded and the soft information of LDPC coded symbols will be calculated and fed to the LDPC decoder in the second step. Although these steps can be jointly performed [6], in this section, we only focus on AFC decoding as the LDPC decoding can be performed separately using the standard decoding algorithms. We use the belief propagation (BP) decoding algorithm which was originally proposed in [10] for sparse signal recovery in compressive sensing and was further modified for binary input signals in [11]. In this decoder, the messages which are conditional probabilities are passed from variable to check nodes and vise versa.

More specifically, as each variable node can be either -1 or $+1$, in each iteration of the BP decoder we calculate the probability that a variable node equals to -1 or $+1$ given a received coded symbol and the known weight coefficients. Let us consider the AFC code shown in Fig. 1 and let $q_{ji}^{(\ell)}(-1)$ and $q_{ji}^{(\ell)}(1)$ denote the messages that are passed from variable node j to check node i in the ℓ^{th} iteration of the BP algorithm. Similarly, the messages which are sent back from check node i to variable node j in the ℓ^{th} iteration of the BP decoder are denoted by $m_{ij}^{(\ell)}(-1)$ and $m_{ij}^{(\ell)}(1)$. In the ℓ^{th} iteration of the BP decoder, message $m_{ij}^{(\ell)}(n)$ is calculated as follows for $n \in \{-1, 1\}$:

$$m_{ij}^{(\ell)}(n) = p(v_j = n|c_i) = p\left(\sum_{j' \in \mathcal{N}(i)\backslash j} v_{j'} w_{i,j'} = c_i - n w_{i,j}\right),$$

where $\mathcal{N}(i)\backslash j$ is the set of all neighbors of check node i except the variable node j. Also, the message $q_{ji}^{(\ell)}(n)$ is calculated as follows.

$$q_{ji}^{(\ell)}(n) = \frac{C_{ji}}{2} \prod_{i' \in \mathcal{M}(j)\backslash i} m_{i'j}^{(\ell)}(n), \ n \in \{0, 1\}$$

where $\mathcal{M}(j)\backslash i$ is the set of all neighbors of variable node j except check node i, and C_{ji} is chosen in a way that $q_{ji}^{(\ell)}(-1) + q_{ji}^{(\ell)}(1) = 1$. After a predefined number of iterations, called T, the final beliefs for variable nodes are obtained as follows:

$$p_j^{(T)}(n) = \frac{C_j}{2} \prod_{i \in \mathcal{M}(j)} m_{ij}^{(T)}(n),$$

where C_j is chosen in a way that $p_j^{(T)}(-1) + p_j^{(T)}(1) = 1$. Variable node j is then decided to be -1 if $p_j^{(T)}(-1) > p_j^{(T)}(1)$; otherwise, it is set to 1. Further details of this decoder can be found in [11].

IV. Error Probability of the AFC code

As stated before the AFC decoder's task is to compute the conditional probability of $\mathbf{b}$ given the entire received sequence $\mathbf{u}$, $p(\mathbf{b}|\mathbf{u}, \mathbf{G})$, with the knowledge of the random generator matrix $\mathbf{G}$, where $\mathbf{u} = \mathbf{G}\mathbf{b} + \mathbf{n}$, $\mathbf{b}$ is the sequence of information symbols, and $\mathbf{n}$ is the zero-mean additive white Gaussian noise vector with variance $\sigma^2\mathbf{I}$. Based on Bayes's rule, we have

$$p(\mathbf{b}|\mathbf{u}, \mathbf{G}) = \frac{p(\mathbf{u}|\mathbf{b}, \mathbf{G}) p(\mathbf{b})}{p(\mathbf{u}|\mathbf{G})},$$

where $p(\mathbf{u}|\mathbf{G}) = \sum_{\mathbf{b} \in \{-1,1\}^k} p(\mathbf{u}|\mathbf{b}, \mathbf{G}) p(\mathbf{b})$ is independent of the message symbols that are transmitted. Also, 2^k possible random vectors $\mathbf{b}$ are equally probable, i.e., $p(\mathbf{b}) = \frac{1}{2^k}$. Furthermore,

$$p(\mathbf{u}|\mathbf{b}, \mathbf{G}) = \prod_{i=1}^{m} (\frac{1}{\sqrt{2\pi\sigma^2}}) e^{-\frac{(u_i - \sum_{j=1}^{k} b_j g_{ij})^2}{2\sigma^2}}.$$

Thus, we have

$$\ln\, p(\mathbf{u}|\mathbf{b}, \mathbf{G}) = -\frac{M}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{M} (u_i - \sum_{j=1}^{k} b_j g_{ij})^2.$$

The maximization of $\ln\, p(\mathbf{u}|\mathbf{b}, \mathbf{G})$ over $\mathbf{b}$ is equivalent to finding the message $\mathbf{b}$ that minimizes the Euclidian distance

$$\hat{\mathbf{b}} = \arg\min_{\mathbf{b}} \sum_{i=1}^{m} (u_i - \sum_{j=1}^{k} b_j g_{ij})^2 = \arg\min_{\mathbf{b}} ||\mathbf{u} - \mathbf{G}\mathbf{b}||_2^2.$$

Let us assume that the message vector $\mathbf{b}$ has been transmitted and $\mathbf{b}'$ is different from $\mathbf{b}$ only in the first l places. Let $p_{l|\mathbf{G}}$ denote the probability that $\mathbf{b}'$ has lower Euclidian distance than that of $\mathbf{b}$ for a given $\mathbf{G}$, then $p_{l|\mathbf{G}}$ can be calculated as follows.

$$\begin{aligned} p_{l|\mathbf{G}} &= p\left(||\mathbf{u} - \mathbf{G}\mathbf{b}')||^2 < ||\mathbf{u} - \mathbf{G}\mathbf{b})||^2 \middle| \mathbf{G}\right) \\ &= p\left(\sum_{i=1}^{m} (n_i - \sum_{j=1}^{k} g_{ij}(b_j - b'_j))^2 < \sum_{i=1}^{m} n_i^2 \middle| \mathbf{G}\right) \\ &= p\left(\sum_{i=1}^{m} (n_i - \sum_{j=1}^{l} 2b_j g_{ij})^2 < \sum_{i=1}^{m} n_i^2 \middle| \mathbf{G}\right) \\ &= p\left(\sum_{i=1}^{m} \left(\sum_{j=1}^{l} 2b_j g_{ij}\right)^2 < 2\sum_{i=1}^{m} n_i \sum_{j=1}^{l} 2b_j g_{ij} \middle| \mathbf{G}\right). \end{aligned}$$

Since n_i's are identical and independent Gaussian random variables with mean zero and the variance σ_n^2, then $\sum_{i=1}^{m} n_i \sum_{j=1}^{l} 2b_j g_{ij}$ is also a Gaussian random variable with zero mean and variance $4\sigma_n^2 \sum_{i=1}^{m} (\sum_{j=1}^{l} b_j g_{ij})^2$. Therefore, $p_{l|\mathbf{G}}$ can be calculated as follows:

$$p_{l|\mathbf{G}} = Q\left(\frac{1}{\sigma_n} \sqrt{\sum_{i=1}^{m} \left(\sum_{j=1}^{l} b_j g_{ij}\right)^2}\right), \quad (3)$$

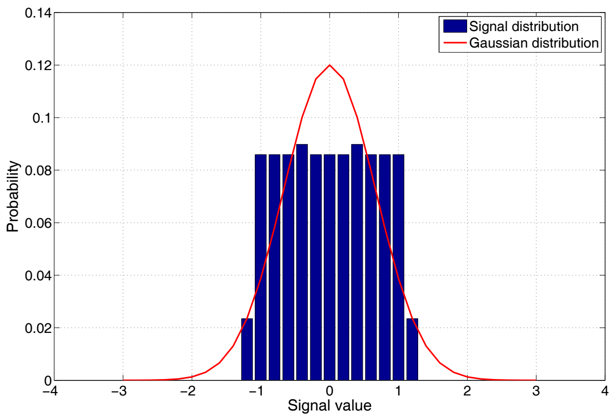
where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{x^2}{2}} dx$. Eq. (3) shows that if $\left(\sum_{j=1}^{l} b_j g_{ij}\right) \neq 0$, p_ℓ is a decreasing function of m, which means that by transmitting more coded symbols the error probability decreases.

V. Optimizing the Weight Set

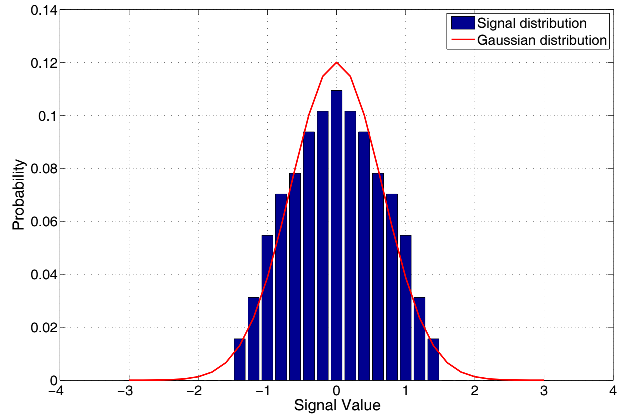
It is important to note that in high SNRs, the decoder's task is to actually solve a system of binary linear equations. To achieve a higher throughput in this case, each coded symbol should be able to uniquely recover all its adjacent information bits. This occurs if and only if the associated linear equation to each coded symbol has a unique solution. Generally, equation $\sum_{i=1}^{d+1} v_i w_i = u$ has a unique solution if exactly one of equations $\sum_{i=1}^{d} v_i w_i = u + w_{d+1}$ and $\sum_{i=1}^{d} v_i w_i = u - w_{d+1}$, has a unique solution; but, not both of them, where $v_i \in \{-1, 1\}$ for $i = 1, ..., d+1$. To make sure that each linear equation associated with each coded symbol has a unique solution, we need to restrict the weights to be among a specific set. Here we show that if the weights satisfy the following condition, each coded symbol can uniquely decode its neighboring information symbols:

$$\sum_{i=1}^{d} (-1)^{n_i} I_i w_i \neq 0, \quad (4)$$

where $n_i \in \{0, 1\}$ and $I_i \in \{0, 1\}$. Let us assume that equation $\sum_{i=1}^{d} v_i w_i = u$ has two different solutions $\mathbf{b}_1$ and $\mathbf{b}_2$, and the weight coefficients satisfy condition (4). It is clear that we have $\sum_{i=1}^{d} (b_{1,i} - b_{2,i}) w_i = 0$. As $b_i \in \{-1, 1\}$, then $(b_{1,i} - b_{2,i}) \in$



(a)



(b)

Fig. 3. Signal distribution for different weight sets. (a) $\mathcal{W}_s = \{1, 2, 4, 8, 16, 32, 64, 128\}$ and (b) $\mathcal{W}_s = \{1/2, 1/3, 1/5, 1/7, 1/11, 1/13, 1/17, 1/19\}$.

$\{-2, 0, 2\}$; thus, at least one of conditions (4) is not satisfied, which contradicts with the assumption. Therefore, as long as the weight coefficients satisfy 4), the binary linear equation associated with each coded symbol has a unique solution.

Moreover, when (4) is satisfied, $\{\left(\sum_{j=1}^{D} b_j g_{i,j}\right)^2\}$ is always larger than zero and so p_l monotonically decreases by increasing m. It is important to note that the proposed code in [11] is a special case of AFC, when input symbols are from set $\{0, 1\}$, each coded symbol has a fixed degree of 8 and the weight set is $\mathcal{W}_s = \{-4, -2, -1, 1, 2, 4\}$. More specifically, each row of the generator matrix has 8 nonzero elements in which 2 of them are equal to -4, two of them are equal to 4, and four others are -1,-2,1, and 2. In this case, the summation over each row of the generator matrix will be zero. Therefore, in Seamless codes [11], $\min_{\mathbf{b}}\{\left(\sum_{j=1}^{D} b_j g_{ij}\right)^2\} = 0$, which results in $p_{l|\mathbf{G}} = 0.5$. This means that even in the noiseless case, a coded symbol connected to some information symbols may not be able to recover its adjacent information symbols, leading to a poor performance in high SNRs.

We further optimize the wight in a way that the output signal distribution follows the Gaussian distribution. The reason behind this optimization is that to achieve the capacity of the Gaussian channel, the input signal distribution should be Gaussian [3]. For this aim, we consider the general form of AFC with degree distribution $\Omega(x)$ and then we find the probability distribution function of the coded symbols. More specifically, for coded symbol c_i of degree d we have:

$$p(c_i = u) = p\left(\sum_{l=1}^{d} w_{i_l} b_{i_l} = u\right), \tag{5}$$

where $B_i = \{b_{i_1}, b_{i_2}, ..., b_{i_d}\}$ is the set of information symbols that are connected to c_i, $\{w_{i_1}, w_{i_2}, ..., w_{i_d}\}$ is the set of weights that are associated with the edges between B_i and c_i, and $1 \le i_l \le k$ for $l \in \{1, 2, ..., d\}$. Since each information symbol is either -1 or 1 with the same probability of 0.5, and weights are chosen uniformly at random from $\mathcal{W}_s$, then $s_{i,l} \triangleq w_{i_l} b_{i_l}$ is uniformly distributed as follows.

$$p(s_{i,l} = v) = \frac{1}{2f}, \;\; v \in \{-a_f, ..., -a_1, a_1, ..., a_f\}. \tag{6}$$

Furthermore, the mean and variance of $s_{i,l}$ are respectively, $m_s = 0$ and $\sigma_s^2 = \frac{1}{f}\sum_{i=1}^{f}(a_i^2)$. Since $s_{i,l}$'s are identical and independent random variables, c_i has mean 0 and variance $d\sigma_s^2$. Then we optimize the weight coefficients in a way that (4) and the following condition are simultaneously satisfied, for the given $\epsilon > 0$ and $\delta > 0$:

$$|p_\delta^{(i)} - q_\delta^{(i)}|^2 \le \epsilon, \quad \text{for } i = 1, 2, ... \tag{7}$$

where $p_\delta^{(i)} = p\left((i-1)\delta \le \sum_{j=1}^{d} b_j w_j < i\delta\right)$ and $q_\delta^{(i)} = Q\left((i-1)\delta\right) - Q(i\delta)$. This optimization problem can be numerically solved for different values of δ and ϵ. Note that (7) ensures that the output signal distribution approaches the Gaussian distribution. For instance, for $\delta = 0.2$ and $\epsilon = 10^{-4}$, the weight set $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{11}, \frac{1}{13}, \frac{1}{17}, \frac{1}{19}\}$ satisfies both conditions (4) and (7). It is important to note that when d is relatively large, according to the central limit theorem, c_i has zero mean Gaussian distribution with variance $d\sigma_s^2$. However, by increasing the code degree, the complexity of the decoder significantly increases; thus, we only consider small degrees and later we show that by a small code degree ($d \approx 8$), we can achieve the capacity of the Gaussian channel in a wide range of SNRs. Fig. 3 shows the AFC coded symbol distribution for different weight sets. As can be seen in these figures, the weight set $\mathcal{W}_s = \{1, 2, 4, 8, 16, 32, 64, 128\}$ which satisfied condition (4) can produce equally spaced signal points which result in a uniform output signal distribution. However, the wight set $\mathcal{W}_s = \{1/2, 1/3, 1/5, 1/7, 1/11, 1/13, 1/17, 1/19\}$ can generate coded symbols with probability distribution very close to the Gaussian one, which can simultaneously satisfy condition (4). Note that for a fair comparison between two weight sets, we have normalized the two weight sets in Fig. 3 in a way that two weight sets have the same variance.

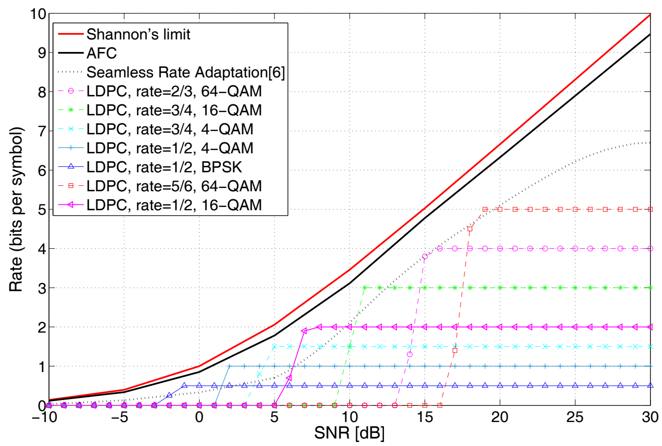


Fig. 4. Achievable rate of AFC versus SNR, when $k = 10000$, BER=10^{-4}, and each coded symbol of AFC has a fixed degree of 8. The LDPC codeword is of length 648 bits.

VI. Simulation Results

For simulation purposes, we consider the standard additive white Gaussian noise (AWGN) channels as $y = x+n$, where x and y are respectively the input and output signals and n is the additive white Gaussian noise. To fully utilize the constellation plane, each two consecutive AFC coded symbols compose one modulation signal by $c_i + \sqrt{-1}c_{i+1}$. We use a rate 0.95 LDPC code which has been originally proposed in [6] for precoding. We also assume that coded symbols have a fixed degree of 8 and the weight set is $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{11}, \frac{1}{13}, \frac{1}{17}, \frac{1}{19}\}$.

Fig. 4 shows the achievable rate of the proposed AFC code in the AWGN channel versus the SNR for BER equals to 10^{-4} and $k = 10000$. As can be seen in this figure, the proposed AFC code can closely approach the capacity of the Gaussian channel in a wide range of SNRs. More specifically, the AFC code achieves a throughput of 1.78 bits/symbols and 9.46 bits/symbols at SNR values 5 dB and 30 dB, respectively. We also shows the achievable rate of the seamless rate adaptation strategy [11] in Fig. 4, which can only achieves 0.7 bits/symbols and 6.7 bits/symbols at SNR values 5 dB and 30 dB, respectively. Fig. 4 also shows the performance of the LDPC code from the high-throughput mode of IEEE 802.11n with different code rate and modulation types. As can be seen in this figure, each LDPC code with a certain rate and modulation order can perform well only in a small range of SNRs. Generally, fixed rate codes and a fixed modulation scheme can be optimized for a specific SNR; thus, they are not optimal for other SNRs. Therefore, the transmitter requires a feedback from the destination to be able to adapt to the channel condition by accordingly changing the code rate and the modulation order, or even the bit-to-signal mapping strategy.

It is important to note that in the noise-less case, each coded symbol can uniquely decode its connected variable nodes; thus, for a AFC code with code degree d, the maximum achievable rate will be $2d$ as each coded symbol can uniquely decode at most d variable nodes. The maximum achievable rate of the AFC code in high SNRs can be increased by increasing the degree of coded symbols and also increasing the weight set size; however, the complexity of the decoder will be significantly increased in these cases. More specifically, to calculate each message, the destination requires to calculate 2^d different probabilities according to the different combinations of the variable node values. This means that the decoding complexity exponentially increases with the code degree. To avoid such a huge complexity, we only consider small code degrees, such as $d = 8$, which has been shown in Fig. 4 that can approach the capacity of the Gaussian channel in a wide range of SNRs.

VII. Conclusion

In this paper, we proposed an analog fountain code that can effectively adapt to the channel conditions without the knowledge of channel conditions. In the proposed code, each coded symbol is a linear combination of randomly selected information symbols with real weighting coefficients. The number of information symbols which are connected to a coded symbols is obtained based on a predefined degree distribution and the weight coefficients are randomly chosen from a finite weight set. We proposed an optimization problem to obtain the optimum weight set in order to minimize the decoding error probability of the AFC code. Simulation results show that the proposed scheme can achieve the Shannon's capacity of the Gaussian channel in a wide range of signal to noise ratios.

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