SSY285 - Home Assignment M1

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Introduction

The task of this assignment is to model a DC-motor with a flywheel and thereafter simulate the system with provided values. Also, the input-output stability of the system will be discussed as well as if the system is minimum phase for different cases of outputs.

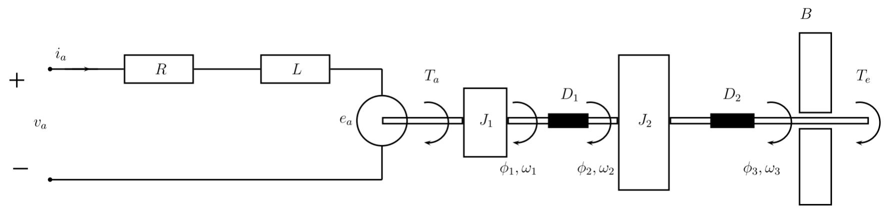

Figure 1: DC-motor with flywheel.

1 Question a)

In order to describe the system completely six linear differential equations of order 1 are required.

$$
\begin{aligned}
J_1\dot{\omega}_1 &= K_t i_a - D_1(\phi_1 - \phi_2) \\
J_2\dot{\omega}_2 &= D_1(\phi_1 - \phi_2) - D_2(\phi_2 - \phi_3) \\
B\omega_3 &= T_e + D_2(\phi_2 - \phi_3) \\
T_a &= K_t i_a \\
e_a &= K_e\omega_1 \\
V_a &= i_a R_a + L\dot{i}_a + e_a
\end{aligned}
$$

The inputs for the systems are the voltage v_a and the external torque T_e. The rest of the parameter describe the rotor current i_a, the induced rotor voltage e_a, the rotor produced torque T_a, the angles ϕ_1, ϕ_2, ϕ_3 and angular speeds ω_1, ω_2, and ω_3. L describes the rotor inductance and R the rotor resistance, while K_e is the coefficient related to e_a. K_t is the rotor torque constant, J_1 is the rotor inertia, and J_2 is the flywheel inertia. $D_1 and D_2$ represent torsional springs on each side of the flywheel axis and B denotes linear friction proportional to the angular speed. Note that this is not the same as the B matrix later on referenced in the state space model.

The external torque T_e is in figure 1 given by the assignment applied in the same direction as the rotor produced torque T_a, however will be later simulated using a negative value as it is interpreted to be a load torque.

With these linear differential equations, it is possible to extract the A and B matrix by solving for all the time differentiated variables.

$$
\dot{x}(t) = \underbrace{\begin{bmatrix} -\frac{R_a}{L} & 0 & 0 & 0 & -\frac{K_e}{L} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{D_2}{B} & -\frac{D_2}{B} & 0 & 0 \\ \frac{K_T}{J_1} & -\frac{D_1}{J_1} & \frac{D_1}{J_1} & 0 & 0 & 0 \\ 0 & \frac{D_1}{J_2} & \frac{-D_1-D_2}{J_2} & \frac{D_2}{J_2} & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_a \\ \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} \frac{1}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{B} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} v_a \\ T_e \end{bmatrix}}_{u(t)} \tag{1}
$$

2 Question b)

Assuming the inductance L very small (≈ 0) this makes i an algebraic variable (since the time-derivative of i disappears) and is therefore no longer a state variable. Therefore the variables that depended on i have to be expressed in the other variables.

$$i = \frac{V_a - K_E \times \omega_1}{R} \tag{2}$$

$$x(t) = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \omega_1 & \omega_2 \end{bmatrix}^T \tag{3}$$

By using the same equations as in Question 1 but with $L = 0$ the following system is derived:

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{D_2}{B} & \frac{-D_2}{B} & 0 & 0 \\ \frac{-D_1}{J_1} & \frac{D_1}{J_1} & 0 & \frac{-K_E K_T}{RJ_1} & 0 \\ \frac{D_1}{J_2} & \frac{-(D_2+D_1)}{J_2} & \frac{D2}{J2} & 0 & 0 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{B} \\ \frac{K_T}{RJ_1} & 0 \\ 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} v_a \\ T_e \end{bmatrix}}_{u(t)} \tag{4}$$

Note here that the B-matrix is not the same as the variable B, which represents the dynamic friction given in the system in the assignment.

3 Question c)

By modelling the output $y(t)$ as:

$$y(t) = Cx(t) + Du(t) \tag{5}$$

and considering the output as (1) $y_1(t) = \Phi_2,\ y_2(t) = \omega_2$ and (2) $y_1(t) = i_a,\ y_2(t) = \omega_3$ the C- and D-matrixes will be defined as:

Case (1):

$$y = \begin{bmatrix} \Phi_2 \\ \omega_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} u(t) \tag{6}$$

Case (2):

$$y = \begin{bmatrix} i_a \\ \omega_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \frac{-K_e}{R} & 0 \\ 0 & \frac{D_2}{B} & \frac{-D_2}{B} & 0 & 0 \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} \frac{1}{R} & 0 \\ 0 & \frac{1}{B} \end{bmatrix}}_{D} u(t) \tag{7}$$

4 Question d)

By extracting and looking the eigenvalues from matrix A in 4 it is evident that the system is input-output stable for all cases since all eigenvalues lie in the left half plane (LHP) and the system is linear and time-invariant. The eigenvalues can be seen in Eq. 8.

$$eig(A) = 1 \times 10^3 \begin{bmatrix} -0.3914 + 1.4791i \\ -0.3914 - 1.4791i \\ 0 \\ -0.2708 \\ -0.9464 \end{bmatrix} \tag{8}$$

Since finding the eigenvalues only requires matrix A the system is input-output stable for both output cases.

5 Question e)

Using the matrices that was calculated in question b) the system was simulated in Simulink, using both the state-space block and also a system built using subsystems (which can be seen in Fig. 2).

By using a modified C matrix it is possible to look at all the different angular velocities. With a zero voltage it is observable from Fig. 3 that no velocities are present at the start. When the voltage of 10 is applied to the system all of the velocities are increasing up to their working point. Notice how ω_1 is quite volatile when the input is applied as it is close to the motor and the dampening from the torsional spring is not quite fast enough. ω_2 and ω_3 on the other hand is slower in the beginning. When the external torque is applied it is observable that it is more or less only ω_3 that gets slowed down. The angles, observed in Fig. 4, are not bounded and therefore increasing linearly.

The current i is heavily dependant on ω_1 and the current. When the voltage is applied the current races to 10 A since the resistance is just 1 . When the angular speed increases the drawn current is decreased.

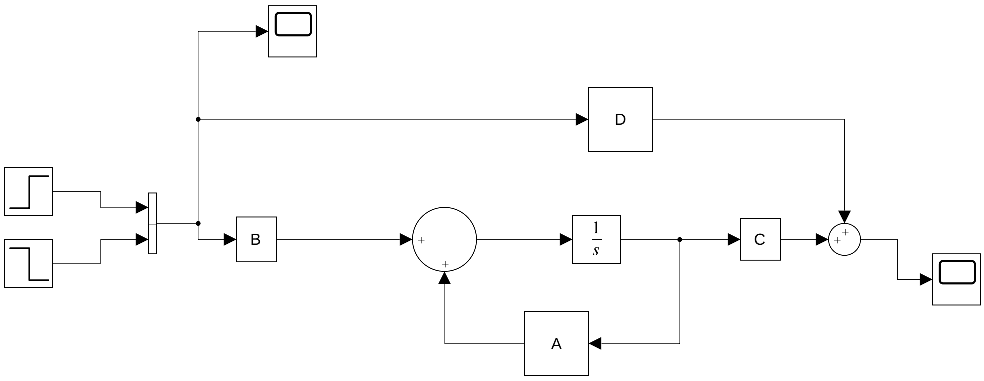

Figure 2: Simulink Block Diagram

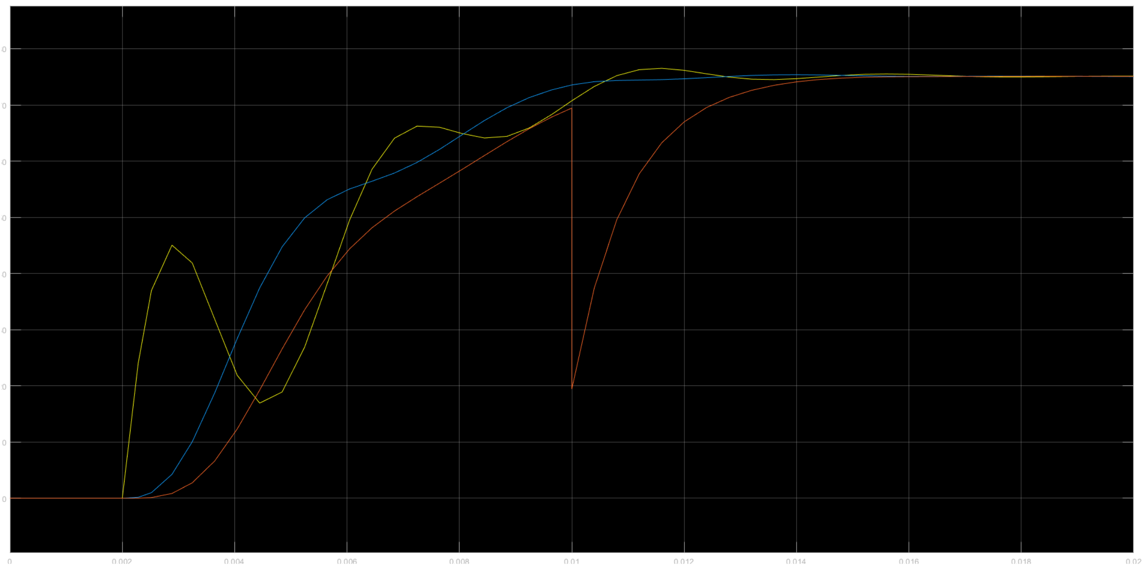

Figure 3: Step response of ω_1, ω_2, and ω_3. The load torque is applied at t = 0.05 s.

Figure 4: The angles of ϕ_1, ϕ_2 and ϕ_3. The load torque is applied at t = 0.05 s.

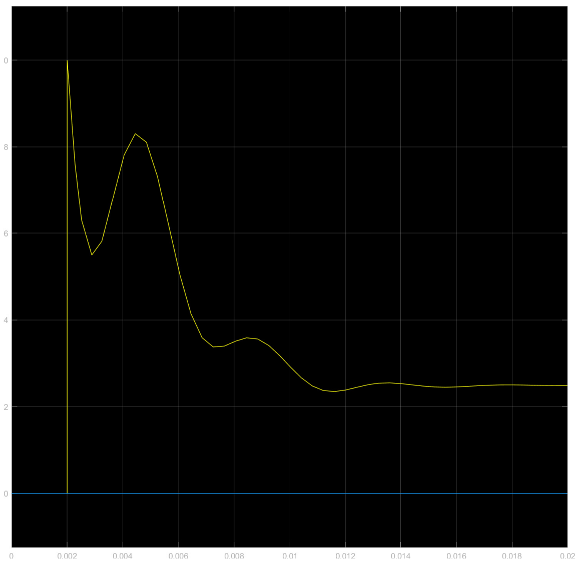

Figure 5: Graph of the current. Voltage is applied at t = 0.002 s.

6 Question f)

To get the transfer function from the state space model 9 can be used.

$$G(s) = C(sI - A)^{-1}B + D \tag{9}$$

Through Matlab's control system toolbox the solved transfer function is found:

From input 1 to output 1:

$$\frac{s(s+955.3)(s+41.75)(s^2+2.92s+2.507e06)}{s(s+946.4)(s+270.8)(s^2+782.8s+2.341e06)} \tag{10}$$

From input 1 to output 2:

$$\frac{5e12s^2(s+946.4)(s+270.8)(s^2+782.8s+2.341e06)}{s^2(s+946.4)^2(s+270.8)^2(s^2+782.8s+2.341e06)^2} \tag{11}$$

The poles are:

$$1 \times 10^3 \begin{bmatrix} -0.3914 + 1.4791i \\ -0.3914 - 1.4791i \\ -0.0000 + 0.0000i \\ -0.2708 + 0.0000i \\ -0.9464 + 0.0000i \end{bmatrix} \tag{12}$$

The transmission zeroes are:

$$1 \times 10^3 \begin{bmatrix} 0.0000 + 1.5843i \\ 0.0000 - 1.5843i \\ 0.0000 + 0.1996i \\ 0.0000 - 0.1996i \\ 0.0000 + 0.0000i \end{bmatrix} \tag{13}$$

Because all the the poles are in the LHP (with one on the zero) the system can be viewed as a minimum phase system.