SSY285 - Home Assignment M1

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Introduction

The task of this assignment is to model a DC-motor with a flywheel and thereafter simulate the system with provided values. Also, the input-output stability of the system will be discussed as well as if the system is minimum phase for different cases of outputs.

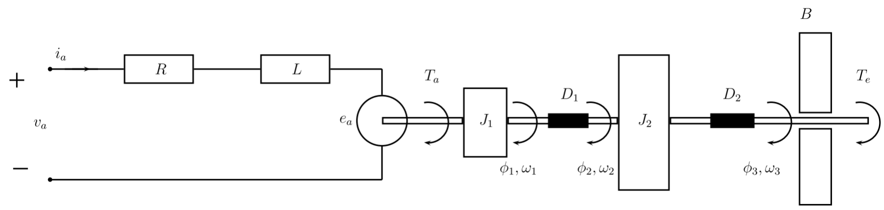

Figure 1: DC-motor with flywheel.

1 Question a)

In order to describe the system completely six linear differential equations of order 1 are required.

$$
\begin{aligned}
J_1\dot{\omega}_1 &= K_t i_a - D_1(\phi_1 - \phi_2) \\
J_2\dot{\omega}_2 &= D_1(\phi_1 - \phi_2) - D_2(\phi_2 - \phi_3) \\
B\omega_3 &= T_e + D_2(\phi_2 - \phi_3) \\
T_a &= K_t i_a \\
e_a &= K_e \omega_1 \\
V_a &= i_a R_a + L\dot{i}_a + e_a
\end{aligned}
$$

The inputs for the systems are the voltage v_a and the external torque T_e is applied to the system.

With these linear differential equations, it is possible to extract the A and B matrix by solving for all the time differentiated variables.

$$
\dot{x}(t) = \underbrace{\begin{bmatrix} -\frac{R_a}{L} & 0 & 0 & 0 & -\frac{K_e}{L} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{D_2}{B} & -\frac{D_2}{B} & 0 & 0 \\ \frac{K_T}{J_1} & -\frac{D_1}{J_1} & \frac{D_1}{J_1} & 0 & 0 & 0 \\ 0 & \frac{D_1}{J_2} & \frac{-D_1-D_2}{J_2} & \frac{D_2}{J_2} & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_a \\ \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} \frac{1}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{B} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} v_a \\ T_e \end{bmatrix}}_{u(t)} \tag{1}
$$

2 Question b)

When assuming the inductance L very small (≈ 0). This makes i an algebraic variable (since the time-derivative of i disappears) and is therefore no longer a state variable. Therefore the variables that depended on i have to be expressed in the other variables.

$$
i = \frac{V_a - K_E \times \omega_1}{R} \tag{2}
$$

$$
x(t) = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \omega_1 & \omega_2 \end{bmatrix}^T \tag{3}
$$

By using the same equations as in Question 1 but with $L = 0$ the following system is derived:

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{D_2}{B} & \frac{-D_2}{B} & 0 & 0 \\ \frac{-D_1}{J_1} & \frac{D_1}{J_1} & 0 & \frac{-K_e}{R}\frac{K_T}{J_1} & 0 \\ \frac{D_1}{J_2} & \frac{-(D_2+D_1)}{J_2} & \frac{D2}{J2} & 0 & 0 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{B} \\ \frac{K_T}{RJ_1} & 0 \\ 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} v_a \\ T_e \end{bmatrix}}_{u(t)} \tag{4}$$

Note here that the B-matrix is not the same as the variable B, which represents the dynamic friction given in the system in the assignment.

3 Question c)

By modelling the output $y(t)$ as:

$$y(t) = Cx(t) + Du(t) \tag{5}$$

and considering the output as (1) $y_1(t) = \Phi_2,\ y_2(t) = \omega_2$ and (2) $y_1(t) = i_a,\ y_2(t) = \omega_3$ the C- and D-matrixes will be defined as:

Case (1):

$$y = \begin{bmatrix} \Phi_2 \\ \omega_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} u(t) \tag{6}$$

Case (2):

$$y = \begin{bmatrix} i_a \\ \omega_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \frac{-K_e}{R} & 0 \\ 0 & \frac{D_2}{B} & \frac{-D_2}{B} & 0 & 0 \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} \frac{1}{R} & 0 \\ 0 & \frac{1}{B} \end{bmatrix}}_{D} u(t) \tag{7}$$

4 Question d)

By extracting and looking the eigenvalues from matrix A in 4 it is evident that the system is input-output stable for all cases since all eigenvalues lie in the left half plane (LHP) and the system is linear and time-invariant. The eigenvalues can be seen in Eq. 8.

$$eig(A) = 1 \times 10^3 \begin{bmatrix} -0.3914 + 1.4791i \\ -0.3914 - 1.4791i \\ 0 \\ -0.2708 \\ -0.9464 \end{bmatrix} \tag{8}$$

Since finding the eigenvalues only requires matrix A the system is input-output stable for both output cases.

5 Question e)

Using the matrices that was calculated in question b the system was simulated in Simulink. Using both the state-space block and also a system built using subsystems (which can be seen in Fig. 2).

By using a modified C matrix it is able to look at all the different angular velocities. With a zero voltage it is observable from Fig. 3 that no velocities are present at the start. When the voltage is applied to 10 all of the velocities are working up to their working point. Notice how ω_1 is quite volatile when the input is applied as it is close to the motor and the dampening from the torsional spring is not quite fast enough. ω_2 and ω_3 on the other hand is slower in the beginning. When the external torque is applied it is observable that it is more or less only ω_3 that gets slowed down.

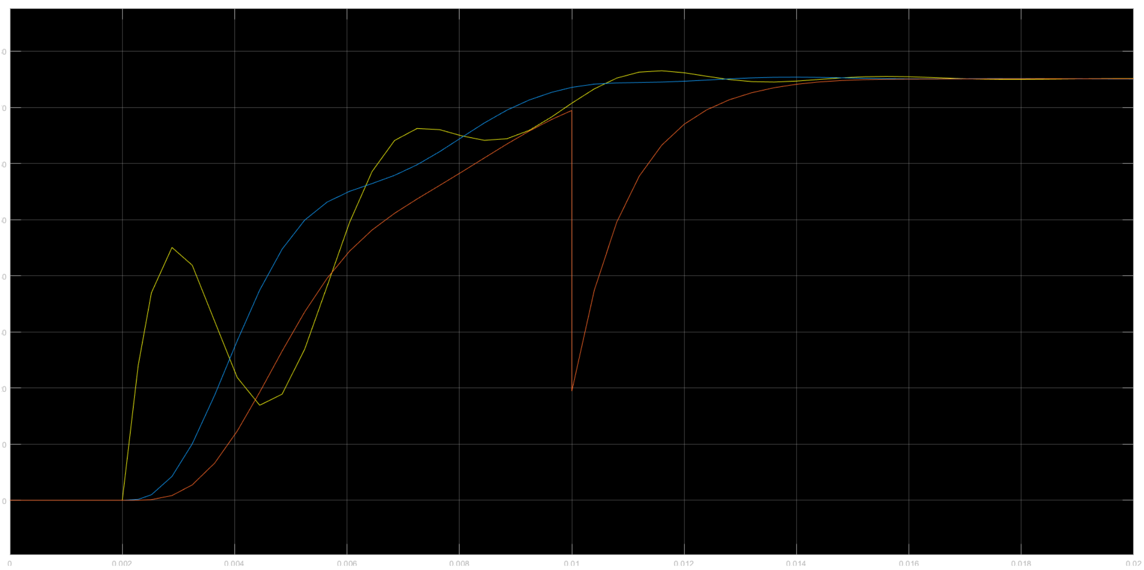

Figure 3: Step response of ω_1, ω_2, and ω_3. The load torque is applied at t = 0.05 s.

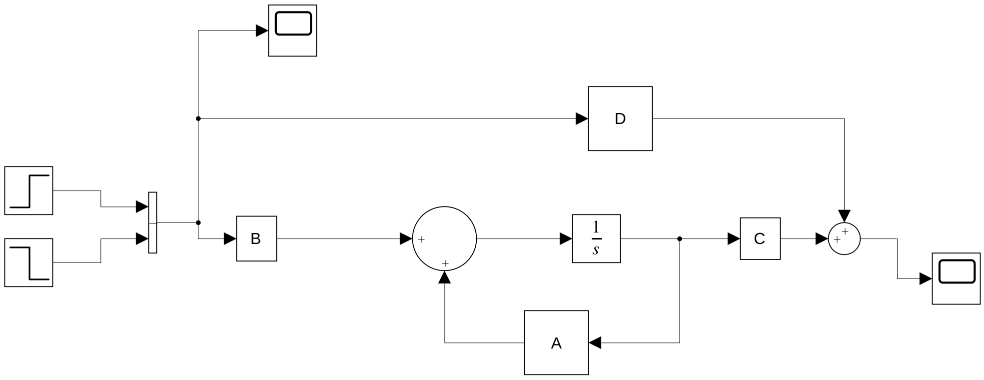

Figure 2: Simulink Block Diagram

6 Question f)

To get the transfer function from the state space model 9 can be used.

$$G(s) = C(sI - A)^{-1}B + D \tag{9}$$

Through matlab's control system toolbox the solved transfer function is found:

From input 1 to output 1:

$$\frac{s(s + 955.3)(s + 41.75)(s^2 + 2.92s + 2.507e06)}{s(s + 946.4)(s + 270.8)(s^2 + 782.8s + 2.341e06)} \tag{10}$$

From input 1 to output 2:

$$\frac{5e12s^2(s + 946.4)(s + 270.8)(s^2 + 782.8s + 2.341e06)}{s^2(s + 946.4)^2(s + 270.8)^2(s^2 + 782.8s + 2.341e06)^2} \tag{11}$$

The poles are:

$$1 \times 10^3 \begin{bmatrix} -0.3914 + 1.4791i \\ -0.3914 - 1.4791i \\ -0.0000 + 0.0000i \\ -0.2708 + 0.0000i \\ -0.9464 + 0.0000i \end{bmatrix} \tag{12}$$

The transmission zeroes are:

$$1 \times 10^3 \begin{bmatrix} 0.0000 + 1.5843i \\ 0.0000 - 1.5843i \\ 0.0000 + 0.1996i \\ 0.0000 - 0.1996i \\ 0.0000 + 0.0000i \end{bmatrix} \tag{13}$$

Because all the the poles are in the LHP (with one on the zero) the system can be viewed as a minimum phase system.