

# (Asynchronous) Temporal Logics for Hyperproperties on Finite Traces

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# Outline

- ▶ Hyperproperties
- ▶ Temporal Logics for Hyperproperties
- ▶ Temporal Logics for Asynchronous Hyperproperties
- ▶ Temporal Logics for Asynchronous Hyperproperties over finite traces (SC-HyperLTL)
- ▶ Model Checking of SC-HyperLTL
- ▶ Conclusions

# Hyperproperties

[Clarkson and Schneider, “Hyperproperties” J. of Computer Security, 2010]

- ▶ Trace properties  $P$  *set of trace*:

**Safety:** My program never reach a **bad** state

**Eventuality:** My process will **eventually** enter in the critical section

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- ▶ Hyperproperties  $H$  of systems *set of set of traces*:

“For any execution with a **secret input**, there is a distinct execution **Observationally equivalent** without the **secret input**. ”

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# Information Flow Examples

**Goguen and Meseguer non interference:** For any execution with a **secret input**, there is a distinct execution **Observationally equivalent** without the **secret input**.

**Observational determinism** traces with the same **initial low input** are indistinguishable for **low users**

# HyperLTL

[Clarkson, Finkbeiner, Koleini, Micinski, Rabe and Sánchez, "Temporal Logics for Hyperproperties. POST'14]

Quantifiers with trace variables:  $\forall x. \varphi$      $\exists x. \varphi$

**Syntax:**

$$\varphi ::= \forall x. \varphi \mid \exists x. \varphi \mid \psi$$

$$\psi ::= p[x] \mid X\psi \mid G\psi \mid F\psi \mid \psi U \psi \mid \psi R \psi$$

**Notes:**

- ▶ HyperLTL starts with quantifiers, then temporal formula (prenex form).
- ▶ Model Checking decidable.

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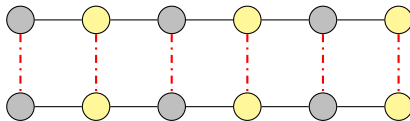
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All executions have the light on at the same time:

$$\forall x \forall y. G(\text{on}[x] \leftrightarrow \text{on}[y])$$



# Information Flow Examples in HyperLTL

**Goguen and Meseguer non interference:** For any execution with a **secret input**, there is a distinct execution **Observationally equivalent** without the **secret input**.

$$\forall x \exists y. G \lambda[y] \wedge \bigwedge_{lo \in LO} G(lo[x] = lo[y])$$

**Observational determinism** traces with the same **initial low input** are indistinguishable for **low users**

$$\forall x \forall y. \bigwedge_{v \in LI} (v[x] = v[y]) \rightarrow G \bigwedge_{v \in LO} (v[x] = v[y])$$



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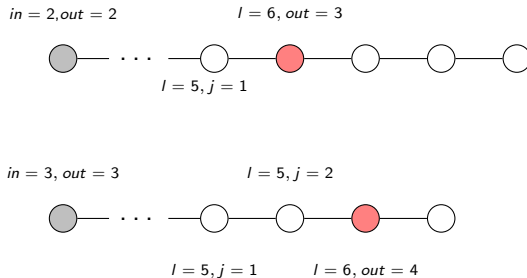
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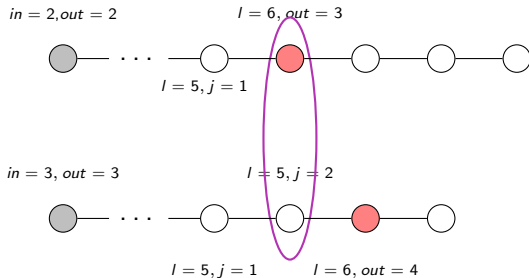


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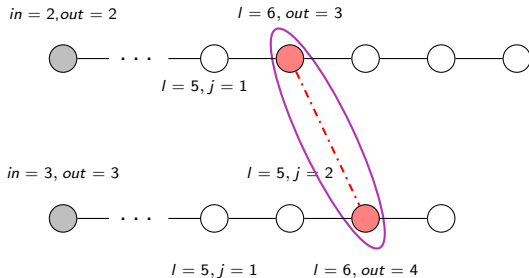
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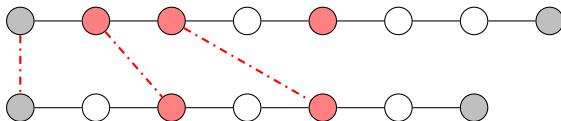


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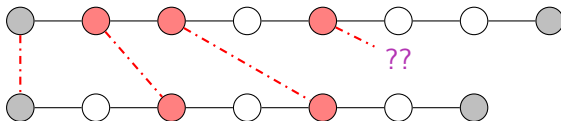
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What is the correct semantics?

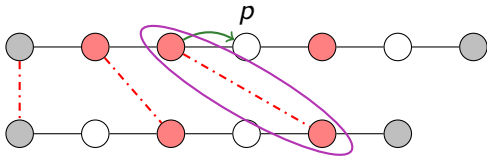
# SC-HyperLTL

$$\begin{array}{l}
 \varphi ::= \overbrace{\forall x.\varphi \mid \exists x.\varphi}^{\text{Quantifiers}} \mid \overbrace{\Gamma.\psi}^{\text{Points of interest}} \\
 \psi ::= \overbrace{p[x] \mid \neg\psi \mid \psi \vee \psi \mid \psi \wedge \psi}^{\text{Propositional}} \mid \overbrace{X\psi \mid N\psi \mid G\psi \mid F\psi \mid \psi U \psi \mid \psi R \psi}^{\text{Future fragment}} \\
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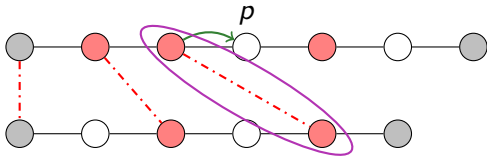
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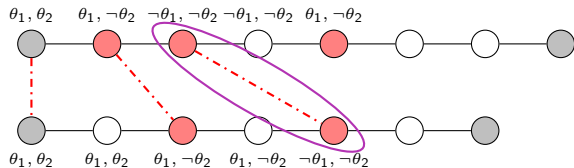
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$\Gamma = \{\theta_1, \dots, \theta_n\}$ :  $\bullet$  iff init or  $\exists \theta_j : \text{chd}(\theta_j)$



## Example properties

**Async GMNI:**

$$\forall x \exists y. LO. \langle y \rangle G \lambda[y] \wedge G \bigwedge_{lo \in LO} (lo[x] = lo[y])$$

**Async observational Determinism:**

$$\forall x \forall y. LO. \bigwedge_{v \in LI} (v[x] = v[y]) \rightarrow G \bigwedge_{v \in LO} (v[x] = v[y])$$

**Reactive Monotonicity:**  $\forall x \forall y. \{out\}. G(out[x] = out[y])$

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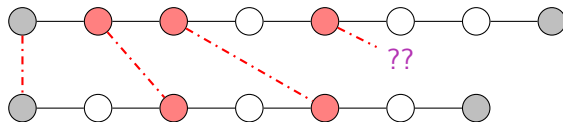
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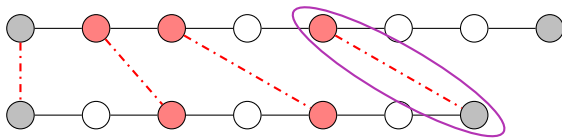


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**Local reasoning:** Don't care if another trace is finished

**Flexibility:** Permit to compare traces with different amount of observations

**Symmetry:** Past and future are treated symmetrically

# Reversability and Past

**Reversability:** For every formula  $\varphi$ ,  $\text{rev}(\text{rev}(\varphi)) \equiv \varphi$ .  
Given trace assignment  $\Pi$ ,  $(\Pi, \Gamma) \models \varphi$  if and only if  $(\Pi^{-1}, \Gamma_R) \models \text{rev}(\varphi)$ .

Validate past and future are treated symmetrically

## Examples with finite semantics

**Async Observational Determinism:** Shortest trace

$$\forall x \forall y. LO. \bigwedge_{v \in LI} (v[x] = v[y]) \rightarrow G \left( \bigwedge_{v \in LO} (v[x] = v[y]) \right)$$

## Examples with finite semantics

**Async Observational Determinism:** Enforce #obs

$$\forall x \forall y. LO. \bigwedge_{v \in LI} (v[x] = v[y]) \rightarrow G \left( \bigwedge_{v \in LO} (v[x] = v[y]) \wedge ((\langle x \rangle N \perp) \leftrightarrow (\langle y \rangle N \perp)) \right)$$

**Pre-post conditions**

$$\forall x \exists y. \emptyset. Pre(x, y) \rightarrow XPost(x, y)$$

# Model Checking

## Stuttering extension:

- ▶ **Idea:** Reduce  $\forall^*/\exists^*$  async to  $\forall^*/\exists^*$  sync
- ▶ Use rewriting technique similar to [Bombardelli, Tonetta “Asynchronous Composition of Local Interface LTL Properties” NFM22]
- ▶ In principle similar to the approach used for A-HLTL [Baumeister, Coenen, Bonakdarpour, Finkbeiner and Sánchez. “A Temporal Logic for Asynchronous Hyperproperties.” CAV’21.]

## Bounded observations:

- ▶ Support  $\forall\exists$  fixing at most  $k$  observation points
- ▶ Traces can be unbounded
- ▶ Reduces to HyperLTL

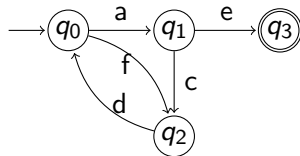


## Model Checking via stuttering encoding: Overall idea

Extend  $K$  with stuttering and halting state

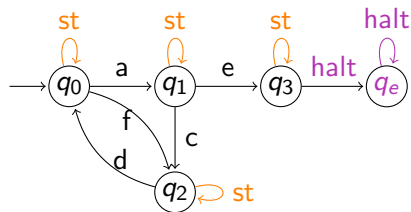
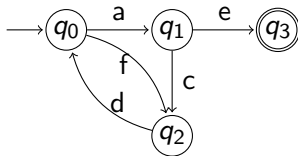
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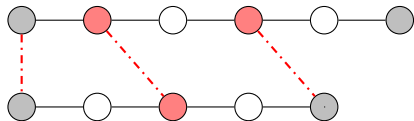
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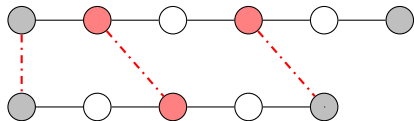
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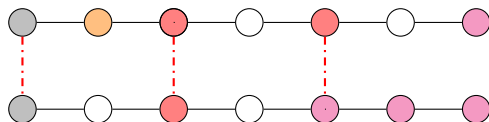
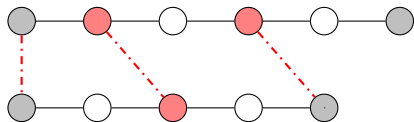
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$$G(\neg end[x] \wedge \neg end[y] \rightarrow (\Phi_{\Gamma}[x] \leftrightarrow \Phi_{\Gamma}[y]))$$

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Rewrite **temporal operators** to evaluate temporal operators at observation points synchronously

$$\langle x \rangle \beta[x] \Rightarrow \mathcal{R}_{st[x] \vee halt[x]}(\beta[x])$$
$$\varphi_1 U \varphi_2 \Rightarrow \mathcal{R}_{\Phi_\Gamma}(\varphi_1 U \varphi_2)$$

$$\Phi_\Gamma := \bigvee_{\theta \in \Gamma} (Y\theta \leftrightarrow \neg\theta) \vee init \vee last$$

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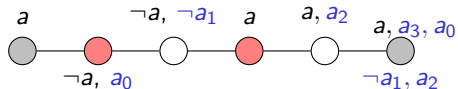
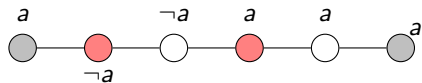
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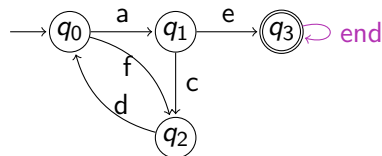
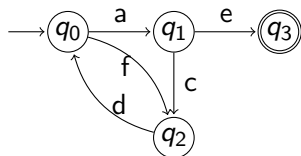
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Evaluate in a BMC-like manner at the end of the traces (sync because of padding)

# Model checking with Bounded observation

Assume at most  $K$  observations but unbounded length of trace

Add past history variables to mimic  $i$ -th observation of a variable  $v$

Padding of  $K$  with end state to ensure same length

Evaluate in a BMC-like manner at the end of the traces (sync because of padding)

$$\begin{array}{ll}
 \rho \llbracket v[x] \rrbracket_i^k & := v_i[x] & \rho \llbracket \neg v[x] \rrbracket_i^k & := \neg v_i[x] \\
 \rho \llbracket \psi_1 \vee \psi_2 \rrbracket_i^k & := \rho \llbracket \psi_1 \rrbracket_i^k \vee \rho \llbracket \psi_2 \rrbracket_i^k & \rho \llbracket \psi_1 \wedge \psi_2 \rrbracket_i^k & := \rho \llbracket \psi_1 \rrbracket_i^k \wedge \rho \llbracket \psi_2 \rrbracket_i^k \\
 \rho \llbracket \mathbf{X}_\Gamma \psi \rrbracket_i^k & := \neg \text{end}_\Gamma^{i+1} \wedge \rho \llbracket \psi \rrbracket_{i+1}^k & \rho \llbracket \mathbf{N}_\Gamma \psi \rrbracket_i^k & := \text{end}_\Gamma^{i+1} \vee \rho \llbracket \psi \rrbracket_{i+1}^k \\
 \rho \llbracket \psi_1 \mathbf{U}_\Gamma \psi_2 \rrbracket_i^k & := \rho \llbracket \psi_2 \rrbracket_i^k \vee (\rho \llbracket \psi_1 \rrbracket_i^k \wedge \rho \llbracket \mathbf{X}^\top \rrbracket_{i+1}^k \wedge \rho \llbracket \psi_1 \mathbf{U}_\Gamma \psi_2 \rrbracket_{i+1}^k) \\
 \rho \llbracket \psi_1 \mathbf{R}_\Gamma \psi_2 \rrbracket_i^k & := \rho \llbracket \psi_2 \rrbracket_i^k \wedge (\rho \llbracket \psi_1 \rrbracket_i^k \vee \rho \llbracket \mathbf{N}^\perp \rrbracket_{i+1}^k \vee \rho \llbracket \psi_1 \mathbf{R}_\Gamma \psi_2 \rrbracket_{i+1}^k)
 \end{array}$$



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# Proof of Concept Implementation (reducing to nuXmv $\forall\forall$ and AutoHyper $\forall\forall\exists$ )

Name	Method	Bound	Tool	Time (sec)	Res
Process $n = 6$	$k$ -bound (Alg.2)	7	nuXmv	4.17	True
Process $n = 6$	$k$ -bound (Alg.2)	7	AutoHyper	MO	-
Process $n = 2$	$k$ -bound (Alg.2)	3	nuXmv	2.82	True
Process $n = 2$	$k$ -bound (Alg.2)	3	AutoHyper	5.54	True
Process $n = 6$	Stuttering (Alg.1)	-	nuXmv	2.65	True
Process $n = 6$ (bug)	$k$ -bound (Alg.2)	7	nuXmv	3.33	False
Process $n = 6$ (bug)	Stuttering (Alg.1)	-	nuXmv	3.02	False
Process q. alt. $n = 2$	$k$ -bound (Alg.2)	3	AutoHyper	34.93	True
Process q. alt. $n = 3$	$k$ -bound (Alg.2)	4	AutoHyper	TO	-
Motivating example	$k$ -bound (Alg.2)	3	nuXmv	34.64	False*
Motivating example	Stuttering (Alg.1)	-	AutoHyper	6.19	True
Battery Sensor	Stuttering (Alg.1)	-	nuXmv	6.03	True
Battery Sensor	$k$ -bound (Alg.2)	4	AutoHyper	TO	-
Battery Sensor	$k$ -bound (Alg.2)	7	nuXmv	11.86	False*
Battery Sensor (bugged)	Stuttering (Alg.1)	-	nuXmv	3.04	True

# Conclusion and Future Work

## Conclusion:

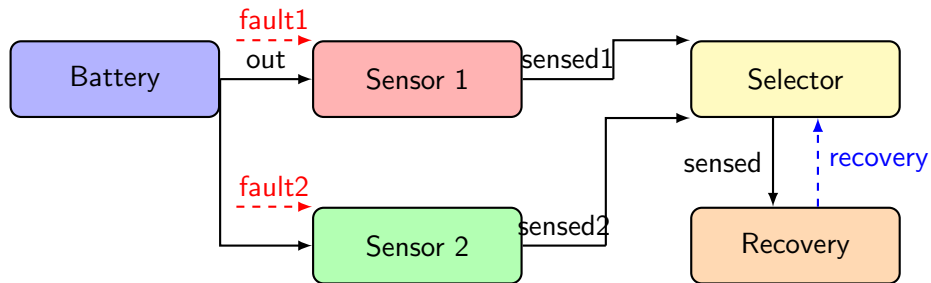
- ▶ Presented **decidable** asynchronous hyperlogics over **finite traces**
- ▶ Validated semantics and showed expressibility over interesting examples
- ▶ Provided a proof of concept tool for the verification of the logics.

## Future works:

- ▶ Simplify Algorithm 2 and investigate more efficient techniques with bounded observations
- ▶ Relax bounded observation requirement
- ▶ Direct reduction of Alg. 1 to  $LTL_f$
- ▶ ...

# Appendix

## FDIR of Battery Sensor



$$\phi_{rec} := \forall x. \forall y. \{sensed\}. G(\phi_{obseq} \wedge \langle x \rangle (faulted[x]) \rightarrow \langle y \rangle F^{\leq d} rcv[y])$$

$$\begin{aligned} \phi_{obseq} &:= H(sensed[x] \leftrightarrow sensed[y]) \\ faulted &:= (\neg chd(sensed) \mathbf{S} fault) \end{aligned}$$