Inverting Astrobee's Force Allocation Module

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RATTLE's inertial parameter estimation algorithm uses as inputs the actuation, i.e, forces and torques, and the measurement data. As actuation input saturation is expected in dynamical systems, the applied or resulting actuation might not always match its commanded values. Therefore, the knowledge of the post-saturation input forces and torques is crucial to ensuring accuracy of parameter estimates in an estimation method based on the system's dynamic model.

Astrobee's FAM sends out the impeller speed and allowable nozzle opening angles to the Propulsion Module Controller. As the saturation is imposed on the flap rotation that controls the opening and *not* on the commanded forces and torques, an inverse mixing operation is conducted to compute the actuation post-saturation. Steps used by the FAM in the forward-mixing operation (transformation of the commanded thrust to servo actuation for nozzle opening), and those implemented during the development of RATTLE for retrieving the post-saturation forces and torques from nozzle angles (which we call the inverse-mixing operation) are detailed here.

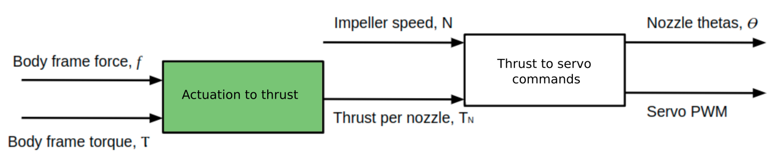

Figure 1: A simplified block diagram illustrating the series of operations carried out by the FAM. Block *Actuation to thrust* converts the forces and torques in the body frame to the thrust per nozzle given in (1), while block *Thrust to servo commands* deals with the underlying physics to yield the nozzle opening angles and the servo PWM signals required to generate the commanded thrust. The operations in block A are known and can be inverted, while inverting Block B, which uses a lookup table, needed further analysis.

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The **forward force allocation** operation takes place as follows. A description of the symbols used is given in table 1 in the order of appearance.

- In block A, the commanded forces and torques in robot body frame, f and $\tau \in \mathbb{R}^3$ are converted to thrust per nozzle $\underline{T_N} \in \mathbb{R}^{12}$. Note that T_N may have a negative thrust value, as indicated by the underline.

$$\underline{T_N} = \begin{bmatrix} D \\ (R \times D) \end{bmatrix}^{\dagger} \begin{bmatrix} f \\ \tau \end{bmatrix} \tag{1}$$

  where $\dagger$ denotes the Moore-Penrose pseudoinverse, and $(... \times ...)$ denotes the horizontal concatenation after computing the cross product of corresponding columns of R and D. Note that D is 3×12, where the columns contain the direction of each of the 12 nozzles. For instance, $[\begin{array}{ccc} 0 & 0 & -1 \end{array}]^T$ indicates that the nozzle is pointing towards the negative z direction. Similarly, R, the result of subtraction of two 3×12 matrices, is also 3×12. The minimum value below zero of $\underline{T_N}$ is added to it to prevent negative thrust from being commanded. That is, if $\underline{T_{N_i}} < 0$, then $T_N = \underline{T_N} + \min(|\underline{T_N}|)$.

- Block B accounts for the impeller and nozzle air flow dynamics to convert the commanded thrust per nozzle to nozzle opening angles. Fig.2 shows the airflow into the plenum through the impeller, and its precise exhaust through the nozzles, creating thrust. The impeller speed N is assigned based on the flight mode chosen at the time of operation. There are four possible preset values, from 0 to 293.12 RPM.

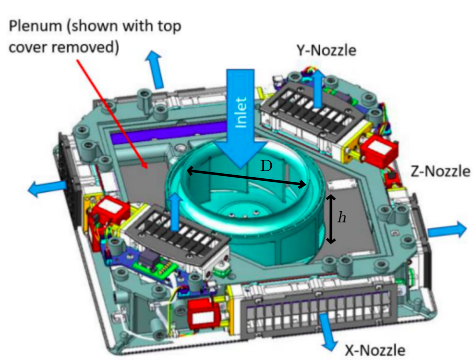

Figure 2: Astrobee propulsion module (plenum and 6 nozzles) showing airflow. Thrust is produced by varying the nozzle opening angle for a constant impeller speed. Image credit: [1]

For each set of six nozzles connected to a single impeller, (i = 1 to 6 and 7 to 12), the following operations are carried out:

1. The plenum rise in pressure, ΔP, is calculated from the corresponding total thrust, T_N, using the following sub-steps:

Symbol	Known constant	Description
D_i	x	Direction of each nozzle
R_i	x	Position of each nozzle - position of center of gravity
N	x	Impeller speed
d	x	Impeller diameter
h	x	Impeller height
ρ	x	Air density
C_{d_i}	x	Nozzle Discharge coefficient, determined through testing
C_{dp}		Pressure output coefficient
A_i		Commanded area per nozzle
S_i		Commanded nozzle opening
W	x	Nozzle width
H	x	Nozzle intake height
L	x	Flap length
θ_i		Commanded opening angle

Table 1: Description of the symbols used in explanation of the FAM, indicating the known parameters. Subscript i denotes that the quantity is defined or calculated for each nozzle, $i \in [1, ..., 12]$. The geometric parameters are indicated in figs. 2 and 3.

(a) The total thrust per nozzle (connected to one impeller), normed by the discharge coefficient and divided by impeller speed square, $\frac{\sum_i \frac{T_{N_i}}{C_{d_i}}}{N^2}$, is mapped to C_{dp} using a lookup table (more on this in section 1).

(b) C_{dp} is then used to calculate the pressure rise, ΔP, in the plenum created by impeller action

$$\Delta P = \rho C_{dp} N^2 d^2 \tag{2}$$

2. Recall that Astrobee generates precise thrust by opening the nozzles following the rise in pressure. Therefore, for each nozzle connected to the impeller, the commanded opening area to generate the required thrust is found as

$$A_i = \frac{T_{N_i}}{2\Delta P C_{d_i}^2} \tag{3}$$

3. This is followed by the commanded nozzle opening per flap. As both flaps of the nozzle will be opened the same amount, the commanded nozzle opening is calculated for one flap by dividing the area by 2 (fig. 3 illustrates the nozzle opening mechanism).

$$S_i = \frac{A_i}{2W} \tag{4}$$

4. Finally, the commanded nozzle opening angle is calculated

$$cos(\vartheta_i) = \frac{H - S_i}{L} \tag{5}$$

The maximum allowed nozzle opening is about 79.91° , and the lowest is 15.68° . After applying the saturation function, $\vartheta_i = max\{\vartheta_{min}, min\{\vartheta_i, \vartheta_{max}\}\}$, the commanded angles are mapped on a scale of 0 to 255 and sent off to be converted to raw servo commands.

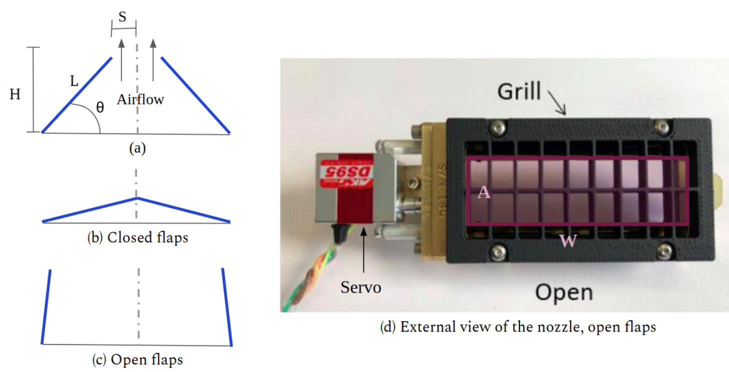

Figure 3: Airflow through the flaps, shown in blue. Physical parameters of the flaps (a), the flaps closed (b) and open all the way through (c), and (d) the exterior of the nozzle showing nozzle width, as well as the commanded opening area with the flaps completely open. Figures (a)-(c) represent a side view of the flaps, as seen from the direction of the servo. Figure (d) was adapted from [1].

1 The lookup table

Since the saturation check takes place after the nozzle angles are calculated, finding the post-saturation forces and torques requires inverting the FAM series of operations. The principal challenge here is posed by eq. 3 - both T_{N_i} and ΔP are unknown, and T_{N_i} is the quantity needed to find f and τ by inverting block A. Further, ΔP for each impeller depends on C_{dp}, which is the result of a look up depending on $\sum_i \frac{T_{N_i}}{C_{d_i}}$, denoted as T for brevity. This lookup table is generated from the impeller delta pressure to flow rate curve[1].

To summarize the lookup generation, for each impeller, a range of flow rate coefficients, C_q, and the output C_p - which is a polynomial in C_q - is generated. Using C_q and C_p, the total thrust divided by N^2 (square of the impeller speed)

[1]https://github.com/nasa/astrobee/blob/master/gnc/pmc/src/fam.cc

is found using the following relation which follows from fluid mechanics:

$$\frac{T}{N^2} = \rho C_q d^3 h\sqrt{2Cp} - 2\rho d^2 A_0 C_p \tag{6}$$

The first component in the equation calculates the thrust, while the second accounts for the leakage; A_0 is the zero thrust area. The lookup table is formed by taking select thrust breakpoints, and the corresponding pressure output Cp coefficient (referred to as C_{dp}). A plot of the lookup table is given in fig.4a.

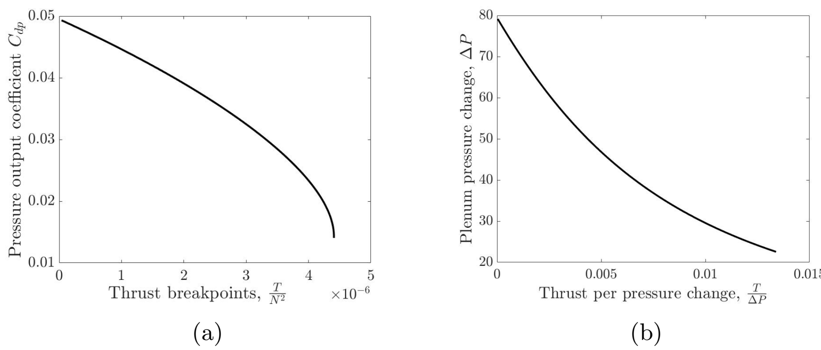

Figure 4: Visualization of total thrust divided by impeller speed $\frac{T}{N^2}$ and the corresponding Cdp values (L), and the lookup table from $\frac{T}{\Delta P}$ to ΔP used for the inverse operation.

2 Finding the lookup from $\frac{T}{\Delta P}$ to ΔP

As mentioned earlier, the main challenge in inverting the FAM is (3), where T_{N_i} and ΔP are both unknown. This is resolved in the inverse FAM by using the lookup table from $\frac{T}{N^2}$ to C_{dp} to obtain the lookup from $\frac{T}{\Delta P}$ to ΔP. This is done using the following steps:

- From the relation in (2), ΔP can be computed from C_{dp} as $\Delta P = \rho C_{dp} N^2 d^2$. Using this in (6) gives,

  $$T = C_q N d^2 h\sqrt{2\rho\Delta P} - 2A_0\Delta P \tag{7}$$

  Therefore, the input points of the lookup table $\frac{T}{N^2}$ are converted to T by mutliplying with N^2, and the output points C_{dp} are converted to ΔP by using (2). This gives us a lookup from T to ΔP.

- Since the relation between the total thrust and the corresponding C_{dp} values is monotonic (fig. 4a), the thrust breakpoints T are divided by ΔP to obtain the lookup from $\frac{T}{\Delta P}$ to ΔP, shown in fig. 4b

2.0.1 Inverting the FAM

For performing the reverse mixing operation, the commanded post-saturation nozzle angles θ_i are first converted to commanded area per nozzle, A_i using eqs. 4 and 5. Then, using eq. 3, the following quantity is found.

$$\frac{\sum_{i=1}^{6} \frac{T_{Ni}}{C_{di}}}{\Delta P} = \sum_{i=1}^{6} 2A_i C_{di} \tag{8}$$

and similarly for the second impeller, with nozzles $i = 7...12$. Next, the new lookup table is used to find ΔP from $\frac{T}{\Delta P}$, followed by calculation of the thrust per nozzle from eq. 3 as given below, and inversion of block A.

$$T_{Ni} = 2A_i \Delta P {C_{d}}_i^2 \tag{9}$$

References

[1] Earl Daley. Astrobee free-flyer nozzle mechanism summary. In *The 45th Aerospace Mechanism Symposium*, number ARC-E-DAA-TN73478, 2020.