University of Lleida

Master's Degree in Informatics Engineering

Higher Polythecnic School

Exercise 5

ICT Project: Communication Services and Security

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1 Problem 1

Let's assume we employ 4 APs[1] at the UNII[2] band. We use 3 spatial streams, a 1/2 coding rate, a transmission rate of 100 Mbps and a 32 QAM[3] modulation.

1. Which is the bit time? Which is the symbol time for each spatial stream?
2. Assuming that the period of the transmitted signal is twice the symbol time and that its bandwidth is defined by the main lobe plus the two side lobes (right and left from the main), which is the signal bandwidth?
3. Propose a channel assignment for each AP considering:
   - all 4 APs interface between them
   - the lowest usable channel is 36
   - to minimize the frequency gap between channels
   - the frequency allocation for Europe http://en.wikipedia.org/wiki/U-NII

1.1 Question 1

By definition, the bit time is the duration of an indiviual one (1) or zero (0) bit information in a digital transmission.

As the transmitter ejects bits at a given transmission rate (V_t) defined in bits per second (bps), the bit time is calculated by dividing 1 by the transmission rate, in other words getting its inverse, so

$$T_b = \frac{1(b)}{V_T(bps)} : T_b = \frac{1}{100 * 10^6} s = \frac{1}{100} \mu s = 1 * 10^{-8} s \tag{1}$$

On the other hand, the symbol time (T_s) is defined as the amount of time spent by a pulse in a digital transmission which fits into the symbol rate (f_s), also known as baud rate or modulation rate, standing as the number of symbols/signaling events across the transmission per unit of time, both inversely proportional terms related as

$$T_s = \frac{1}{f_s} s \tag{2}$$

But, the key is understanding that each symbol may encode one or several binary digits or 'bits' because of the modulation.

So, regarding this case where it is used a 32 QAM modulation there will be n bits encoded in each symbol, resulting as

$$q = 2^n : 32 = 2^n : n = 5b \tag{3}$$

[1] Access Points
[2] Unlicensed National Information Infrastructure
[3] Quadrature Amplitude Modulation

Besides that, there is a coding rate applied over the baseband signal which will provide a redundancy based on a

$$\frac{1}{r} \text{ factor, where } r = 2$$

meaning that for each bit of information will be added on more.

Furthermore, it must be taken into account the number of spatial streams (M) on which the signal will be distributed which corresponds to 3.

As a matter of fact, in the figure 1 you may see an overview of this specific case procedure, and now takes sense all the previous stuff.

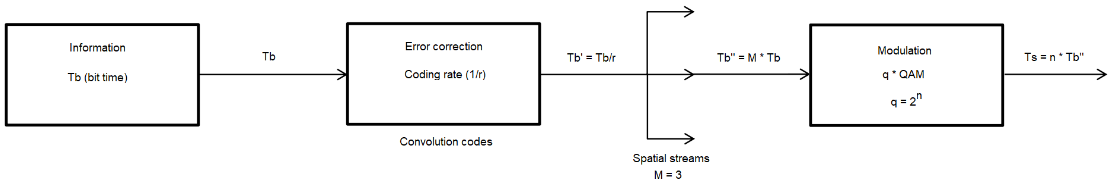


Figure 1: Baseband signal transformation

The above figure shows a basic representation of how a baseband signal is transformed in order to be transmitted over the air, first applying an error correction based on a coding rate, then a spatial distribution following the MIMO[4] methodology and lastly a QAM modulation for each of the spatial streams.

In this way, we should understand how T_s can be expressed from the initial T_b as follows,

$$T_s = n * T_b'' = n * M * Tb' = n * M * \frac{T_b}{r} \tag{4}$$

Finally, we apply those values to the formula,

$$T_s = 5 * 3 * \frac{1}{100 * 10^6} * \frac{1}{2} = 7.5 * 10^{-8}s = 75ns$$

and in addition, the symbol rate stands as,

$$f_s = \frac{1}{T_s} : f_s = \frac{1}{7.5 * 10^{-8}} * 10^6 = 13.\widehat{33}MBd$$

[4]Multiple-Input and Multiple-Output

1.2 Question 2

Assuming that the period of the transmitted signal is twice the symbol time, is applied as

$$T_0 = 2T_s : T_0 = 2 * 75ns = 150ns \quad (5)$$

so therefore, the fundamental frequency stands for

$$f_0 = \frac{1}{T_0} : f_0 = \frac{1}{150 * 10^{-9}} = 6.\widehat{66}MHz \quad (6)$$

Then, its bandwidth is defined by the main lobe plus the two side lobes (right and left from the main), meaning the problem is based on a $\frac{sin(2\pi f_0)}{n}$ signal type, represented as follows

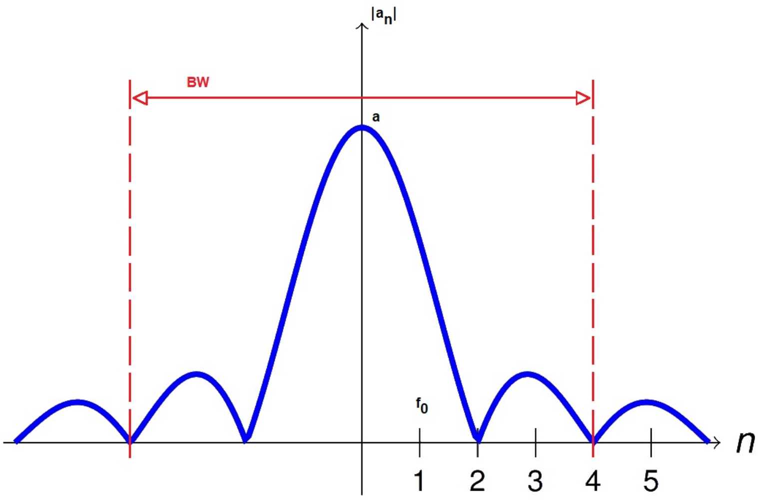


Figure 2: Signal representation over frequency terms

where you may see the lobes mentioned by the statement.

Lastly, since we are calculating the bandwidth this will be

$$BW^5 = n * f_0 \quad (7)$$

where n is the number of componets between the expected bandwith, all resulting as

$$BW = 8 * 6.\widehat{66} = 53.\widehat{33}MHz$$

[5] Bandwidth

1.3 Question 3

We start at channel 36, from which we must assign the following three channels considering the frequency range equivalent to 53.33MHz in order to minimize the total gap between them.

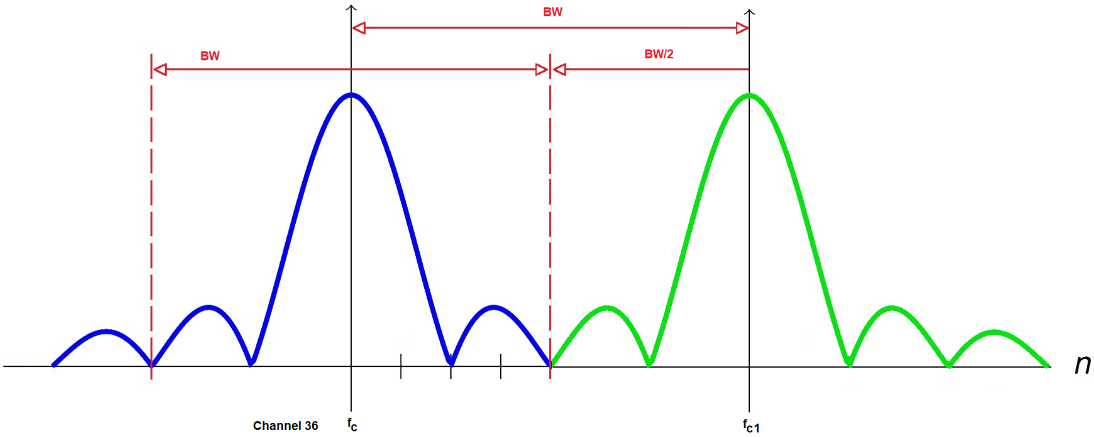


Figure 3: Channel assignment

The above figure represents how the signals will interact showing that there must be at least $BW = 5.\widehat{33}MHz$ between the corresponding channels frequency carriers to not interfere.

Therefore to assign the channels without interference, we must consult the below Europe UNII[6] allocation standards, as one of the statement constraints to get a proposed assignment.

	U-NII-1				U-NII-2A			
Channel	36	40	44	48	52	56	60	64
Frequency (MHz)	5180	5200	5220	5240	5260	5280	5300	5320

	U-NII-2C										
Channel	100	104	108	112	116	120	124	128	132	136	140
Frequency (MHz)	5500	5520	5540	5560	5580	5600	5620	5640	5660	5680	5700

Table 1: U-NII Channels - Europe 40/20MHz

[6] https://en.wikipedia.org/wiki/U-NII

Since the objective of this task is to assign the 3 rest APs repsecting the European standard and ensuring a minimum frequency gap, but avoiding interference, one proposal might be as follows

AP	**Band**	**Channel**	**Frequency (MHz)**	**Frequency gap +/-**
1	U-NII-1	36	5180	5126 - 5234
2	U-NII-1	48	5240	5186 - 5294
3	U-NII-2A	60	5300	5246 - 5354
4	U-NII-2B	100	5500	5446 - 5554

Table 2: APs channel allocations proposition

2 Problem 2

Consider a 802.11b Channel 1 with a 22 MHz perfect pass-band at transmiters and receivers. BPSK[7] at 6 Mbps is employed. Assume the following two transmissions getting the receiver with the same power:

1. A periodic signal (101010 ...) at channel 1
2. A periodic signal (100100 ...) at channel X (interferent)

Plot the received base-band signal during two bits time considering two cases; X = 3 and X = 6 and detail the procedure to obtain the received base-band signal.

Scenario

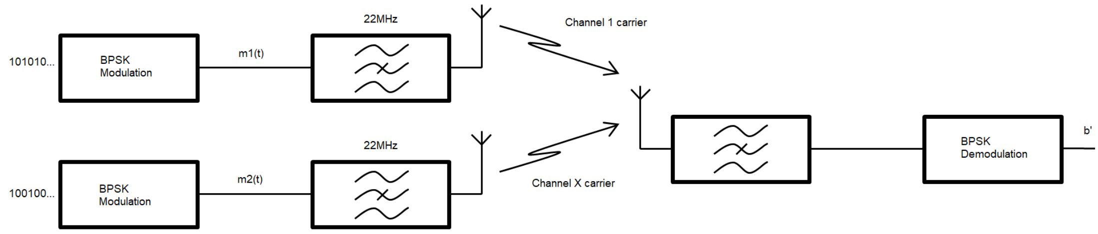


Figure 4: Wireless communication scenario

The main purpose of this problem is to understand how works a wireless transmission, starting by transforming the baseband signal to a low pass filtered and modulated one in order to fit in the air.

In addition, are proposed two interference situations introducing the concepts of channels, because the idea is work on what happens if two signals interfere depending in which channel are working. Will the received signal be the same in both cases?

As you may see in figure 4, there are two baseband signals that need to be modulated using BPSK method and then low pass filtered because we must adjust the bandwidth used.

As a matter of fact, the first signal will be using channel 1 and then, stands two cases for the second signal. The first one using channel 3 and the second channel 6.

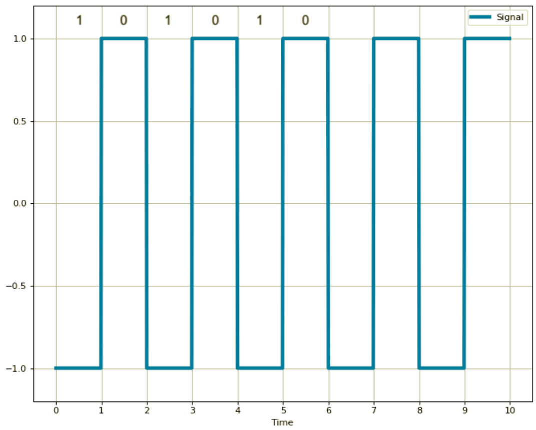
In the next sections, are detailed all the procedure followed to compute as Fourier componentes the previous signals, as well as the modulation and filtering done, but also how in both cases become the interferred signal.

[7] Binary Phase Shift Keying

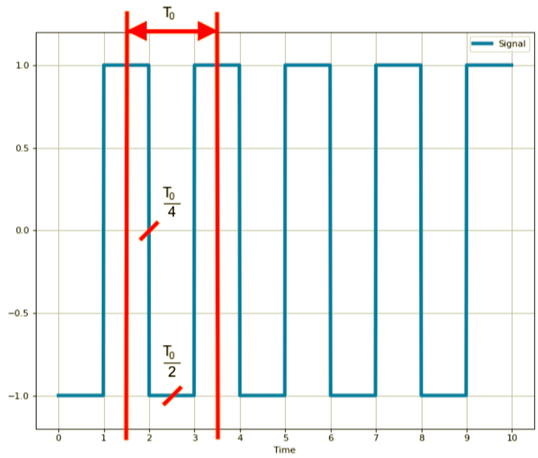
2.1 Signal 1

First of all, the following figure shows the way we consider to represent the signal $s(t)$. As you may see, the 1s are negative voltage and 0s positive voltatge, which principle is also followed on the second signal representation.

Besides, to make the signal even as calculating the Fourier components is much easier, we moved the coordinates and getting a period $T_0 = 2T_b$.



(a) Original



(b) Coordinates moved

Figure 5: Signal 101010... representation

where $T_b = \frac{1}{6*10^6} = \frac{1}{6}\mu s = 1.\widehat{66} * 10^{-7}s$ and the analytic expression of $s(t)$ results as:

$$s(t) = \begin{Bmatrix} -1 & 0 \leq t < \frac{T_0}{4} \\ 1 & \frac{T_0}{4} \leq t < \frac{3T_0}{4} \\ -1 & \frac{T_0}{2} \leq t < T_0 \end{Bmatrix} \tag{8}$$

Then, we will follow the Fourier representation of periodic signals because any can be represented as an addition of weighted basic periodic signals such as cosinus and sinus.

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n * cosWnt + \sum_{n=1}^{\infty} b_n * sinWnt \tag{9}$$

where the coefficients computation is based on:

$$\begin{aligned} a_0 &= \frac{2}{T_0} \int_{T_0} s(t)\, dt \\ a_n &= \frac{2}{T_0} \int_{T_0} s(t) * cosWnt\, dt \\ b_n &= \frac{2}{T_0} \int_{T_0} s(t) * sinWnt\, dt \end{aligned} \tag{10}$$

So, the Fourier coefficients are computed as follows:

$$\boldsymbol{a_0} = \frac{2}{T_0}\int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} s(t)\,dt = \frac{4}{T_0}\int_{0}^{\frac{T_0}{2}} s(t)\,dt = 0$$

$$\boldsymbol{b_n} = \frac{2}{T_0}\int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} s(t) * sinWnt\,dt = 0$$

$$\boldsymbol{a_n} = \frac{2}{T_0}\int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} s(t) * cosWnt\,dt = \frac{4}{T_0}\int_{0}^{\frac{T_0}{2}} s(t) * sinWnt\,dt =$$

$$= \frac{4}{T_0}\left(\int_{0}^{\frac{T_0}{4}} (1) * cosWnt\,dt + \int_{\frac{T_0}{4}}^{\frac{T_0}{2}} (-1) * cosWnt\,dt\right) = \frac{4}{T_0}\left(\left[\frac{1}{Wn}sinWnt\right]_0^{\frac{T_0}{4}} - \left[\frac{1}{Wn}sinWnt\right]_{\frac{T_0}{4}}^{\frac{T_0}{2}}\right) =$$

$$= \frac{4}{T_0Wn}\left(sinWn\frac{T_0}{4} - sin0 - sinWn\frac{T_0}{2} + sinWn\frac{T_0}{4}\right) = \frac{4}{T_0Wn}\left(2sinWn\frac{T_0}{4} - sinWn\frac{T_0}{2}\right) =$$

$$= \frac{2}{\pi n}\left(2sin\frac{\pi n}{2} - sin\pi n\right) = \frac{2}{\pi n} * 2sin\frac{\pi n}{2} = \boldsymbol{\frac{4sin\frac{\pi n}{2}}{\pi n}}$$

Therefore, the signal can be expressed only with a_n coefficients as:

$$s(t) = \sum_{n=1}^{\infty} \frac{4sin\frac{\pi n}{2}}{\pi n}$$

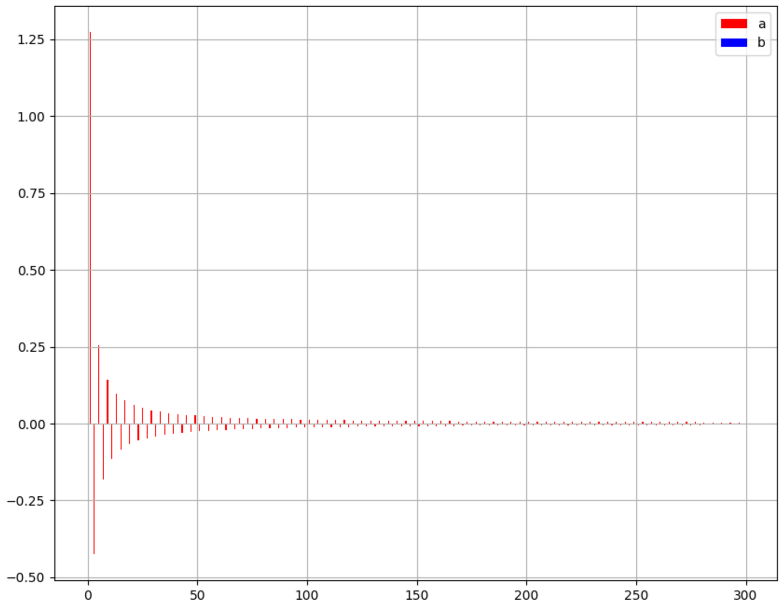


Figure 6: Coefficients signal 1 as Fourier frequency terms

Low pass filter

As the signals must be filtered in order to fit in the air, meaning we only have a maximum bandwidth of 22MHz we must select a maximum number n of components to be transmitted.

In this way, the signal is characterized by $T_0 = 2T_b$ where $T_b = 1.\widehat{66} * 10^{-7}s$, so

$$T_0 = 2 * 1.\widehat{66} * 10^{-7} = 3.\widehat{33} * 10^{-7}$$

Thus,

$$f_0 = \frac{1}{T_0} = \frac{1}{3.\widehat{33} * 10^{-7}} = 3MHz$$

which is the distance in frequency terms between the components, therefore as considering the filter prior the modulation

$$\frac{22}{2}MHz \geq 3MHz * n = 11MHz \geq *n$$

$$\boldsymbol{n} = 3$$

resulting only from a_0 to a_3 frequency components will pass through

$\boldsymbol{n}$	$\boldsymbol{a_n}$
0	0
1	$\frac{4}{\pi}$
2	0
3	$-\frac{4}{3\pi}$

Table 3: Frequency components that will pass through the low pass filter - signal 1

Modulation

At this point, the signal must be modulated since baseband signals don't propagate well on air and thus the frequency spectrum should be moved to a higher frequency range to make more efficient the transmission.

Specifically by this signal, the components will be moved arround the carrier frequency of channel 1 which is computed in the section 2.4 standing as

$$f_{c1} = 2.412GHz$$

In addition the resulting modulated signal will become applying the Fourier series

$$m(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n cos W_c t + b_n sin W_c t] \text{, where } W_c = 2\pi f_c \tag{11}$$

So, signal 1 will perform as:

$$m(t) = s(t) * cos W_c t \tag{12}$$

meaning that the modulated signal become as:

$$m(t) = \frac{4}{\pi} cos W_c t - \frac{4}{3\pi} cos W_c t$$

<u>**Demodulation**</u>

Finally, to demodulate it is need to fold in a half the modulated signal, and sum the coefficients that are in the same frequency.

But also aplying the Fourier series formula again, and a low pass filter using a 2 times higher frequency than f_c in order to removed the higher frequency values,

$$m(t) * cos^2 W_c t = m'(t) \left[\frac{1 + cos 2W_c t}{2} \right]$$

where m'(t) is the b'(t), the signal/information received

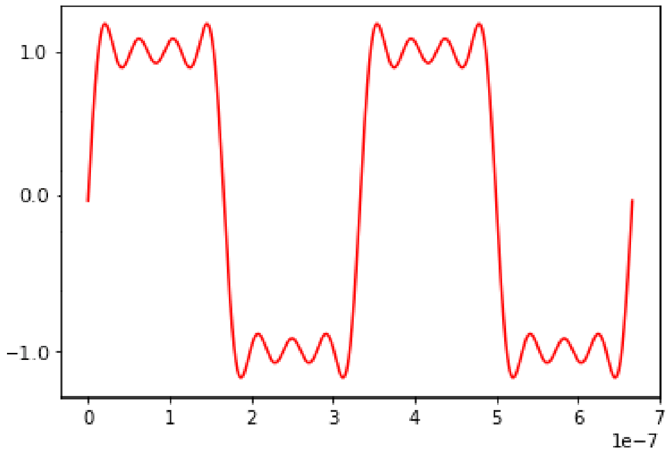
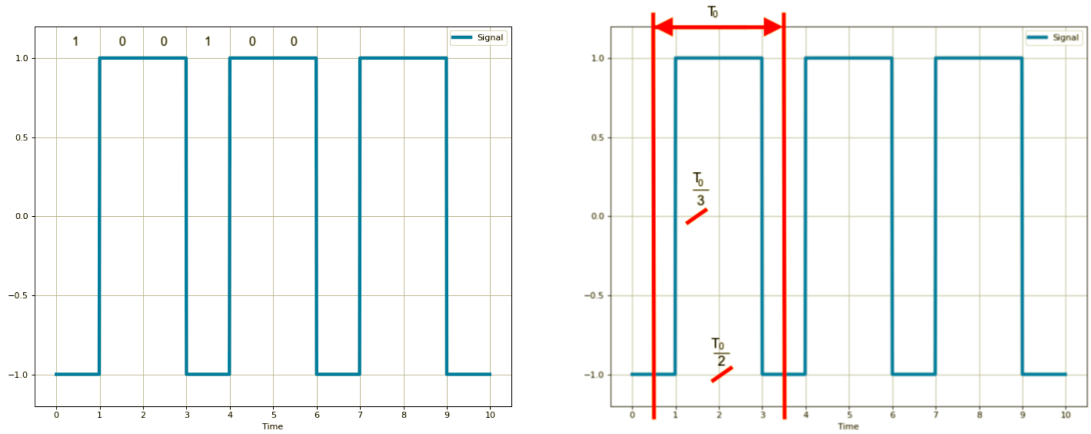


Figure 7: Signal 1 with no interference

2.2 Signal 2

We will follow the same procedure for signal 2. So, we moved the coordinates and got a period $T_0 = 3T_b$.



(a) Original (b) Coordinates moved

Figure 8: Signal 100100... representation

where $T_b = \frac{1}{6*10^6} = \frac{1}{6}\mu s = 1.\widehat{66} * 10^{-7}s$ and the analytic expression of $s(t)$ results as:

$$s(t) = \begin{Bmatrix} -1 & 0 \leq t < \frac{1T_0}{6} \\ 1 & \frac{1T_0}{6} \leq t < \frac{5T_0}{6} \\ -1 & \frac{5T_0}{6} \leq t < T_0 \end{Bmatrix} \tag{13}$$

Then, the Fourier coefficients can be computed as:

$$\boldsymbol{a_0} = \frac{2}{T_0}\int_{T_0} s(t)\,dt = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} s(t)\,dt =$$

$$= \frac{4}{T_0}\left[\int_0^{\frac{T_0}{3}} s(t)\,dt + \int_{\frac{T_0}{3}}^{\frac{T_0}{2}} s(t)\,dt\right] = \frac{4}{T_0}\left[0 + \int_{\frac{T_0}{3}}^{\frac{T_0}{2}} 1(t)\,dt\right] = \boldsymbol{\frac{2}{3}}$$

$$\boldsymbol{b_n} = \frac{2}{T_0}\int_{T_0} s(t) * sinWnt\,dt = 0$$

$$\boldsymbol{a_n} = \frac{2}{T_0}\int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} s(t) * cosWnt\,dt = \frac{4}{T_0}\int_{0}^{\frac{T_0}{2}} s(t) * sinWnt\,dt =$$

$$= \frac{4}{T_0}\left(\int_0^{\frac{T_0}{6}} -1 \cdot cosW_nt\,dt + \int_{\frac{T_0}{6}}^{\frac{T_0}{2}} 1 \cdot cosW_nt\,dt\right) = \frac{4}{T_0}\left(\left[-\frac{1}{Wn}sinWnt\right]_0^{\frac{T_0}{6}} + \left[\frac{1}{Wn}sinWnt\right]_{\frac{T_0}{6}}^{\frac{T_0}{2}}\right) =$$

$$= \frac{4}{T_0Wn}\left(-sinWn\frac{T_0}{6} + sin0 - sinWn\frac{T_0}{2} + sinWn\frac{T_0}{6}\right) = \frac{4}{T_0Wn}\left(-2sinWn\frac{T_0}{6} + sinWn\frac{T_0}{2}\right)$$

$$= \frac{2}{\pi n}\left(-2sin\frac{\pi n}{3} + sin\pi n\right) = \frac{2}{\pi n} * -2sin\frac{\pi n}{3} = \boldsymbol{-\frac{4}{\pi n}\sin\frac{\pi n}{3}}$$

Therefore, the signal can be expressed only with a_n coefficients as:

$$s(t) = \sum_{n=1}^{\infty} -\frac{4}{\pi n}\sin\frac{\pi n}{3}$$

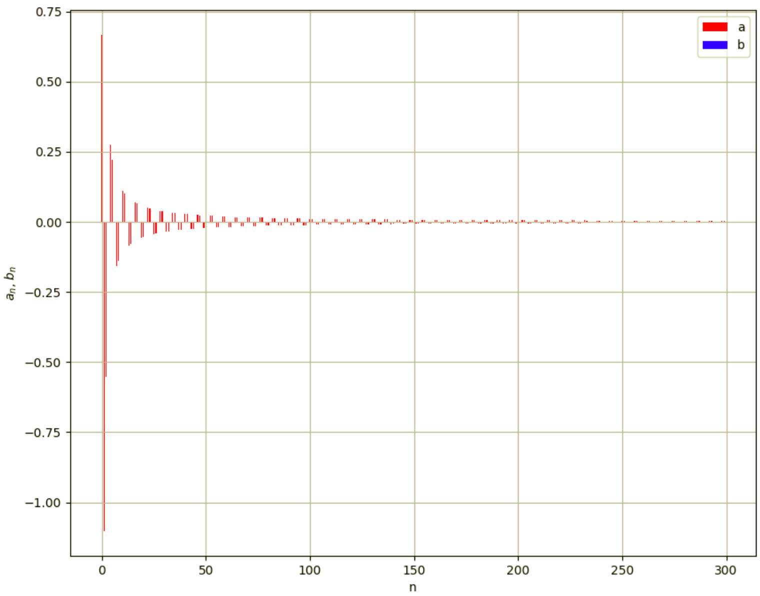


Figure 9: Coefficients signal 2 as Fourier frequency terms

Low pass filter

In this case, the signal is characterized by $T_0 = 3T_b$ where $T_b = 1.\widehat{66} * 10^{-7}s$, so

$$T_0 = 3 * 1.\widehat{66} * 10^{-7} = 5 * 10^{-7}s$$

Thus,

$$f_0 = \frac{1}{T_0} = \frac{1}{5 * 10^{-7}} = 2MHz$$

which is the distance in frequency terms between the components, therefore as considering the filter prior the modulation

$$\frac{22}{2}MHz \geq 2MHz * n = 11MHz \geq *n$$

$$\boldsymbol{n} = 5$$

resulting only from a_0 to a_5 frequency components will pass through

$\boldsymbol{n}$	$\boldsymbol{a_n}$
0	$\frac{2}{3}$
1	$-\frac{2\sqrt{3}}{\pi}$
2	$-\frac{\sqrt{3}}{\pi}$
3	0
4	$\frac{\sqrt{3}}{2\pi}$
5	$\frac{2\sqrt{3}}{5\pi}$

Table 4: Frequency components that will pass through the low pass filter - signal 2

Modulation

In this case, the modulation depends for each channel case where its carrier frequencies are computed in the section 2.4 resulting as:

$$f_{c3} = 2.422GHz$$

$$f_{c6} = 2.437GHz$$

And following same procedure of introduced for signal 1 to get the modulate signal, signal 2 will perform as:

$$m(t) = \frac{2}{3} - \frac{2\sqrt{3}}{\pi}cosW_ct - \frac{\sqrt{3}}{\pi}cosW_ct + \frac{\sqrt{3}}{2\pi}cosW_ct + \frac{2\sqrt{3}}{5\pi}cosW_ct$$

Demodulation

As regards the demodulation, the procedure is basically the same explained for signal 1 and you may consult how looks like the received signal with no interference.

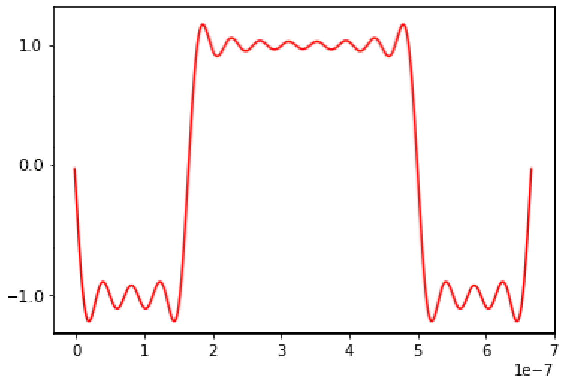


Figure 10: Signal 2 received with no interference

2.3 Interference

In this section the first signal will be interfered by both cases in channel 3 and channel 6 with the aim of comparing which case further distorts the signal.

Taking into account that the interference performs as a sum of the same frequency coefficients.

<u>1st Case: channel 3</u>

In this case, as the channels perform in relative near frequencies there are some components from signal 1 interferred.

So in conclusion, there is a difference beween the transmitted signal and the received one related to the channel distribution used.

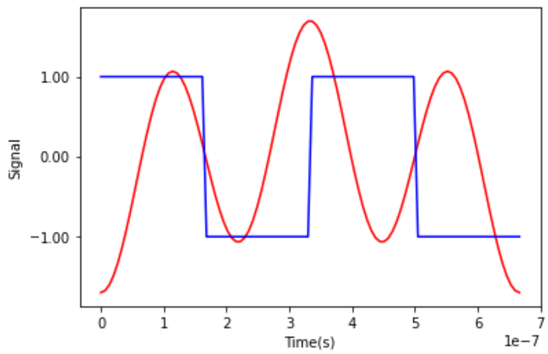


Figure 11: Signal 1 received with interference - channel 3

The above figure presents the signal 1 once the receiver demodulate it after being interferred during the transmission.

2nd Case: channel 6

In this case, as the carrier frequencies are far away there is no interference between the components, so basically the signal 1 sent is the same received.

All by when checking mathematically the components, not even the farthest from signal 1, which is a_3 is interferred. So, definetly the receiver will receive the signal without distorsion.

So basically, e.g if the farthest component from signal 1 is at $a_3 = 2.412GHz + 3 * 3MHz = 2.421GHz$ of frequency and the one from signal 2 is at $a_5 = 2.437GHz - 5 * 2MHz = 2.427GHz$ is proved there is no interference between the signals.

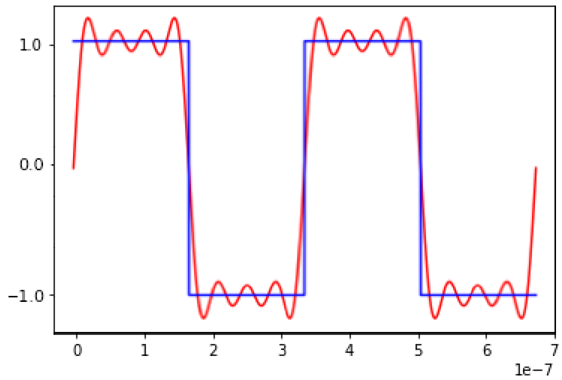


Figure 12: Signal 1 received with no inteference - channel 6

2.4 Channels frequencies

In order to modulate the signals at specific channels we must know the Carrier frequency (f_c) for each one. In this case, the f_{c1}, f_{c3} and f_{c6} frequencies.

The next figure shows the channel distribution where it can be seen which ones interfere most, the distance between them as $5MHz$ and the $BW = 22MHz$.

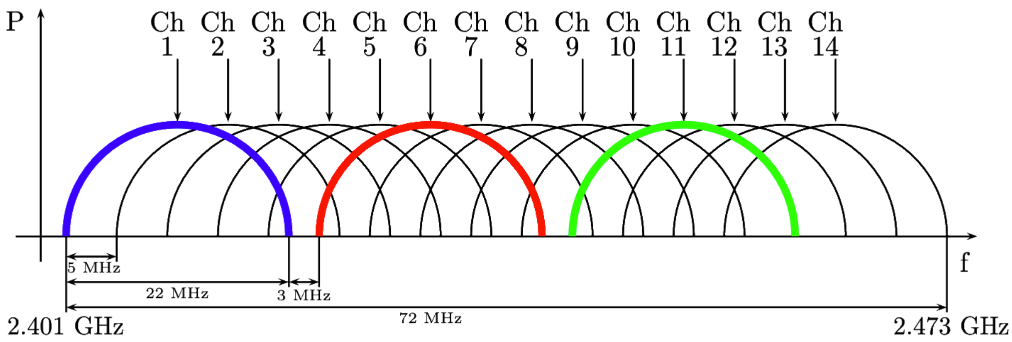


Figure 13: Channel distribution

Hence, to compute the carrier frequencies we must apply basic mathematics as these ones start at 2.401GHz.

So,

$$\boldsymbol{f_{c1}} = 2.401 + \frac{22MHz}{2} = 2.401 + 11MHz = 2.412GHz$$
$$\boldsymbol{f_{c3}} = f_{c1} + 2 * 5MHz = 2.422GHz$$
$$\boldsymbol{f_{c6}} = f_c1 + 5 * 5MHz = 2.437GHz$$

References

[1] Wireless Networks - Communication Services and Security - César Fernández Camón

[2] Plotting python script by César Fernández Camón

[3] Bit-time Meaning

[4] Symbol rate article - Wikipedia