COLORINGS OF KNOTS

LUCAS MEYERS

ABSTRACT. Empty

1. INTRODUCTION

2. DIAGRAMS AND COLORING

3. THE KNOT GROUP AND D_6

4. n-COLORINGS

Prior to this point we have talked about tricolourings of knots. However there is a natural way to extend our notion of colouring so that we may work with n colors as opposed to limiting ourself to 3.

When it came to tricolouring knots we assigned arcs either blue, red, or green. However we could just as easily assigned numbers 0,1, or 2 to arcs and had our condition instead be that if we have a crossing as in figure

Need a crossing

That the equation $2x - y - z \equiv 0 \mod p$. We restrict ourselves to odd primes for technical reasons. Then a knot K is p-colourable if we can assign a value between 0 and $p - 1$ to each arc in such a way that each crossing satisfies the above equation and at least two different colors are used.

The proof that this is an invariant is precisely the same as it is for showing that tricolourability is an invariant.

The extension of the Knot group definition of colouring is even faster to extend. Let $G := \pi_1(\mathbb{R}^3 \setminus K, *)$. Then we define an n-colouring of K as a representation ρ of G into the dihedral group D_{2n}. As before this is clearly a knot invariant as there is no mention of a diagram. Moreover the proof that the representations of G in D_{2n} correspond to colorings of diagrams follows similarly.

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References

[1] Kung, J.P.S., Critical problems, in *Matroid Theory* (eds. J. E. Bonin, J.G. Oxley, and B. Servatius), *Contemporary Mathematics* **197**, Amer. Math. Soc., Providence, 1996, pp. 1–127.

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