Problem 1 (1.1.21).

Proof. Let $x \in G$ have order odd $2n+1$. Then $x^{2n+1} = e$ and if we multiply by x on both sides we get $x^{2n+2} = x^{2(n+1)} = x$.

Therefore if x has odd order then it $x^{2k} = x$ for some $k \in \mathbb{Z}_{>0}$. □

Problem 2 (1.1.25).

Proof. Suppose that G is a group such that for all $x \in G$ that $x^2 = 1$. This implies that for any element $x^{-1} = x$. Then we have $xy = (xy)^{-1} = y^{-1}x^{-1} = yx$ which shows that G is commutative. □

Problem 3 (1.1.35).

Proof. Let $x \in G$ have order n. Then consider x^m. Using the division algorithm with m and n we can rewrite m as $qn + d$ where $0 \leq d < n$. This implies that

$$x^m = x^{qn+d} = x^{qn}x^d = (x^n)^q x^d = e^q x^d = x^d$$

As such $x \in \{e, x^1, \ldots, x^{n-1}\}$. □

Problem 4 (1.3.6).

Proof. □

Problem 5 (1.3.9).

Proof. □

Problem 6 (1.3.13).

Proof. □

Problem 7 (1.5.2).

Proof. □