Problem 1 (5.2.1)**.** *Find the isomorphism classes of Abelian groups of order 200.*

Proof. □

Problem 2 (5.2.2)**.** *Find the invariant factors and the elementary divisors of the Abelian group*

$$G = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_9 \times \mathbb{Z}_5 \times \mathbb{Z}_5$$

Proof. □

Problem 3 (5.2.4)**.**

Proof. Let G be a finite group and p a prime factor of $|G|$. Prove that the number of order p elements in G is congruent to -1 modulo p. □

Problem 4 (5.3.2)**.** *Let G be a finite group and $N_1, \dots, N_n$ normal subgroups of G such that $G = N_1 \cdots N_n$ and $|G| = |N_1| \cdots |N_n|$. Prove that G is the internal direct product of G.*

Proof. □

Problem 5 (5.5.1)**.** *Let G be a group, H, K subgroups of G, and $H \trianglelefteq G$. Let $\varphi : K \to Aut(H)$ be the homomorphism associated with the conjugate action of K on H. Then the following statements are equivalent:*

1. *$\phi : H \rtimes_\varphi K \to G$ defined by $\phi(h, k) = hk$ is an isomorphism.*
2. *Every element $g \in G$ can be written as $g = hk$ with $h \in H$ and $k \in K$ in a unique way.*
3. *$G = HK$ and $H \cap K = \{e\}$.*

Proof. □

Problem 6 (5.5.4)**.**

Proof. (a) For any positive integer n, prove that $\mathrm{Aut}(\mathbb{Z}_n) \cong \mathbb{Z}_n^*$.

(b) For any primes $p < q$, if $p|q-1$, there exists a monomorphism $\varphi : \mathbb{Z}_p \to \mathrm{Aut}(\mathbb{Z}_q)$ and $\mathbb{Z}_q \rtimes_\varphi \mathbb{Z}_p$ is a non-abelain group of order pq.

□

Problem 7 (5.5.11(book))**.** *Classify groups of order 28 (there are four isomorphism types).*

Proof. □