Problem 1. *Construct several examples of homotopic and non-homotopic maps.*

Problem 2. *Show that the relation of fixed-endpoint homotopy is an equivalence relation.*

Proof. □

Problem 3. *Construct some examples of paths which are fixed-endpoint homotopic, and some which are not.*

Problem 4. *a) Show that any convex open subset of $\mathbb{R}^n$ is contractible.*

b) Show that a contractible space is path connected.

c) Show that if Y is contractible, then all maps

$$f : X \to Y$$

are homotopic.

d) Show that if X is contractible and Y is path-connected, then all maps

$$f : X \to Y$$

are homotopic. What happens if we remove the path-connectedness assumption?

Proof. □

Problem 5. *Check that the fundamental group of a pointed space (X, x_0) is a group.*

Proof. a)

b)

c)

d)

□

Problem 6. *Show that if x_0, x_1 are in the same path component of a space X, then $\pi_1(X, x_0) \simeq \pi_1(X, x_1)$.*

Proof. □