

Problem 1 (1). *What familiar space is the quotient Δ -complex of a 2-simplex $[v_0, v_1, v_2]$ obtained by identifying the edges $[v_0, v_1]$ and $[v_1, v_2]$, preserving the ordering of vertices?*

Use the following drawing of X as our guide for the boundary maps.

Insert pretty picture here

Since we are identifying edges e_1 and e_2 this also induces the identification of v_1, v_2 , and v_3 . We'll call them e and v respectively. Then:

$$\begin{aligned}\partial f &= 2e + e_3 \\ \partial e &= v - v = 0 \\ \partial e_3 &= v - v = 0\end{aligned}$$

Then for $H_2(X)$ we have $\text{im } \partial_3 = 0$ and $\ker \partial_2 = 0$. Thus $H_2(X) \cong 0$. For H_1 we have $\text{im } \partial_2 = \langle 2e + e_3 \rangle$ and $\ker \partial_1 = \langle e, e_3 \rangle$. Then we have

$$H_1(X) \cong \langle e, e_3 \rangle / \langle 2e + e_3 \rangle \cong \langle e, e_3 | 2e = -e_3 \rangle \cong \langle e \rangle \cong \mathbb{Z}$$

Finally $H_0(X) = \mathbb{Z}$ as there is only a single connected component. Therefore the homology for X is

$$H_p(X) = \begin{cases} \mathbb{Z} & p = 0, 1 \\ 0 & p \geq 2 \end{cases}$$

It's probably a Möbius strip.

Problem 2 (4). *Compute the simplicial homology groups of the triangular parachute obtained from Δ^2 by identifying its three vertices to a single point.*

Use the picture below to guide the boundary map for X .

Insert pretty picture here

The boundary map for our components will be:

$$\begin{aligned}\partial f &= e_1 + e_2 + e_3 \\ \partial e_1 &= v - v = 0 \\ \partial e_2 &= v - v = 0 \\ \partial e_3 &= v - v = 0\end{aligned}$$

For $H_2(X)$ we have $\text{im } \partial_3 = 0$ and $\ker \partial_2 = 0$. Thus $H_2(X) \cong 0$.

For $H_1(X)$ we have $\text{im } \partial_2 = \langle e_1 + e_2 + e_3 \rangle$ and $\ker \partial_1 = \langle e_1, e_2, e_3 \rangle$. Thus

$$H_1(X) = \langle e_1, e_2, e_3 \rangle / \langle e_1 + e_2 + e_3 \rangle \cong \langle e_1, e_2, e_3 | e_1 + e_2 = -e_3 \rangle \cong \langle e_1 + e_2 \rangle \cong \mathbb{Z} \oplus \mathbb{Z}$$

Finally there is only a single connected component so $H_0(X) \cong \mathbb{Z}$.

Therefore the homology for X is

$$H_p(X) = \begin{cases} \mathbb{Z} & p = 0 \\ \mathbb{Z} \oplus \mathbb{Z} & p = 1 \\ 0 & p \geq 2 \end{cases}$$

Problem 3 (6). *Compute the simplicial homology groups of the Δ -complex obtained from $n+1$ 2-simplicies $\Delta_0^2, \dots, \Delta_n^2$ by identifying all three edges of Δ_0^2 to a single edge, and for $i > 0$ identifying the edges $[v_0, v_1]$ and $[v_1, v_2]$ of Δ_i^2 to a single edge and the edge of $[v_0, v_2]$ to the edge $[v_0, v_1]$ of Δ_{i-1}^2 .*

We'll use the following diagram for the space X to define the homology.

Insert a picture here.

The values for the boundary maps are

$$\begin{aligned}\partial e_i &= v_0 - v_0 = 0 \\ \partial f_0 &= 3e_0 \\ \partial f_i &= 2e_i + e_{i-1} \quad 1 \leq i \leq n\end{aligned}$$

For $H_2(X)$ we have that $\text{im } \partial_3 = 0$. For $\ker \partial_2$ start with the matrix

$$\begin{pmatrix} 3 & 1 & 0 & \cdots & 0 \\ 0 & 2 & 1 & \cdots & \vdots \\ \vdots & 0 & & & \end{pmatrix}$$