

Problem 1 (2.2.35). Use the Mayer-Vietoris sequence to show that a nonorientable closed surface, or more generally a finite simplicial complex X for which $H_1(X)$ contains torsion, cannot be embedded as a subspace of \mathbb{R}^3 in such a way as to have a neighborhood homeomorphic to the mapping cylinder of some map from a closed orientable surface to X . [This assumption on a neighborhood is in fact not needed if one deduces the result from Alexander duality in 3.3]

Proof.

□

Problem 2 (2.2.38). Show that the commutative diagram

$$\begin{array}{ccccccc}
 \cdots & \longrightarrow & C_{n+1} & & B_n & \longrightarrow & C_n & & B_{n-1} & \longrightarrow & \cdots \\
 & & \downarrow & \searrow & \uparrow & \downarrow & \downarrow & \searrow & \uparrow & & \\
 & & & & A_n & & & & A_{n-1} & & \\
 & & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & & \\
 \cdots & \longrightarrow & E_{n+1} & & D_n & \longrightarrow & E_n & & D_{n-1} & \longrightarrow & \cdots
 \end{array}$$

with the two sequences across the top and bottom exact, gives rise to an exact sequence

$$\cdots \rightarrow E_{n+1} \rightarrow B_n \rightarrow C_n \oplus D_n \rightarrow E_n \rightarrow B_{n-1} \rightarrow \cdots$$

where the maps are obtained from those in the previous diagram in the obvious way, except that $B_n \rightarrow C_n \oplus D_n$ has a minus sign in one coordinate.

Proof.

□

Problem 3 (2.B.1). Compute $H_i(S^n \setminus X)$ when X is the subspace of S^n homeomorphic to $S^k \vee S^\ell$ or to $S^k \amalg S^\ell$.

Proof.

□