Problem 1 (2.1.17). *(a) Compute the homology groups $H_n(X, A)$ when X is S^2 or $S^1 \times S^1$ and A is a finite set of points in X.*

(b) Compute the groups $H_n(X, A)$ and $H_n(X, B)$ for X a closed orientable surface of genus two with A and B the circles shown (A is circle around join of Tori and B is around tube of right tori). [What are X/A and X/B.]

Proof. $\square$

Problem 2 (2.2.40). *From the long exact sequence of homology groups associated to the short exact sequence of chain complexes $0 \to C_i(X) \xrightarrow{n} C_i(X) \to C_i(X; \mathbb{Z}_n) \to 0$ deduce immediately that there are short exact sequences*

$$0 \to H_i(X)/nH_i(X) \to H_i(X; \mathbb{Z}_n) \to n - Torsion(H_{i-1}(X)) \to 0$$

where $n-Torsion(G)$ is the kernel of the map $G \xrightarrow{n} G$, $g \mapsto ng$. Use this to show that $\widetilde{H}_i(X; \mathbb{Z}_p) = 0$ for all i and all primes p iff $\widetilde{H}_i(X)$ is a vector space over $\mathbb{Q}$ for all i.

Proof. $\square$

Problem 3 (2.2.43(a)). *Show that a chain complex of a free abelian groups C_n splits as a direct sum of subcomplexes $0 \to L_{n+1} \to K_n \to 0$ with at most two nonzero terms. [Show that the short exact sequence $0 \to \ker \partial \to C_n \to im\ \partial \to 0$ splits and take $K_n = \ker \partial$.]*

Proof. $\square$

Problem 4 (2.3.4). *Show that the wedge axiom for homology theories follows from the other axioms in the case of finite wedge sums.*

Proof. $\square$

Problem 5 (2.B.5). *Let S be an embedded k-sphere in S^n for which there exists a disk $D^n \subset S^n$ intersecting S in the disk $D^k \subset D^n$ defined by the first k coordinates of D^n. Let $D^{n-k} \subset D^n$ be the disk defined by the last $n-k$ coordinates, with boundary sphere S^{n-k-1}. Show that the inclusions $S^{n-k-1} \hookrightarrow S^n \setminus S$ induces an isomorphism on homology groups.*

Proof. $\square$

Problem 6 (2.B.10). *Use the transfer sequence for the covering $S^\infty \to \mathbb{R}P^\infty$ to compute $H_n(\mathbb{R}P^\infty; \mathbb{Z}_2)$.*

Proof. $\square$