Problem 1 (22). *Prove by induction on dimension the following facts about the homology of a finite-dimensional CW complex X, using the observation that X^n/X^{n-1} is a wedge sum of n-spheres.*

(a) *If X has dimension n then $H_i(X) = 0$ for $i > n$ and $H_n(X)$ is free.*

(b) *$H_n(X)$ is free with basis in bijective correspondence with the n-cells if there are no cells of dimension $n-1$ or $n+1$.*

(c) *If X has k n-cells then $H_n(X)$ is generated by at most k elements.*

Proof. □

Problem 2 (26). *Show that $H_1(X, A)$ is not isomorphic to $\widetilde{H}_1(X/A)$ if $X = [0, 1]$ and A is the sequence $1, \frac{1}{2}, \frac{1}{3}, \ldots$ together with its limit 0. [See Example 1.25.]*

Proof. □

Problem 3 (27). *Let $f : (X, A)(Y, B)$ be a map that both $f : X \to Y$ and the restriction $f : A \to B$ are homotopy equivalences.*

(a) *Show that $f_* : H_n(X, A) \to H_n(Y, B)$ is an isomorphism for all n.*

(b) *For the inclusion $f : (D^n, S^{n-1}) \to (D^n, D^n - \{0\})$, show that f is not a homotopy equivalence of pairs —there is no $g : (D^n, D^n - \{0\}) \to (D^n, S^{n-1})$ such that fg and gf are homotopic to the identity through maps of pairs. [Observe that a homotopy equivalence of pairs $(X, A) \to (Y, B)$ is also a homotopy equivalence for the pairs obtained by replacing A and B by their closures.]*

Proof. □