Problem 1 (11).

Proof. Let A be a retract of X. Then there is a map such that $r : X \to A$ which when composed with inclusion we have $r \circ i = id_A$. However when we look at the induced maps on homology we see that $r_* \circ i_* = id_{A*}$. As such i_* has a left inverse and it then follows that i_* is injective. □

Problem 2 (12). *Show that chain homotopy of chain maps is an equivalence relation.*

Proof. Let $f_\# \sim g_\#$ denote that $f_\#, g_\#$ are chain homotopic. Now we verify that chain homotopy is an equivalence relation.

- Let P be the trivial map. Then $\partial P + P\partial = 0 = f_\# - f_\#$. Thus $\sim$ is reflexive.
- Suppose that $f_\# \sim g_\#$ with prism P. Then for $-P$ we have that

  $$(\partial - P) + (-P)\partial = -(\partial P + P\partial) = -(f_\# - g_\#) = g_\# - f_\#$$

  Thus $\sim$ is reflexive.
- Finally let $f_\# \sim g_\#$ and $g_\# \sim h_\#$ with prism P and Q respectively. Then

  $$\partial(P + Q) + (P + Q)\partial = \partial P + P\partial + \partial Q + Q\partial = f_\# - g_\# + g_\# - h_\# = f_\# - h_\#$$

  Thus $\sim$ is transitive.

Therefore, since it is reflexive, symmetric, and transitive it is an equivalence relation. □

Problem 3 (14).

Proof. □