Problem 1 (28a). *Use the Mayer-Vietoris sequence to compute the homology groups of the space obtained from a torus $S^1 \times S^1$ by attaching a Möbius band via a homeomorphism from the boundary circle of the Möbius band to the circle $S^1 \times \{x_0\}$ in the torus.*

Proof. ☐

Problem 2 (29). *The surface M_g of genus g, embedded in $\mathbb{R}^3$ in the standard way, bounds a compact region R. Two copies of R, glued together by the identity map between boundary surfaces M_g, form a closed 3-manifold X. Compute the homology groups of X into two copies of R. Also compute the relative groups $H_i(R, M_g)$.*

Proof. ☐

Problem 3 (30). *For the mapping torus T_f of a map $f : X \to X$, we constructed in Example 2.48 a long exact sequence*

$$\cdots \longrightarrow H_n(X) \xrightarrow{\mathrm{id}-f_*} H_n(X) \longrightarrow H_n(T_f) \longrightarrow H_{n-1}(X) \longrightarrow \cdots$$

Use this to compute the homology of the mapping tori of the following maps:

(a) A reflection $S^2 \to S^2$

(b) A map $S^2 \to S^2$ of degree 2.

(c) The map $S^1 \times S^1 \to S^1 \times S^1$ that is the identity on one factor and a reflection on the other.

(d) The map $S^1 \times S^1 \to S^1 \times S^1$ that is a reflection on each factor.

(e) The map $S^1 \times S^1 \to S^1 \times S^1$ that interchanges the two factors and then reflects one of the factors.

Proof. ☐