

Problem 1 (2.2.36). Show that $H_i(X \times S^n) \cong H_i(X) \oplus H_{i-n}(X)$ for all i and n , where $H_i = 0$ for $i < 0$ by definition. Namely, show $H_i(X \times S^n) \cong H_i(X) \oplus H_i(X \times S^n, X \times \{x_0\})$ and $H_i(X \times S^n, X \times \{x_0\}) \cong H_{i-1}(X \times S^{n-1}, X \times \{x_0\})$ [For the latter isomorphism the relative Mayer-Vietoris sequence yields an easy proof].

Proof.

□

Problem 2 (2.3.3). Show that if \tilde{h} is a reduced homology theory then $\tilde{h}_n(\text{point}) = 0$ for all n . Deduce that there are suspension isomorphisms $\tilde{h}_n(X) \cong \tilde{h}_{n+1}SX$ for all n .

Proof.

□

Problem 3 (2.B.3). Let $(D, S) \subset (D^n, S^{n-1})$ be a pair of subspaces homeomorphic to (D^k, S^{k-1}) , with $D \cap S^{n-1} = S$. Show that the inclusion $S^{n-1} - S \hookrightarrow D^n - D$ induces an isomorphism on homology. [Glue two copies of (D^n, D) to the two ends of $(S^{n-1} \times I, S \times I)$ to produce a k -sphere in S^n and look at the Mayer-Vietoris sequence for the complement of this k -sphere.]

Proof.

□