**Problem 1** (1). *What familiar space is the quotient $\Delta$-complex of a 2-simplex $[v_0, v_1, v_2]$ obtained by identifying the edges $[v_0, v_1]$ and $[v_1, v_2]$, preserving the ordering of vertices?*

Use the following drawing of $X$ as our guide for the boundary maps.
**Insert pretty picture here**
Since we are identifying edges $e_1$ and $e_2$ this also induces the identification of $v_1, v_2$, and $v_3$. We'll call them $e$ and $v$ respectively. Then:

$$\partial f = 2e + e_3$$
$$\partial e = v - v = 0$$
$$\partial e_3 = v - v = 0$$

Then for $H_2(X)$ we have $\mathrm{Im}\partial_3 = 0$ and $\ker \partial_2 = 0$. Thus $H_2(X) \cong 0$.
For $H_1$ we have $\mathrm{Im}\partial_2 = \langle 2e + e_3 \rangle$ and $\ker \partial_1 = \langle e, e_3 \rangle$. Then we have

$$H_1(X) \cong \langle e, e_3 \rangle / \langle 2e + e_3 \rangle \cong \langle e, e_3 | 2e = -e_3 \rangle \cong \langle e \rangle \cong \mathbb{Z}$$

Finally $H_0(X) = \mathbb{Z}$ as there is only a single connected component.
Therefore the homology for $X$ is

$$H_p(X) = \begin{cases} \mathbb{Z} & p = 0, 1 \\ 0 & p \geq 2 \end{cases}$$

It's probably a Möbius strip.

**Problem 2** (4). *Compute the simplicial homology groups of the triangular parachute obtained from $\Delta^2$ by identifying its three vertices to a single point.*

Use the picture below to guide the boundary map for $X$.
**Insert pretty picture here**
The boundary map for our components will be:

$$\partial f = e_1 + e_2 + e_2$$
$$\partial e_1 = v - v = 0$$
$$\partial e_2 = v - v = 0$$
$$\partial e_3 = v - v = 0$$

For $H_2(X)$ we have $\mathrm{Im}\partial_3 = 0$ and $\ker \partial_2 = 0$. Thus $H_2(X) \cong 0$.
For $H_1(X)$ we have $\mathrm{Im}\partial_2 = \langle e_1 + e_2 + e_3 \rangle$ and $\ker \partial_1 = \langle e_1, e_2, e_3 \rangle$. Thus

$$H_1(X) = \langle e_1, e_2, e_3 \rangle / \langle e_1 + e_2 + e_3 \rangle \cong \langle e_1, e_2, e_3 | e_1 + e_2 = -e_3 \rangle \cong \langle e_1 + e_2 \rangle \cong \mathbb{Z} \oplus \mathbb{Z}$$

Finally there is only a single connected component so $H_0(X) \cong \mathbb{Z}$.
Therefore the homology for $X$ is

$$H_p(X) = \begin{cases} \mathbb{Z} & p = 0 \\ \mathbb{Z} \oplus \mathbb{Z} & p = 1 \\ 0 & p \geq 2 \end{cases}$$

**Problem 3** (6)**.** *Compute the simplicial homology groups of the* $\Delta$*-complex obtained from* $n+1$ 2-*simplicies* $\Delta_0^2, \cdots, \Delta_n^2$ *by identifying all three edges of* $\Delta_0^2$ *to a single edge, and for* $i > 0$ *identifying the edges* $[v_0, v_1]$ *and* $[v_1, v_2]$ *of* $\Delta_i^2$ *to a single edge and the edge of* $[v_0, v_2]$ *to the edge* $[v_0, v_1]$ *of* $\Delta_{i-1}^2$.