Problem 1 (3.2.15). For a fixed coefficient field F, define the **Pointcaré series** of a space X to be the formal power series $p(t) = \sum_i a_i t^i$ where a_i is the dimension of $H^i(X; F)$ as a vector space over F, assuming this dimension is finite for all i. Show that $p(X \times Y) = p(X)p(Y)$. Compute the Poincaré series for $S^n, \mathbb{R}P^n, \mathbb{R}P^\infty, \mathbb{C}P^n$, and $\mathbb{C}P^\infty$.

Proof. □

Problem 2 (3.2.16). Show that if X and Y are finite CW complexes such that $H^*(X; \mathbb{Z})$ and $H^*(Y; \mathbb{Z})$ contain no elements of order a power of a given prime p, then the same is true for $X \times Y$. [Apply Theorem 3.15 with coefficients in various fields.]

Proof. □

Problem 3 (3.3.3). Show that every covering space of an orientable manifold is an orientable manifold.

Proof. □