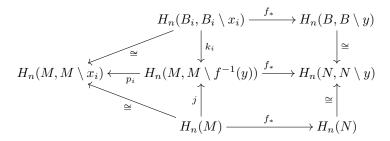
Problem 1 (3.3.8). For a map  $f: M \to N$  between connected closed orientable n-manifolds, suppose there is a ball  $B \subset N$  such that  $f^{-1}(B)$  is the disjoint union of balls  $B_i$  each mapped homeomorphically by f onto B. Show the degree of f is  $\sum_i \epsilon_i$  where  $\epsilon_i$  is +1 or -1 according to whether  $f: B_i \to B$  preserves or reverses local orientations induced from given fundamental classes [M] and [N].

*Proof.* Let  $y \in B$  and  $\{x_1, \ldots, x_m\} = f^{-1}(y)$  where  $x_i \in B_i$ . Note that there must be finitely many  $B_i$ s or the sum in the problem is not well defined. We then have the following adaption of the commutative diagram from the proof of Prop. 2.30 in Hatcher



As in the proposition the upper two arrows come from excision. We will now show that the lower two arrows marked as isomorphisms are actually isomorphisms. Insert proof here.  $\Box$ 

Problem 2 (3.3.9). Show that a p-sheeted covering space projection  $M \to N$  has degree  $\pm p$ , when M and N are connected closed orientable manifolds.

*Proof.* Since covering space projections are local homeomorphisms we can apply the previous problem to this by considering a point  $y \in N$  and its p-preimages. As such it will suffice to show that the local degree of each point in  $f^{-1}(y)$  agrees.

Suppose that not all of the local degrees agree for some subset of N. Then partition M into  $M_+$  and  $M_-$  denoting the points of M where f preserves and reverses local orientation respectively. It's clear from the definition that  $M_+ \cap M_- = \emptyset$ . Moreover both  $M_+$  and  $M_-$  are open as given a point  $x \in M$  f(x) has an open neighborhood U that is oriented and each disjoint sheet in  $f^{-1}(U)$  must either have orientation preserved or reversed. Thus  $M_+$  and  $M_-$  form a partition of M which contradicts our assumption that M was connected.

Therefore given a point  $y \in N$  the local degree must be  $\pm p$  and by the previous problem the degree of f is then  $\pm p$ .

Problem 3 (3.3.10). Show that for a degree 1 map  $f: M \to N$  of connected closed orientable manifolds, the induced map  $f_*: \pi_1 M \to \pi_1 N$  is surjective, hence also  $f_*: H_1(M) \to H_1(N)$ . [Lift f to the covering space  $\widetilde{N} \to N$  corresponding to the subgroup  $\operatorname{Im} f_* \subset \pi_1 N$ , then consider the two cases that this covering is finite sheeted or infinite sheeted.]

$$\square$$