

Problem 1 (3.1.10). For the lens space $L_m(\ell_1, \dots, \ell_n)$ defined in Example 2.43, compute the cohomology groups using the cellular cochain complex and taking coefficients in $\mathbb{Z}, \mathbb{Q}, \mathbb{Z}_m$, and \mathbb{Z}_p for p prime. Verify that the answers agree with those given by the universal coefficient theorem.

Proof.

□

Problem 2 (3.1.11). Let X be a Moore space $M(\mathbb{Z}_m, n)$ obtained from S^n by attaching a cell e^{n+1} by a map of degree m .

- (a) Show that the quotient map $X \rightarrow X/S^n = S^{n+1}$ induces the trivial map on $\tilde{H}_i(-; \mathbb{Z})$ for all i , but not on $H^{n+1}(-; \mathbb{Z})$. Deduce that the splitting in the universal coefficient theorem for cohomology cannot be natural.
- (b) Show that the inclusion $S^n \hookrightarrow X$ induces the trivial map on $\tilde{H}^i(-; \mathbb{Z})$ for all i , but not on $H_n(-; \mathbb{Z})$.

Proof.

□