Problem 1 (3.2.1). Assuming as known the cup product structure on the torus $S^1 \times S^1$, compute the cup product structure in $H^*(M_g)$ for M_g the closed orientable surface of genus g by using the quotient map from M_g to a wedge sum of g tori.

Proof. □

Problem 2 (3.2.2). Using the cup product $H^k(X, A : R) \times H^\ell(X, B; R) \to H^{k+\ell}(X, A \cup B; R)$, show that if X is the union of contractible open subsets A and B, then all cup products of positive-dimensional classes in $H^*(X; R)$ are zero. This applies in particular if X is a suspension. Generalize to the situation that X is a union of n contractible open subsets, to show that the n-fold cup products of positive dimensional classes are zero.

Proof. □

Problem 3 (3.2.4). Apply the Lefschetz fixed point theorem to show that every map $f : \mathbb{C}P^n \to \mathbb{C}P^n$ has a fixed point if n is even, using the fact that $f^* : H^*(\mathbb{C}P^n; \mathbb{Z}) \to H^*(\mathbb{C}P^n; \mathbb{Z})$ is a ring homomorphism. When n is odd show there is a fixed point unless $f^*(\alpha) = -\alpha$, for α a generator of $H^2(\mathbb{C}P^n; \mathbb{Z})$. [See Exercise 3 in §2.C for an example of a map without fixed points in this exceptional case.]

Proof. □