Problem 1 (3.3.1). Show that there exist nonorientable 1-dimensional manifolds if the Hausdorff condition is dropped form the definition of a manifold.

Proof. □

Problem 2 (3.3.11). If M_g denotes the closed orientable surface of genus g, show that degree 1 maps $M_g \to M_h$ exist iff $g \geq h$.

Proof. □

Problem 3 (3.3.16). Show that $(\alpha \frown \varphi) \frown \psi = \alpha \frown (\varphi \smile \psi)$ for all $\alpha \in C_k(X;R)$, $\varphi \in C^l(X;R)$, and $\psi \in C^m(X;R)$. Deduce that cap product makes $H_*(X;R)$ a right $H^*(X;R)$-module.

Proof. □

Problem 4 (3.3.17). Show that a direct limit of exact sequences is exact. More generally show that homology commutes with direct limits: If $\{C_\alpha, f_{\alpha\beta}\}$ is a directed system of chain complexes, with the maps $f_{\alpha\beta} : C_\alpha \to C_\beta$ chain maps, then $H_n(\varinjlim C_\alpha) = \varinjlim H_n(C_\alpha)$.

Proof. □

Problem 5 (3.3.20). Show that $H_c^0(X;G) = 0$ if X is path-connected and noncompact.

Proof. □

Problem 6 (3.3.25). Show that if a closed orientable manifold M of dimension $2k$ has $H_{k-1}(M;\mathbb{Z})$ torsion-free, then $H_k(M;\mathbb{Z})$ is also torsion-free.

Proof. □

Problem 7 (3.3.32). Show that a compact manifold does not retract onto its boundary.

Proof. □

Problem 8 (3.3.33). Show that if M is a compact contractible n-manifold then ∂M is a homology $(n-1)$-sphere.

Proof. □