Problem 1 (3.3.8). For a map $f : M \to N$ between connected closed orientable n-manifolds, suppose there is a ball $B \subset N$ such that $f^{-1}(B)$ is the disjoint union of balls B_i each mapped homeomorphically by f onto B. Show the degree of f is $\sum_i \epsilon_i$ where ϵ_i is $+1$ or -1 according to whether $f : B_i \to B$ preserves or reverses local orientations induced from given fundamental classes $[M]$ and $[N]$.

Proof. Let $y \in B$ and $\{x_1, \ldots, x_m\} = f^{-1}(y)$ where $x_i \in B_i$. Note that there must be finitely many B_is or the sum in the problem is not well defined. We then have the following adaption of the commutative diagram from the proof of Prop. 2.30 in Hatcher

$$\begin{tikzcd}
 & H_n(B_i, B_i \setminus x_i) \arrow[r, "f_*"] \arrow[dl, "\cong"'] \arrow[d, "k_i"] & H_n(B, B \setminus y) \arrow[d, "\cong"] \\
H_n(M, M \setminus x_i) & H_n(M, M \setminus f^{-1}(y)) \arrow[l, "p_i"'] \arrow[r, "f_*"] & H_n(N, N \setminus y) \\
 & H_n(M) \arrow[u, "j"] \arrow[ul, "\cong"] \arrow[r, "f_*"] & H_n(N) \arrow[u, "\cong"']
\end{tikzcd}$$

As in the proposition the upper two arrows come from excision. We will now show that the lower two arrows marked as isomorphisms are actually isomorphisms. **Insert proof here**. $\square$

Problem 2 (3.3.9). Show that a p-sheeted covering space projection $M \to N$ has degree $\pm p$, when M and N are connected closed orientable manifolds.

Proof. Since covering space projections are local homeomorphisms we can apply the previous problem to this by considering a point $y \in N$ and its p-preimages. As such it will suffice to show that the local degree of each point in $f^{-1}(y)$ agrees.

Suppose that not all of the local degrees agree for some subset of N. Then partition M into M_+ and M_- denoting the points of M where f preserves and reverses local orientation respectively. It's clear from the definition that $M_+ \cap M_- = \emptyset$. Moreover both M_+ and M_- are open as given a point $x \in M$ $f(x)$ has an open neighborhood U that is oriented and each disjoint sheet in $f^{-1}(U)$ must either have orientation preserved or reversed. Thus M_+ and M_- form a partition of M which contradicts our assumption that M was connected.

Therefore given a point $y \in N$ the local degree must be $\pm p$ and by the previous problem the degree of f is then $\pm p$. $\square$

Problem 3 (3.3.10). Show that for a degree 1 map $f : M \to N$ of connected closed orientable manifolds, the induced map $f_* : \pi_1 M \to \pi_1 N$ is surjective, hence also $f_* : H_1(M) \to H_1(N)$. [Lift f to the covering space $\widetilde{N} \to N$ corresponding to the subgroup $\operatorname{Im} f_* \subset \pi_1 N$, then consider the two cases that this covering is finite sheeted or infinite sheeted.]

Proof. $\square$