Problem 1. *On an open set* $U \subset \mathbb{R}^{\ltimes}$ *show that the exterior derivative* d *is the only operator* $d : \Omega^p(U) \to \Omega^{p+1}(U)$ *satisfying:*

(a) $d(\omega + \eta) = d\omega + \eta$

(b) $\omega \in \Omega^p(U), \eta \in \Omega^q(U) \Rightarrow d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^p \omega \wedge d\eta$

(c) $f \in \Omega^0(U) \Rightarrow df X) = X(f)$

(d) $f \in \Omega^0(U) \Rightarrow d(df) = 0$

Deduce that d *is independent of the coordinate system used to define it.*

Proof. $\square$

Problem 2. *On the unit circle* S^1 *in the plane, let* $\theta = \arctan(y/x)$ *be the usual polar coordinate. Show that* $d\theta$ *makes sense on* S^1 *and is a closed 1-form which is not exact.*

Proof. $\square$

Problem 3. *For* $\omega \in \Omega^1(M)$, *verify the special case* $d\omega(X, Y) = X(\omega(Y)) - Y(\omega(X)) - \omega([X, Y])$ *of the invariant formula mentioned above Definition 2.4.*

Proof. $\square$