Problem 1. *Consider the real valued function* $f(x,y,z) = (2-(x^2+y^2)^{1/2})^2+z^2$ *on* $\mathbb{R}^3\setminus\{(0,0,z)\}$. *Show that* 1 *is a regular value of* f. *Identify the manifold* $M = f^{-1}(1)$.

First we calculate the partials

$$\frac{\partial f}{\partial x} = x\left(2 - \frac{4}{(x^2+y^2)^{1/2}}\right), \quad \frac{\partial f}{\partial y} = y\left(2 - \frac{4}{(x^2+y^2)^{1/2}}\right), \quad \frac{\partial f}{\partial z} = 2z$$

Then the critical points of f are those for which f_* is not surjective.
The manifold is the torus T^2.

Problem 2. *Show that the manifold* M *of Problem 1 is transverse to the surface*

$$N = \{(x,y,z) \in \mathbb{R}^3 | x^2+y^2 = 4\}$$

Identify the manifold $M \cap N$.

The manifold is two disjoint copies of S^1.

Problem 3. *Show that the manifold* M *of Problem 1 is not transverse to the surface*

$$N = \{(x,y,z) \in \mathbb{R}^3 | x^2+y^2 = 1\}$$

Is $M \cap N$ *a manifold?*

It is a manifold and it is a single copy of S^1.