Problem 1. *Deduce from de Rham's Theorem that if* $U \subset \mathbb{R}^3$ *is open and* $H^1(U) = 0$ *then any vector field* $\vec{F}$ *on* U *with curl* $\vec{F} = 0$ *is a gradient field.*

Proof. If there was a vector field with zero curl that was not a gradient field that would imply that the $H^1(U) \neq 0$. As such the above property must hold for any vector field. □

Problem 2. *Deduce from de Rham's Theorem that if* $U \subset \mathbb{R}^3$ *is open and* $H^2(U) = 0$ *then any vector field* $\vec{F}$ *on* U *with div* $\vec{F} = 0$ *has the form* $\vec{F} =$ *curl* $\vec{G}$ *for some vector field* $\vec{G}$ *on* U*.*

Proof. Similar to above if there were such a vector field that would imply that $H^2(U) \neq 0$. As such if a vector field has zero divergence then it must be the curl of some vector field. □