Problem 1. *Consider the 3-sphere as the set of unit quaternions*

$$\{x + iy + jz + kw | x^2 + y^2 + z^2 + w^2 = 1\}$$

Let $\phi : U \to \mathbb{R}^3$ *be* $\phi(x + iy + jz + kw) = (y, z, w) \in \mathbb{R}^3$ *where*

$$U = \{x + iy + jz + kw \in S^3 | x > 0\}$$

Consider ϕ *as a chart. For each* $q \in S^3$*, define a chart* ψ_q *by* $\psi_q(p) = \phi(q^{-1}p)$*. Show that this set of charts is an atlas for a smooth structure on* S^3*.*

Proof. □

Problem 2. *Let* X *be a copy of the real line* $\mathbb{R}$ *and let* $\phi : X \to \mathbb{R}$ *be* $\phi(x) = x^3$*. Taking* ϕ *as a chart, this defines a smooth structure on* X*. Prove or disprove the following statements:*

1. X *is diffeomorphic to* $\mathbb{R}$*;*
2. *the identity map* $X \to \mathbb{R}$ *is a diffeomorphism;*
3. ϕ *together with the identity map comprise an atlas;*
4. *on the one-point compactification* X^+ *of* X*,* ϕ, ψ *give an atlas, where* $\psi(x) = 1/x$*, for* $x \neq 0, \infty$*, and* $\psi(\infty) = 0$*. (*ψ *is defined on* $X^+ \setminus \{0\}$*.)*

Proof. □