

**Problem 1.** Consider the real valued function  $f(x, y, z) = (2 - (x^2 + y^2)^{1/2})^2 + z^2$  on  $\mathbb{R}^3 \setminus \{(0, 0, z)\}$ . Show that 1 is a regular value of  $f$ . Identify the manifold  $M = f^{-1}(1)$ .

*Proof.*

□

**Problem 2.** Show that the manifold  $M$  of Problem 1 is transverse to the surface

$$N = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 4\}$$

Identify the manifold  $M \cap N$ .

*Proof.*

□

**Problem 3.** Show that the manifold  $M$  of Problem 1 is not transverse to the surface

$$N = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$$

Is  $M \cap N$  a manifold?

*Proof.*

□