Problem 1. *On the 2-sphere, consider the flow*

$$\theta(t, \langle x, y, z\rangle) = \langle x, y\cos(t) - z\sin(t), y\sin(t) + z\cos(t)\rangle$$

Find the vector field on S^2 induced by this flow.

Problem 2. *Consider the vector field $\xi(x) = x$ on $\mathbb{R}$. Show that ξ is the tangent field to a flow, and find the flow. (Hint: In classical notation, this vector field corresponds to the initial value problem $dy/dt = y, y(0) = x$.)*

Problem 3 (4)**.** *If X and Y vector fields on M then XY makes sense as an operator on smooth real valued functions on M. Show that $[X, Y] = XY - YX$ is a vector field. (This is called the "Lie bracket" of X and Y. Sometimes it is defined with opposite sign.) Also show t hat XY itself is not a vector field.*

Proof. □

Problem 4 (5)**.** *Show that the Klein Bottle has an everywhere nonzero vector field. Describe the resulting flow.*