**Technical Document:**

**Optimization Models for Capital Budgeting**

**Nuclear Plant Asset Management Tool: NuPAT**

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**1.** **OVERVIEW**

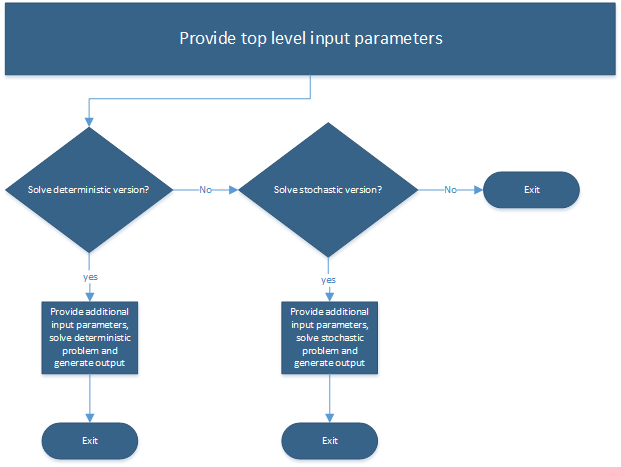
This is a technical document for **NuPAT**, a nuclear plant asset management tool. The document provides: a) the required inputs; b) the obtained outputs; and, c) the workflow needed to move from a) to b). The document describes this workflow in two ways. First, the mathematical optimization model is given, and here the required inputs are specified by sets and parameters, and the outputs are specified by the model’s decision variables. This abstract mathematical description is accompanied by flowcharts, and more detailed enumerated lists, to help make the step-by-step workflow concrete. Second, we specify the form of the inputs via the required data structures to use Pyomo, a Python-based open-source software package for expressing and solving optimization models; see Hart et al. (2011, 2017). We further detail model implementation and model output using Pyomo via an example.

The tool we envision has two sub-tools. The user will choose whether to use the sub-tool to solve a capital budgeting model in which project costs and available resources are assumed to be known, or whether to use the second sub-tool in which these key problem parameters are modeled as being uncertain through a user-specified probability distribution. We refer to the two corresponding optimization models in these two sub-tools as a **deterministic capital budgeting model** and a **stochastic capital budgeting model**.

Aside from handling the issue of uncertainty in modeling project costs and available resources, the required inputs and other options are common to the deterministic and stochastic capital budgeting models. For this reason, we first describe the simpler deterministic model and its inputs, outputs, and workflow. Then we generalize to the stochastic setting.

Importantly, even if a user wishes to assume that project costs and available resources are uncertain, we anticipate that the user would start by using NuPAT’s *deterministic* capital budgeting tool. Within the deterministic tool, the user can carry out “what-if” analysis; i.e., the user can employ the tool multiple times varying key uncertainties in a parametric way to understand how the selected portfolio of projects, and associated net present value, changes. Insights obtained from such what-if analysis will help facilitate understanding of the output of NuPAT’s *stochastic* capital budgeting sub-tool.

We detail further options that the user has in what follows, and just briefly indicate that these involve problem specifications such as: the number of years in the planning horizon; the set of candidate projects; options for how to perform certain projects; the number and type of resources that projects consume; and, so-called piggy-backing options, in which selection of one project allows selection of another project that would otherwise be unavailable or more expensive. Figure 1 shows the general steps of the NuPAT tool.



*Figure 1: High-level overview of the NuPAT tool with sub-tools corresponding to deterministic and stochastic capital budgeting models.*

**2. DETERMINISTIC CAPITAL BUDGETING TOOL**

We consider a capital budgeting problem for a nuclear generation station, with possible extension to a larger fleet of plants. Due to limited resources, we can only select a subset from several candidate capital projects. Our goal is to maximize overall net present value (NPV). In doing so, we must respect resource limits and capture key structural and stochastic dependencies of the system. Example projects include upgrading a steam turbine, refurbishing or replacing a set of reactor coolant pumps, and replacing a set of feed-water heaters. We provide a specific example that illustrates the steps of the model and optimization algorithm.

**2.1. Deterministic Capital Budgeting Optimization Model: Mathematical Formulation**

*Indices and sets:*

candidate projects

options for selecting project , e.g., initiate project in year or and in a standard

(three year) or in an expedited (two year) manner

option for project can be selected only if option is selected for

project

types of resources, e.g., capital funds, O&M funds, labor-hours, time during outage

time periods (years)

*Data:*

reward (revenue less financial cost) in year of selecting project via option

available budget for a resource of type in year

consumption of resource of type in year if project is performed via option

*Decision variables:*

takes value 1 if project is selected via option

*Optimization model formulation:*

The decision variables, , indicate whether we choose to do project by means. Restated, if , then we recommend doing project via option , and taken together these decision variables produce a schedule for performing projects over time. The set of available options, , typically (more below) includes the “do-nothing” option, and the first constraint ensures that we choose exactly one option from the available set for each project. Even if we select the do-nothing option for a project, it induces an NPV, , which can may be negative, representing growing O&M costs, losses in plant efficiency, etc. The second structural constraint ensures that the budget of each resource is respected in each year . The third structural constraint captures piggybacking situations in which option for project (which may have cheaper costs) may be selected only if project-option pair is also selected. The objective function includes the NPV for each project-option pair, , and the correct NPV is selected by the 0-1 decision variable, .

We note that sometimes there are projects that must be done, e.g., for safety and/or regulatory reasons. This can be handled within the formulation just given, without introducing additional constructs. The set typically includes a do-nothing option for each project, but when project must be done, we simply do not include the do-nothing option.

The deterministic model can be repeatedly solved by changing the input values. This will allow for what-if sensitivity analysis to identify the crucial drivers behind the optimal project selection decision.

**2.2. Deterministic Capital Budgeting Workflow:**

1. Input: data structure

* Candidate projects: I [Set]
* Project options: J [Set]
* Project-option pairs: IJ [Set within IxJ]
* Resources: K [Set]
* Time periods: T [Set]
* NPV: r[i,j]: two-dimensional real array on IJ
* Budget: b[k,t]: two-dimensional real array on KxT
* Cost: c[i,j,k,t]: four-dimensional real array on IJxKxT

2. Output:

* Yes-no project-option pairs: x: two-dimensional binary array on IJ
* Overall NPV for portfolio of selected projects: NPV: scalar, real

**2.3. Deterministic Capital Budgeting Example:**

|  |  |
| --- | --- |
| Project # | Description |
| 1 | Feedwater heater replacement |
| 2 | Emergency sump overhaul |
| 3 | Transformer replacement |
| 4 | Replace turbine-hall crane |
| 5 | Replacement of four reactor coolant pumps |
| 6 | Corrosion mitigation of buried piping |
| 7 | Refurbish turbine governors |
| 8 | Service circulation water pumps |
| 9 | Maintain RAOC system |
| 10 | Replace piping in high-pressure coolant injection system |
| 11 | Replace chiller |
| 12 | Vessel-head welding remediation |
| 13 | Maintain main condenser |
| 14 | Replace instrumentation and control cables |
| 15 | Upgrade power feeds to large cranes |
| 16 | Replace moisture separator reheater |

*Table 1: Example projects*

**Candidate projects:**

Set I :=

FeedwaterHeater

EmergencySump

…

CranePowerFeed

MoisterSeparatorReheater

;

**Project options:**

Set J := PlanA PlanB DoNothing ;

This set includes all possible options for any project. If a plan could be done in a piggybacking manner, i.e., contingent on another project, then that could be included as another option in this set, but we haven’t included that in this example. In the current example we associate PlanA with the initial timing of the projects. PlanB can be considered as the timing option where we shift some of the projects starting time one year in the future.

**Project-option pairs:**

Set IJ :=

FeedwaterHeater PlanA

FeedwaterHeater PlanB

FeedwaterHeater DoNothing

EmergencySump PlanA

EmergencySump DoNothing

…

CranePowerFeed PlanA

CranePowerFeed PlanB

MoisterSeparatorReheater PlanA

;

Note that there are “Plan A”, “Plan B”, and “Do nothing” options for the first project, “Plan A” and “Do nothing” options for the second project, and there is no “Do nothing” option for the final two projects. The first nine projects include do-nothing options, and the last seven projects are projects that must be done for safety reasons, and hence none of these include the do-nothing option. Note that there are two options for how the penultimate project can be performed, but there’s only one option for the final project. Similarly, the second project has no “Plan B” option.

**Resources:**

Set K := CapitalFunds OandMFunds ;

The example distinguishes two colors of money, capital funds and O&M funds. Additional types of resources could include labor-hours and time during outage.

**Time periods:**

Set T := year1 year2 year3 year4 year 5;

**NPV:**

param r :=

FeedwaterHeater PlanA 2.315

FeedwaterHeater PlanB 2.205

FeedwaterHeater DoNothing -1.101

EmergencySump PlanA 0.824

EmergencySump DoNothing -0.400

…

CranePowerFeed PlanA -0.246

CranePowerFeed PlanB -0.258

MoisterSeparatorReheater PlanA -20.155

;

**Budget:**

param b :=

CapitalFunds year1 0.665

CapitalFunds year2 4.686

CapitalFunds year3 6.725

CapitalFunds year4 0.539

CapitalFunds year5 0.500

OandMFunds year1 0.000

OandMFunds year2 2.027

OandMFunds year3 4.917

OandMFunds year4 3.320

OandMFunds year5 1.683

;

**Cost:**

param c :=

FeedwaterHeater PlanA CapitalFunds year1 0.219

FeedwaterHeater PlanA CapitalFunds year2 0.257

FeedwaterHeater PlanA OandMFunds year3 0.085

FeedwaterHeater PlanB CapitalFunds year2 0.219

FeedwaterHeater PlanB CapitalFunds year3 0.257

FeedwaterHeater PlanB OandMFunds year4 0.085

FeedwaterHeater DoNothing OandMFunds year1 0.000

FeedwaterHeater DoNothing OandMFunds year2 0.000

FeedwaterHeater DoNothing OandMFunds year3 0.000

FeedwaterHeater DoNothing OandMFunds year4 0.000

FeedwaterHeater DoNothing OandMFunds year5 0.000

…

;

The displayed rows concern the first project. Plan A amounts to doing the project over years 1 and 2 with O&M costs incurred in year 3, and plan B is identical except shifted to years 2 and 3 with O&M costs incurred in year 4. The costs incurred in all years that not listed are zero, i.e., this format allows for sparse data entry. The FeedwaterHeater-DoNothing pair incurs no costs for capital funds (not listed), and similarly incurs no O&M costs (listed explicitly so it is clear these costs truly are meant to be zero) but incurs a negative NPV of -1.101 as indicated earlier.

**2.4. Deterministic Capital Budgeting Input Data:**

We show numerical results for three different cases:

*Case 1*: This is the simplest case, where optimal project selection is done using PlanA and DoNothing options and one resource based on CapitalFunds. The input file for this case is titled “Deterministic\_Input\_Case1.dat”. It has the following format:

Input file “Deterministic\_Input\_Case1.dat” starts here:

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Data in AMPL format

set T := year1 year2 year3 year4 year5;

set I :=

FeedwaterHeater

EmergencySump

Transformer

TurbineHallCrane

ReactorCoolantPumps

MitigateBuriedPiping

RefurbishTurbineGovernors

ServiceCircWaterPumps

MaintainRAOC

ReplaceCoolantInjectPiping

ReplaceChiller

VessellHeadWeldRemediate

MaintainCondensor

ReplaceConrolCables

CranePowerFeed

MoisterSeparatorReheater

;

set J := PlanA DoNothing;

set K := CapitalFunds;

set IJ:=

FeedwaterHeater PlanA

EmergencySump PlanA

Transformer PlanA

TurbineHallCrane PlanA

ReactorCoolantPumps PlanA

MitigateBuriedPiping PlanA

RefurbishTurbineGovernors PlanA

ServiceCircWaterPumps PlanA

MaintainRAOC PlanA

ReplaceCoolantInjectPiping PlanA

ReplaceChiller PlanA

VessellHeadWeldRemediate PlanA

MaintainCondensor PlanA

ReplaceConrolCables PlanA

CranePowerFeed PlanA

MoisterSeparatorReheater PlanA

FeedwaterHeater DoNothing

EmergencySump DoNothing

Transformer DoNothing

TurbineHallCrane DoNothing

ReactorCoolantPumps DoNothing

MitigateBuriedPiping DoNothing

RefurbishTurbineGovernors DoNothing

ServiceCircWaterPumps DoNothing

MaintainRAOC DoNothing

;

param r:=

FeedwaterHeater DoNothing -1.101

EmergencySump DoNothing -0.400

Transformer DoNothing -10.100

TurbineHallCrane DoNothing -30.200

ReactorCoolantPumps DoNothing -0.300

MitigateBuriedPiping DoNothing -2.500

RefurbishTurbineGovernors DoNothing -2.000

ServiceCircWaterPumps DoNothing -0.250

MaintainRAOC DoNothing -0.050

FeedwaterHeater PlanA 2.315

EmergencySump PlanA 0.824

Transformer PlanA 22.459

TurbineHallCrane PlanA 60.589

ReactorCoolantPumps PlanA 0.667

MitigateBuriedPiping PlanA 5.173

RefurbishTurbineGovernors PlanA 4.003

ServiceCircWaterPumps PlanA 0.582

MaintainRAOC PlanA 0.122

ReplaceCoolantInjectPiping PlanA -2.870

ReplaceChiller PlanA -0.322

VessellHeadWeldRemediate PlanA -0.279

MaintainCondensor PlanA -3.996

ReplaceConrolCables PlanA -0.102

CranePowerFeed PlanA -0.246

MoisterSeparatorReheater PlanA -20.155

;

param b:=

CapitalFunds year1 11.000

CapitalFunds year2 13.0

CapitalFunds year3 13.0

CapitalFunds year4 11.4

CapitalFunds year5 11.5

;

param c:=

FeedwaterHeater PlanA CapitalFunds year1 0.219

FeedwaterHeater PlanA CapitalFunds year2 0.257

FeedwaterHeater PlanA CapitalFunds year3 0.085

EmergencySump PlanA CapitalFunds year3 0.122

EmergencySump PlanA CapitalFunds year4 0.103

EmergencySump PlanA CapitalFunds year5 0.013

Transformer PlanA CapitalFunds year1 5.044

Transformer PlanA CapitalFunds year2 1.839

TurbineHallCrane PlanA CapitalFunds year1 6.740

TurbineHallCrane PlanA CapitalFunds year2 6.134

TurbineHallCrane PlanA CapitalFunds year3 10.442

ReactorCoolantPumps PlanA CapitalFunds year1 0.425

MitigateBuriedPiping PlanA CapitalFunds year1 2.125

RefurbishTurbineGovernors PlanA CapitalFunds year1 2.387

RefurbishTurbineGovernors PlanA CapitalFunds year2 0.190

RefurbishTurbineGovernors PlanA CapitalFunds year3 0.012

RefurbishTurbineGovernors PlanA CapitalFunds year4 2.383

RefurbishTurbineGovernors PlanA CapitalFunds year5 0.192

ServiceCircWaterPumps PlanA CapitalFunds year2 0.950

MaintainRAOC PlanA CapitalFunds year1 0.030

MaintainRAOC PlanA CapitalFunds year2 0.030

MaintainRAOC PlanA CapitalFunds year3 0.688

ReplaceCoolantInjectPiping PlanA CapitalFunds year2 0.200

ReplaceCoolantInjectPiping PlanA CapitalFunds year3 0.763

ReplaceCoolantInjectPiping PlanA CapitalFunds year4 0.739

ReplaceCoolantInjectPiping PlanA CapitalFunds year5 2.539

ReplaceChiller PlanA CapitalFunds year1 0.347

ReplaceChiller PlanA CapitalFunds year2 0.347

VessellHeadWeldRemediate PlanA CapitalFunds year1 0.300

VessellHeadWeldRemediate PlanA CapitalFunds year2 0.300

MaintainCondensor PlanA CapitalFunds year1 4.025

MaintainCondensor PlanA CapitalFunds year2 0.297

MaintainCondensor PlanA CapitalFunds year2 4.025

MaintainCondensor PlanA CapitalFunds year3 0.297

ReplaceConrolCables PlanA CapitalFunds year1 0.081

ReplaceConrolCables PlanA CapitalFunds year2 0.032

ReplaceConrolCables PlanA CapitalFunds year2 0.081

ReplaceConrolCables PlanA CapitalFunds year3 0.032

CranePowerFeed PlanA CapitalFunds year1 0.095

CranePowerFeed PlanA CapitalFunds year2 0.095

CranePowerFeed PlanA CapitalFunds year3 0.095

CranePowerFeed PlanA CapitalFunds year2 0.095

CranePowerFeed PlanA CapitalFunds year3 0.095

CranePowerFeed PlanA CapitalFunds year4 0.095

MoisterSeparatorReheater PlanA CapitalFunds year1 5.488

MoisterSeparatorReheater PlanA CapitalFunds year2 5.665

MoisterSeparatorReheater PlanA CapitalFunds year3 0.501

MoisterSeparatorReheater PlanA CapitalFunds year4 6.803

MoisterSeparatorReheater PlanA CapitalFunds year5 6.778

;

Note that this is the example in Koc et al. (2009), but instead of using the nine projects with positive NPV values, we have extended the flexibility of the model by allowing to enter all sixteen projects, and for the mandatory projects there is no DoNothing option. In other words, the last seven projects must be selected.

*Case 2*: This case extends the analysis done in Case 1 by allowing a second option, PlanB, and a second resource OandMFunds. PlanB shifts the starting time for some of the projects to one year later. The costs and budgets are split between the two available resources CapitalFunds and OandMFunds. Note that this use case is new, and there are no published results to compare. However, we use the same Pyomo code that replicated the results from Table 2 from Koc et al. (2009), and so the output is reliable.

The input file for Case 2 is titled “Deterministic\_Input\_Case2.dat” and is shown next:

## Data in AMPL format

set T := year1 year2 year3 year4 year5;

set I :=

FeedwaterHeater

EmergencySump

Transformer

TurbineHallCrane

ReactorCoolantPumps

MitigateBuriedPiping

RefurbishTurbineGovernors

ServiceCircWaterPumps

MaintainRAOC

ReplaceCoolantInjectPiping

ReplaceChiller

VessellHeadWeldRemediate

MaintainCondensor

ReplaceConrolCables

CranePowerFeed

MoisterSeparatorReheater

;

set J := PlanA PlanB DoNothing;

set K := CapitalFunds OandMFunds;

set IJ:=

FeedwaterHeater PlanA

EmergencySump PlanA

Transformer PlanA

TurbineHallCrane PlanA

ReactorCoolantPumps PlanA

MitigateBuriedPiping PlanA

RefurbishTurbineGovernors PlanA

ServiceCircWaterPumps PlanA

MaintainRAOC PlanA

ReplaceCoolantInjectPiping PlanA

ReplaceChiller PlanA

VessellHeadWeldRemediate PlanA

MaintainCondensor PlanA

ReplaceConrolCables PlanA

CranePowerFeed PlanA

MoisterSeparatorReheater PlanA

FeedwaterHeater PlanB

Transformer PlanB

TurbineHallCrane PlanB

ReactorCoolantPumps PlanB

MitigateBuriedPiping PlanB

ServiceCircWaterPumps PlanB

MaintainRAOC PlanB

ReplaceChiller PlanB

VessellHeadWeldRemediate PlanB

MaintainCondensor PlanB

ReplaceConrolCables PlanB

CranePowerFeed PlanB

FeedwaterHeater DoNothing

EmergencySump DoNothing

Transformer DoNothing

TurbineHallCrane DoNothing

ReactorCoolantPumps DoNothing

MitigateBuriedPiping DoNothing

RefurbishTurbineGovernors DoNothing

ServiceCircWaterPumps DoNothing

MaintainRAOC DoNothing

;

param r:=

FeedwaterHeater PlanA 2.315

EmergencySump PlanA 0.824

Transformer PlanA 22.459

TurbineHallCrane PlanA 60.589

ReactorCoolantPumps PlanA 0.667

MitigateBuriedPiping PlanA 5.173

RefurbishTurbineGovernors PlanA 4.003

ServiceCircWaterPumps PlanA 0.582

MaintainRAOC PlanA 0.122

ReplaceCoolantInjectPiping PlanA -2.870

ReplaceChiller PlanA -0.322

VessellHeadWeldRemediate PlanA -0.279

MaintainCondensor PlanA -3.996

ReplaceConrolCables PlanA -0.102

CranePowerFeed PlanA -0.246

MoisterSeparatorReheater PlanA -20.155

FeedwaterHeater PlanB 2.199

Transformer PlanB 21.336

TurbineHallCrane PlanB 57.559

ReactorCoolantPumps PlanB 0.633

MitigateBuriedPiping PlanB 4.914

ServiceCircWaterPumps PlanB 0.553

MaintainRAOC PlanB 0.116

ReplaceChiller PlanB -0.339

VessellHeadWeldRemediate PlanB -0.293

MaintainCondensor PlanB -4.196

ReplaceConrolCables PlanB -0.108

CranePowerFeed PlanB -0.258

FeedwaterHeater DoNothing -1.101

EmergencySump DoNothing -0.400

Transformer DoNothing -10.100

TurbineHallCrane DoNothing -30.200

ReactorCoolantPumps DoNothing -0.300

MitigateBuriedPiping DoNothing -2.500

RefurbishTurbineGovernors DoNothing -2.000

ServiceCircWaterPumps DoNothing -0.250

MaintainRAOC DoNothing -0.050

;

param b:=

CapitalFunds year1 11.000

CapitalFunds year2 10.974

CapitalFunds year3 8.083

CapitalFunds year4 8.080

CapitalFunds year5 9.817

OandMFunds year1 0.000

OandMFunds year2 2.026

OandMFunds year3 4.917

OandMFunds year4 3.320

OandMFunds year5 1.683

;

param c:=

FeedwaterHeater PlanA CapitalFunds year1 0.219

FeedwaterHeater PlanA CapitalFunds year2 0.257

FeedwaterHeater PlanA OandMFunds year3 0.085

FeedwaterHeater PlanB CapitalFunds year2 0.219

FeedwaterHeater PlanB CapitalFunds year3 0.257

FeedwaterHeater PlanB OandMFunds year4 0.085

FeedwaterHeater DoNothing OandMFunds year1 0.000

FeedwaterHeater DoNothing OandMFunds year2 0.000

FeedwaterHeater DoNothing OandMFunds year3 0.000

FeedwaterHeater DoNothing OandMFunds year4 0.000

FeedwaterHeater DoNothing OandMFunds year5 0.000

EmergencySump PlanA CapitalFunds year3 0.122

EmergencySump PlanA CapitalFunds year4 0.103

EmergencySump PlanA CapitalFunds year5 0.003

EmergencySump PlanA OandMFunds year5 0.010

Transformer PlanA CapitalFunds year1 5.000

Transformer PlanA OandMFunds year1 0.044

Transformer PlanA OandMFunds year2 1.839

Transformer PlanB CapitalFunds year2 5.000

Transformer PlanB OandMFunds year2 0.044

Transformer PlanB OandMFunds year3 1.839

TurbineHallCrane PlanA CapitalFunds year1 6.000

TurbineHallCrane PlanA CapitalFunds year2 6.134

TurbineHallCrane PlanA CapitalFunds year3 5.442

TurbineHallCrane PlanA OandMFunds year1 0.740

TurbineHallCrane PlanA OandMFunds year3 5.000

TurbineHallCrane PlanB CapitalFunds year2 6.000

TurbineHallCrane PlanB CapitalFunds year3 6.134

TurbineHallCrane PlanB CapitalFunds year4 5.442

TurbineHallCrane PlanB OandMFunds year2 0.740

TurbineHallCrane PlanB OandMFunds year4 5.000

ReactorCoolantPumps PlanA CapitalFunds year1 0.425

ReactorCoolantPumps PlanB CapitalFunds year2 0.425

MitigateBuriedPiping PlanA CapitalFunds year1 2.125

MitigateBuriedPiping PlanA OandMFunds year2 2.122

MitigateBuriedPiping PlanB CapitalFunds year2 2.125

MitigateBuriedPiping PlanB OandMFunds year3 2.122

RefurbishTurbineGovernors PlanA CapitalFunds year1 2.000

RefurbishTurbineGovernors PlanA CapitalFunds year2 0.190

RefurbishTurbineGovernors PlanA CapitalFunds year3 0.012

RefurbishTurbineGovernors PlanA CapitalFunds year4 0.000

RefurbishTurbineGovernors PlanA CapitalFunds year5 0.092

RefurbishTurbineGovernors PlanA OandMFunds year1 0.387

RefurbishTurbineGovernors PlanA OandMFunds year2 0.000

RefurbishTurbineGovernors PlanA OandMFunds year3 0.000

RefurbishTurbineGovernors PlanA OandMFunds year4 2.383

RefurbishTurbineGovernors PlanA OandMFunds year5 0.100

ServiceCircWaterPumps PlanA OandMFunds year2 0.950

ServiceCircWaterPumps PlanB OandMFunds year3 0.950

MaintainRAOC PlanA CapitalFunds year1 0.030

MaintainRAOC PlanA CapitalFunds year2 0.030

MaintainRAOC PlanA OandMFunds year3 0.688

MaintainRAOC PlanB CapitalFunds year2 0.030

MaintainRAOC PlanB CapitalFunds year3 0.030

MaintainRAOC PlanB OandMFunds year4 0.688

ReplaceCoolantInjectPiping PlanA CapitalFunds year2 0.200

ReplaceCoolantInjectPiping PlanA CapitalFunds year3 0.763

ReplaceCoolantInjectPiping PlanA CapitalFunds year4 0.739

ReplaceCoolantInjectPiping PlanA CapitalFunds year5 2.539

ReplaceChiller PlanA CapitalFunds year1 0.347

ReplaceChiller PlanB CapitalFunds year2 0.347

VessellHeadWeldRemediate PlanA CapitalFunds year1 0.300

VessellHeadWeldRemediate PlanB CapitalFunds year2 0.300

MaintainCondensor PlanA CapitalFunds year1 4.025

MaintainCondensor PlanA CapitalFunds year2 0.297

MaintainCondensor PlanB CapitalFunds year2 4.025

MaintainCondensor PlanB CapitalFunds year3 0.297

ReplaceConrolCables PlanA CapitalFunds year1 0.081

ReplaceConrolCables PlanA CapitalFunds year2 0.032

ReplaceConrolCables PlanB CapitalFunds year2 0.081

ReplaceConrolCables PlanB CapitalFunds year3 0.032

CranePowerFeed PlanA CapitalFunds year1 0.095

CranePowerFeed PlanA CapitalFunds year2 0.095

CranePowerFeed PlanA CapitalFunds year3 0.095

CranePowerFeed PlanA CapitalFunds year2 0.095

CranePowerFeed PlanA CapitalFunds year3 0.095

CranePowerFeed PlanA CapitalFunds year4 0.095

MoisterSeparatorReheater PlanA CapitalFunds year1 5.488

MoisterSeparatorReheater PlanA CapitalFunds year2 5.665

MoisterSeparatorReheater PlanA CapitalFunds year3 0.501

MoisterSeparatorReheater PlanA CapitalFunds year4 6.803

MoisterSeparatorReheater PlanA CapitalFunds year5 6.778

;

**2.5. Deterministic Capital Budgeting Output:**

*Case 1 Output:* The Pyomo optimization model using the CBC solver produced the following output:

Result - Optimal solution found

Objective value: 68.43200000

Enumerated nodes: 0

Total iterations: 0

Time (CPU seconds): 0.01

Time (Wallclock seconds): 0.01

Total time (CPU seconds): 0.02 (Wallclock seconds): 0.02

# ==========================================================

# = Solver Results =

# ==========================================================

# ----------------------------------------------------------

# Problem Information

# ----------------------------------------------------------

Problem:

- Name: unknown

Lower bound: 68.432

Upper bound: 68.432

Number of objectives: 1

Number of constraints: 22

Number of variables: 26

Number of nonzeros: 71

Sense: minimize

# ----------------------------------------------------------

# Solver Information

# ----------------------------------------------------------

Solver:

- Status: ok

User time: -1.0

Termination condition: optimal

Error rc: 0

Time: 0.126662015914917

# ----------------------------------------------------------

# Solution Information

# ----------------------------------------------------------

Solution:

- number of solutions: 0

number of solutions displayed: 0

**Variable x**

**('MaintainCondensor', 'PlanA') 1.0**

**('ReplaceConrolCables', 'PlanA') 1.0**

**('RefurbishTurbineGovernors', 'DoNothing') 1.0**

**('TurbineHallCrane', 'DoNothing') 1.0**

**('EmergencySump', 'PlanA') 1.0**

**('RefurbishTurbineGovernors', 'PlanA') 0.0**

**('FeedwaterHeater', 'PlanA') 1.0**

**('ReactorCoolantPumps', 'DoNothing') 0.0**

**('CranePowerFeed', 'PlanA') 1.0**

**('ServiceCircWaterPumps', 'DoNothing') 0.0**

**('MitigateBuriedPiping', 'DoNothing') 1.0**

**('ReplaceCoolantInjectPiping', 'PlanA') 1.0**

**('FeedwaterHeater', 'DoNothing') 0.0**

**('Transformer', 'DoNothing') 1.0**

**('EmergencySump', 'DoNothing') 0.0**

**('MaintainRAOC', 'PlanA') 0.0**

**('MoisterSeparatorReheater', 'PlanA') 1.0**

**('MitigateBuriedPiping', 'PlanA') 0.0**

**('ReplaceChiller', 'PlanA') 1.0**

**('Transformer', 'PlanA') 0.0**

**('TurbineHallCrane', 'PlanA') 0.0**

**('MaintainRAOC', 'DoNothing') 1.0**

**('VessellHeadWeldRemediate', 'PlanA') 1.0**

**('ServiceCircWaterPumps', 'PlanA') 1.0**

**('ReactorCoolantPumps', 'PlanA') 1.0**

Interpretation of the solution: We will compare this solution with the results in Table 2 from Koc et al. (2009). The first row of the table is the one that we are replicating here. The following table summarizes the output from the deterministic Pyomo code:

|  |  |  |
| --- | --- | --- |
|  | Project number from Koc et al. (2009) | Selected by Deterministic Pyomo Code, Value of 1 = YES |
| FeedwaterHeater | 1 | 1 |
| EmergencySump | 2 | 1 |
| ReactorCoolantPumps | 5 | 1 |
| ServiceCircWaterPumps | 8 | 1 |
| ReplaceCoolantInjectPiping | 10 | 1 |
| ReplaceChiller | 11 | 1 |
| VessellHeadWeldRemediate | 12 | 1 |
| MaintainCondensor | 13 | 1 |
| ReplaceConrolCables | 14 | 1 |
| CranePowerFeed | 15 | 1 |
| MoisterSeparatorReheater | 16 | 1 |

Projects 1,2 5 and 8 were selected as well as all the seven mandatory projects (Projects 10 – 16). The obtained results are identical to the results from Table 2 in the Koc et al. (2009), and can be used as a benchmark.

*Case 2 Output*: The Pyomo optimization model using the CBC solver produced the following output:

Result - Optimal solution found

Objective value: 60.85200000

Enumerated nodes: 0

Total iterations: 0

Time (CPU seconds): 0.02

Time (Wallclock seconds): 0.02

Total time (CPU seconds): 0.08 (Wallclock seconds): 0.08

# ==========================================================

# = Solver Results =

# ==========================================================

# ----------------------------------------------------------

# Problem Information

# ----------------------------------------------------------

Problem:

- Name: unknown

Lower bound: 60.852

Upper bound: 60.852

Number of objectives: 1

Number of constraints: 27

Number of variables: 38

Number of nonzeros: 110

Sense: minimize

# ----------------------------------------------------------

# Solver Information

# ----------------------------------------------------------

Solver:

- Status: ok

User time: -1.0

Termination condition: optimal

Error rc: 0

Time: 0.18752288818359375

# ----------------------------------------------------------

# Solution Information

# ----------------------------------------------------------

Solution:

- number of solutions: 0

number of solutions displayed: 0

**Variable x**

**('MaintainCondensor', 'PlanA') 1.0**

**('ReplaceConrolCables', 'PlanA') 0.0**

**('MaintainCondensor', 'PlanB') 0.0**

**('ReplaceConrolCables', 'PlanB') 1.0**

**('RefurbishTurbineGovernors', 'DoNothing') 1.0**

**('TurbineHallCrane', 'DoNothing') 1.0**

**('ReactorCoolantPumps', 'PlanB') 0.0**

**('EmergencySump', 'PlanA') 1.0**

**('RefurbishTurbineGovernors', 'PlanA') 0.0**

**('ServiceCircWaterPumps', 'PlanB') 0.0**

**('FeedwaterHeater', 'PlanA') 1.0**

**('ReactorCoolantPumps', 'DoNothing') 0.0**

**('CranePowerFeed', 'PlanB') 0.0**

**('CranePowerFeed', 'PlanA') 1.0**

**('ServiceCircWaterPumps', 'DoNothing') 0.0**

**('MitigateBuriedPiping', 'DoNothing') 0.0**

**('TurbineHallCrane', 'PlanB') 0.0**

**('ReplaceCoolantInjectPiping', 'PlanA') 1.0**

**('FeedwaterHeater', 'DoNothing') 0.0**

**('Transformer', 'DoNothing') 1.0**

**('EmergencySump', 'DoNothing') 0.0**

**('MaintainRAOC', 'PlanB') 0.0**

**('MaintainRAOC', 'PlanA') 1.0**

**('VessellHeadWeldRemediate', 'PlanB') 0.0**

**('MoisterSeparatorReheater', 'PlanA') 1.0**

**('MitigateBuriedPiping', 'PlanA') 0.0**

**('Transformer', 'PlanB') 0.0**

**('ReplaceChiller', 'PlanA') 1.0**

**('Transformer', 'PlanA') 0.0**

**('ReplaceChiller', 'PlanB') 0.0**

**('TurbineHallCrane', 'PlanA') 0.0**

**('MaintainRAOC', 'DoNothing') 0.0**

**('MitigateBuriedPiping', 'PlanB') 1.0**

**('VessellHeadWeldRemediate', 'PlanA') 1.0**

**('FeedwaterHeater', 'PlanB') 0.0**

**('ServiceCircWaterPumps', 'PlanA') 1.0**

**('ReactorCoolantPumps', 'PlanA') 1.0**

Interpretation of the solution:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Project number from Koc et al. (2009) | Option Selected | Selected by Deterministic Pyomo Code |
| FeedwaterHeater | 1 | PlanA | 1 |
| EmergencySump | 2 | PlanA | 1 |
| ReactorCoolantPumps | 5 | PlanA | 1 |
| MitigateBuriedPiping | 6 | PlanB | 1 |
| ServiceCircWaterPumps | 8 | PlanA | 1 |
| MaintainRAOC | 9 | PlanA | 1 |
| ReplaceCoolantInjectPiping | 10 | PlanA | 1 |
| ReplaceChiller | 11 | PlanA | 1 |
| VessellHeadWeldRemediate | 12 | PlanA | 1 |
| MaintainCondensor | 13 | PlanA | 1 |
| ReplaceConrolCables | 14 | PlanB | 1 |
| CranePowerFeed | 15 | PlanA | 1 |
| MoisterSeparatorReheater | 16 | PlanA | 1 |

We can see how change in the timing of projects and adding different fund resource changes the optimal solution. The original optimal benchmark solution was to select projects 1, 2, 5, and 8. Now we can select projects 1, 2, 5, 6, 8, and 9. The change in solution is driven by starting project 6 in year 2 (under PlanB) instead of year 1 (under PlanA).

**2.6. Deterministic Capital Budgeting Pyomo Code:**

Pyomo code implementing the above model and producing shown results is:

#File name: Model\_Deterministic\_options\_resources.py

from pyomo.environ import \*

from pyomo.core import \*

from pyomo.opt import SolverFactory

# Create a solver

opt = SolverFactory('cbc')

model = AbstractModel()

# Input data

model.T = Set() # parameter for time

model.I = Set(ordered=True) # parameter for projects

model.J = Set() # parameter for options for selecting project i

model.K = Set() # parameter for types of resources

model.IJ = Set(dimen=2) # parameter for possible combinations between projects and options

model.r = Param(model.IJ,default=0) # parameter for NPVs

model.b = Param(model.K,model.T,default=0) # parameter for budget

model.c = Param(model.IJ,model.K,model.T,default=0) # parameter for costs

# Variables

model.x = Var(model.IJ, domain=NonNegativeIntegers, bounds=(0,1)) # variable x, 1 if project i is selected via option j

#Objective function

def obj\_rule(model): #objective function max the overall NPV

return sum(model.r[ij]\*model.x[ij] for ij in model.IJ)

model.z = Objective(rule=obj\_rule, sense=maximize)

# Constraints

def NodesIn\_init(model, project\_i): # here we create the set of options j for given project i

retval = []

for (i,j) in model.IJ:

if i == project\_i:

retval.append(j)

return retval

model.NodesIn = Set(model.I, initialize=NodesIn\_init)

def constraintX(model, i): #sum of all x[i,j]=1 for all j

return sum(model.x[i,j] for j in model.NodesIn[i]) == 1

model.constraintX = Constraint(model.I,rule=constraintX)

def constraintCX(model, k,t): #budget constraint

return sum(model.c[ij,k,t]\*model.x[ij] for ij in model.IJ) <=model.b[k,t]

model.constraintCX = Constraint(model.K,model.T,rule=constraintCX)

# Create a model instance and optimize

instance = model.create\_instance('Deterministic\_input\_Case2.dat')

results = opt.solve(instance, tee=True)

# Printing the solution

results.write()

instance.solutions.load\_from(results)

for v in instance.component\_objects(Var, active=True):

print ("Variable",v)

varobject = getattr(instance, str(v))

for index in varobject:

print (" ",index, varobject[index].value)

**REFERENCES**

A. Koc and D.P. Morton, “Prioritization via stochastic optimization”, *Management Science* 61, 586-603 (2015).

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