Digital Circuits: Homeworks #2 Solutions

1. **Truth Table.**
   Construct a truth table of following Boolean expressions

   (a) $X = AB + \bar{B}C + CA$.

   (b) $X = (A + B)(B + \bar{C})(C + A)$.

   Solution: Truth Table

   (a) $X = AB + \bar{B}C + CA$.

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

   (b) $X = (A + B)(B + \bar{C})(C + A)$.

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

2. **Standard Forms of Boolean Expressions**

   (a) Convert $X = (A + C)(CD + AC)$ to sum-of-product (SOP) form.

   (b) Convert $X = (A + C)(CD + AC)$ to product-of-sum (POS) form.

   (c) Convert $X = \overline{AB(CD + \bar{E}F)(\overline{AB} + \overline{CD})}$ to sum-of-product (SOP) form.

   Solution: Standard Forms of Boolean Expressions.

(a) We have

$$X = ACD + CCD + AAC + CAC \tag{1}$$
$$= ACD + CD + AC + AC \tag{2}$$
$$= ACD + CD + AC \tag{3}$$
$$= AC + CD \tag{4}$$

(b) It is clear that

$$X = (A + C)C(D + A). \tag{5}$$

Note that this can be further simplifed as

$$X = C(A + D). \tag{6}$$

(c) We have

$$X = \overline{\overline{AB}(CD + \bar{E}F)(\overline{AB} + \overline{CD})} \tag{7}$$
$$= AB + \overline{CD + \bar{E}F} + \overline{\overline{AB} + \overline{CD}} \tag{8}$$
$$= AB + \overline{CD}\,\overline{\bar{E}F} + ABCD \tag{9}$$
$$= AB + (\bar{C} + \bar{D})(E + \bar{F}) + ABCD \tag{10}$$
$$= AB + \bar{C}E + \bar{D}E + \bar{C}\bar{F} + \bar{D}\bar{F} + ABCD. \tag{11}$$

3. **Karnaugh Map**
   Let $X = A\bar{B} + B\bar{C} + CD + AC\bar{D}$.

   (a) Develop a truth table of X

   (b) Use a Karnaugh map to reduce X to a minimum SOP form.

   (c) Use a Karnaugh map to reduce X to a minimum POS form.

Solution: Karnaugh Map.

(a) $X = A\bar{B} + B\bar{C} + CD + AC\bar{D}$.

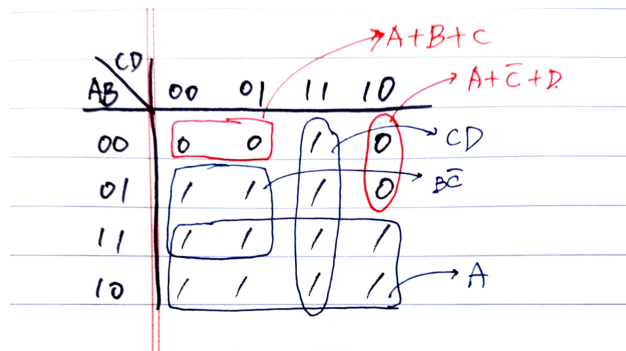| A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Figure 1: Problem 3

(b) It is not hard to show that $X = A + B\bar{C} + CD$.

(c) It is not hard to show that $X = (A + B + C)(A + \bar{C} + D)$.

4. **Karnaugh Map 2**
   Let $X = (\bar{A} + B)(\bar{A} + \bar{B} + \bar{C})(B + \bar{C} + D)(A + \bar{B} + C + \bar{D})$.

   (a) Develop a truth table of X

   (b) Use a Karnaugh map to reduce X to a minimum SOP form.

   (c) Use a Karnaugh map to reduce X to a minimum POS form.

   Solution: Karnaugh Map 2.

   (a) $X = (\bar{A} + B)(\bar{A} + \bar{B} + \bar{C})(B + \bar{C} + D)(A + \bar{B} + C + \bar{D})$.

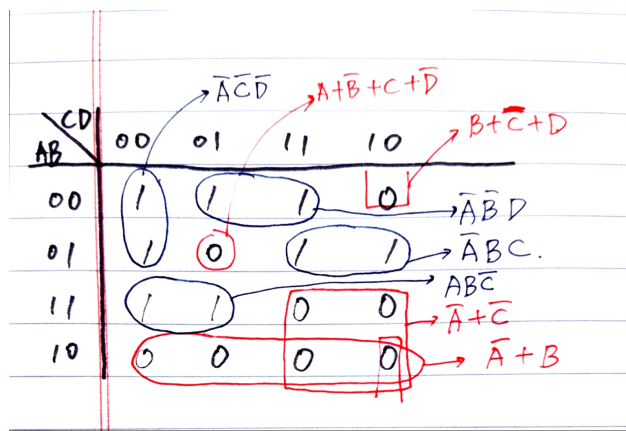| A	B	C	D	X
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Figure 2: Problem 4

(b) It is not hard to show that

$$X = AB\bar{C} + \bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}D + \bar{A}BC. \tag{12}$$

(c) It is not hard to show that

$$X = (\bar{A} + B)(\bar{A} + \bar{C})(B + \bar{C} + D)(A + \bar{B} + C + \bar{D}). \tag{13}$$

5. **Don't Care!**
   For the following truth table, answer the following questions. Note that "x" means don't care.

A	B	C	D	X
0	0	0	0	x
0	0	0	1	x
0	0	1	0	0
0	0	1	1	0
0	1	0	0	x
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	x
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

(a) Draw a K-map (show all 0s, 1s, and x's).

(b) Derive a minimum SOP expression using K-map.

(c) Derive a minimum POS expression using K-map.
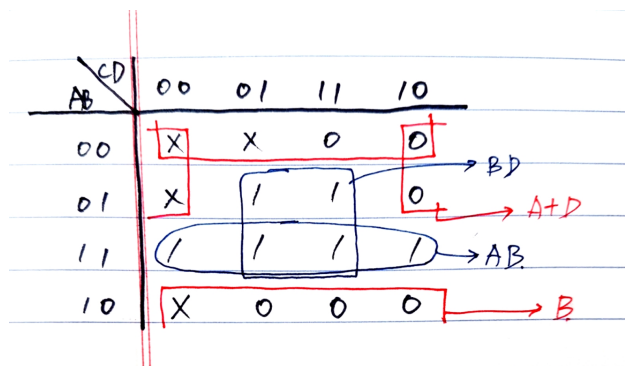
Solution: Don't Care!

(a) K-map:

Figure 3: Problem 5

(b) It is not hard to show that

$$X = AB + BD. \tag{14}$$

(c) It is not hard to show that

$$X = B(A + D) \tag{15}$$