CSC343 Assignment Three: Part II

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We have relation R with attributes $IJKLMNOP$ and functional dependencies:

$$S_P = \{M \to IJL, J \to LI, JN \to KM, M \to J, KLN \to M, K \to IJL, IJ \to K\}$$

(a) To find a minimal basis, we first split the RHS of functional dependencies into one attribute:

$$M \to I$$

$$M \to J$$

$$M \to L$$

$$J \to L$$

$$J \to I$$

$$JN \to K$$

$$JN \to M$$

$$M \to J$$

$$KLN \to M$$

$$K \to I$$

$$K \to J$$

$$K \to L$$

$$IJ \to K$$

Next, for each FD with more than one attribute on the LHS, we see if we can remove excessive attributes and get an FD that follow from S:

$$M \to I$$

$$M \to J$$

$$M \to L$$

$$J \to L$$

$$J \to I$$

$$J \to K$$

$$JN \to M$$

$$M \to J$$

$$KN \to M$$

$$K \to I$$

$$K \to J$$

$$K \to L$$

$$J \to K$$

Lastly, for each FD obtained above, we remove excessive FDs if it is implied other FDs:

$$M \to J$$

$$KN \to M$$

$$K \to IK \to J$$
$$K \to L$$
$$J \to K$$

(b) We first find the set of attributes that did not appear on the RHS of the FDs. This gives us $\{N, O, P\}$. Next, we find the attributes that only appeared on the RHS, these attributes cannot be in any key since we can always remove it and still get the same closure. This gives us $\{I, L\}$. Then we have to manually check all other possibilities since these tricks can only take us so far. Thus we check:

$$NOPJ^+ = \{IJKLMNOP\}$$
$$NOPK^+ = \{IJKLMNOP\}$$
$$NOPM^+ = \{IJKLMNOP\}$$

Thus, we have found all keys for R. All other superkeys are supersets of these three keys, which are trivial.

(c) To perform 3NF-synthesis, we first combine FDs with the same LHS:

$$M \to J$$
$$KN \to M$$
$$K \to IJL$$
$$J \to K$$

We define three new relations based on FDs: $R_1 = \{MJ\}$, $R_2 = \{KNM\}$. $R_3 = \{KIJL\}$. Since these three relations do not contain $\{OP\}$, none of the relations are a superkey for R. So we add a new relation that is a key: $R_4 = \{JNOP\}$. Therefore, we obtain our lossless, dependency-preserving final answer:

$R_1 = \{JM\}$, with FD $M \to J$

$R_2 = \{KMN\}$, with FD $KN \to M$

$R_3 = \{IJKL\}$ with FDs $J \to K, K \to I, K \to J, K \to L$

$R_4 = \{JNOP\}$ with no FD.

(d) Since every LHS of the FDs are superkeys, as we can clearly see for $R_1 = \{JM\}$, M is a superkey. For $R_2 = \{KMN\}$, KN is a superkey. For $R_3 = \{IJKL\}$, J, K are both superkeys. Thus, the relations satisfies BCNF and does not allow redundancy.

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We have relation T with attributes $CDEFGHIJ$ and functional dependencies:

$$S_T = \{C \to EH, DEI \to F, F \to D, EH \to CJ, J \to FGI\}$$

(a) $C \to EH$, $C^+ = \{CDEFGHIJ\}$, not a violation.

$DEI \to F$, $DEI^+ = \{DEIF\}$, this is a violation.

$F \to D$, $F^+ = \{FD\}$, this is a violation.

$EH \to CJ$, $EH^+ = \{CDEFGHIJ\}$, not a violation.

$J \to FGI$, $J^+ = \{DFGIJ\}$, this is a violation.

(b) We choose to decompose using $DEI \to F$, $DEI^+ = \{DEIF\}$.

$DEI^+ = \{DEIF\}$ violates BCNF, thus we decompose to relation $T_1 = \{DEIF\}, T_2 = \{CDEGHIJ\}$ and projecting FDs onto T_1, T_2:

For T_1, $F \to D$ violates BCNF since F is not a superkey, thus we decompose it further: $T_3 = \{FD\}, T_4 = \{EIF\}$. Now, T_3 has FD $F \to D$, which satisfies BCNF. T_4 has no FD, which also satisfies BCNF.

For T_2, we can see that $J^+ = \{DGIJ\}$, since T_2 does not contain F anymore. So we get the FD $J \to \{DGI\}$ which violates BCNF since J is not a superkey. So we decompose it further into: $T_5 = \{J, I, D, G\}, T_6 = \{C, E, H, J\}$. Since $I^+ = I, D^+ = D, G^+ = G$ and all supersets of I, D, G form no functional dependency, T_5 has only one FD $J \to IDG$, which satisfies BCNF. Next, for T_6, since C and EH are both superkeys, $C \to EH$ and $EH \to CJ$ are both FDs that clearly satisfy BCNF. Lastly, we put everything in alphabetical order and obtain the final answer:

$T_3 = \{DF\}$, the projection of S_T onto T_3 is $\{F \to D\}$

$T_4 = \{EFI\}$, the projection of S_T onto T_4 is None.

$T_5 = \{D, G, I, J\}$, the projection of S_T onto T_5 is $\{J \to DGI\}$

$T_6 = \{C, E, H, J\}$, the projection of S_T onto T_6 is $\{C \to EHJ, EH \to CJ\}$

This decomposition is lossless and redundancy-preventing.