PHY408 Lab Three: Filter Design

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1 A Notch Digital Filter

Collaborators: Will Zeng.

1. In complex analysis, a pole is a special singularity meaning the pole of a function is where the function denominator becomes 0. Whereas zeros is when the numerator becomes 0. Thus, this gives us the poles of this system: $z = p, z = p^*$, which can rewritten as $z = (1 + \epsilon)e^{\frac{-i2\pi f_0}{f_s}}$ and $z = (1 + \epsilon)e^{\frac{i2\pi f_0}{f_s}}$. And the zeros of this system: $z = q = e^{\frac{-i2\pi f_0}{f_s}}, z = q^* = e^{\frac{i2\pi f_0}{f_s}}$. Since ϵ is positive, the poles are located outside of the unit circle and the zeros are located on the circle. Thus the system is stable. Next, we prove the filter is real by showing any real input time series passed through the filter remains real. That is, for all input $z \in R, z = z^*$, $W^*(z) = M\frac{z-q^*}{z-p^*}\frac{z-q}{z-p} = W(z)$, so the filter is real.

2. As the lab handout instructed, we set $f_s = 12$, $f_0 = 1$, $M = 1.04$ and $\epsilon = 0.04$. The power spectrum $|W(f)|^2$ is plotted below:

1
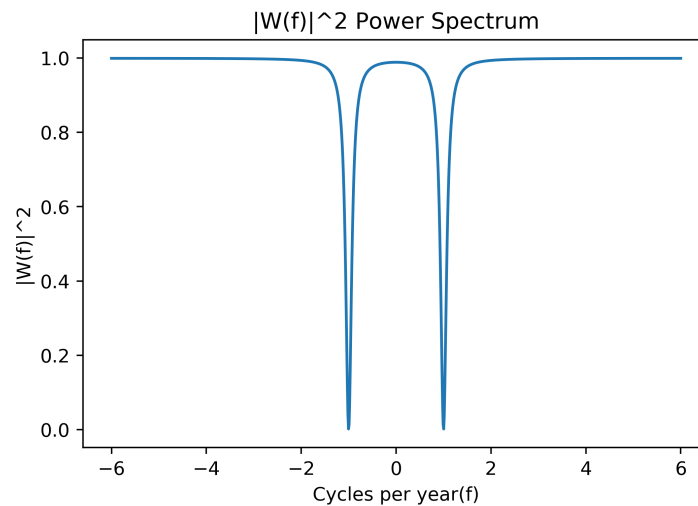
3. If we want to make the notches sharper, we can simply decrease ϵ. This is essentially bringing the poles closer to the zeros. The f_{fwhm} value of this notch filter can be observed when we zoom in on the plot from question 2:
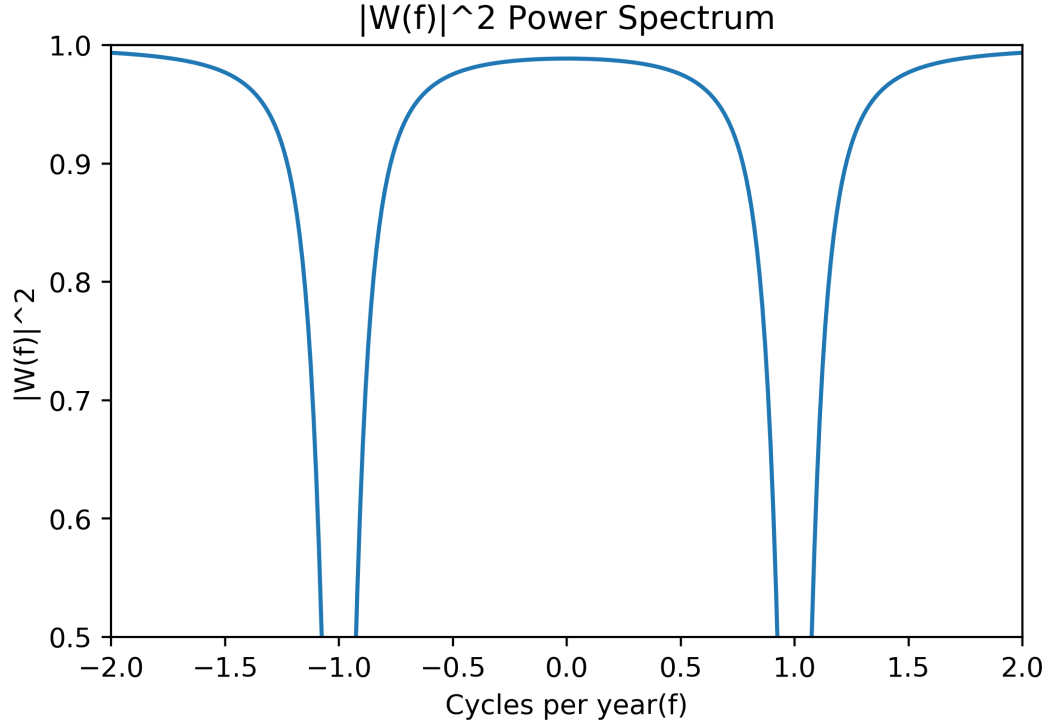
As we can see, the points where $y = 0.5$ lie on the x-axis are about $-1.1 - 0.9$ approximately. Thus, the FWHM is about 0.2 cycles per year.

2 Implementing the Notch Filter

Collaborators: Will Zeng, Shenzhi Wang

1. We can rewrite the notch filter in 1.2 as:

$$M\frac{z - q}{z - p}\frac{z - q^*}{z - p^*} = \frac{(z - q)(z - q^*)}{(z - p)(z - p^*)} = \frac{z^2 - 2Re(q) + |q|^2}{z^2 - 2Re(p) + |p|^2}$$

Dividing both numerator and denominator by $|p|^2$ gives the filter in form of $N(z)$ and $D(z)$. Then the
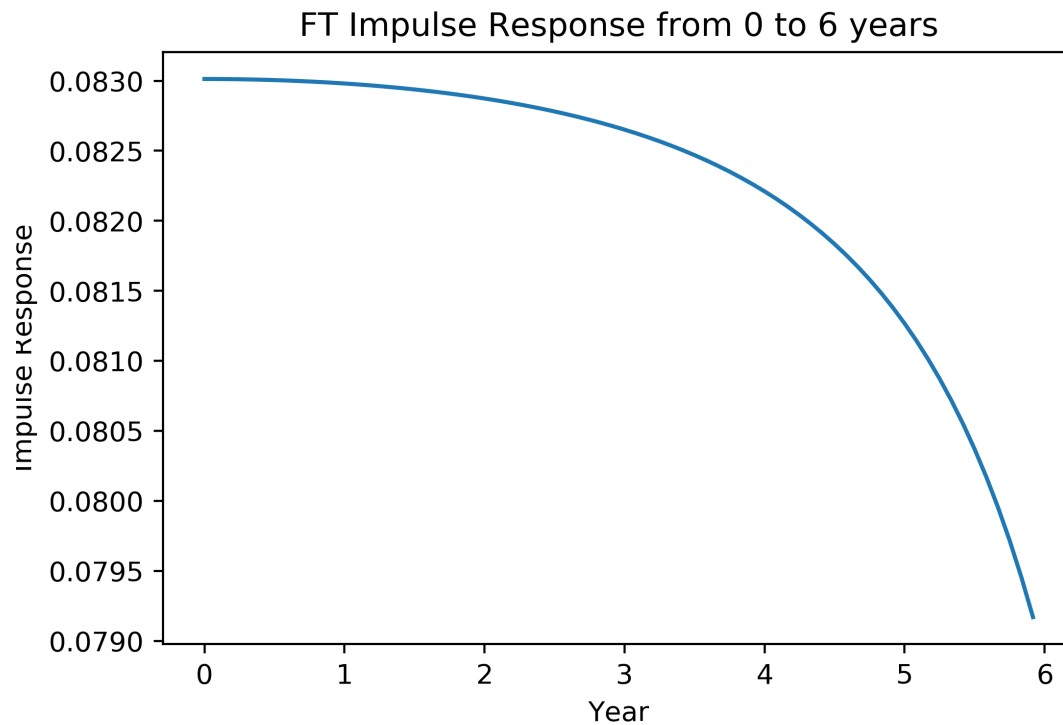
values for a; b; c; B; C:

$$a = M\frac{|q|^2}{|p|^2}$$

$$b = -M\frac{2Re(p)}{|p|^2}$$

$$c = M\frac{1}{|p|^2}$$

$$B = -\frac{2Re(p)}{|p|^2}$$

$$C = \frac{1}{|p|^2}$$

2. See code file for the ratFilter function

3. Continuing from Question 1, we can can calculate $|p^2| = 1.0816$ $a = c = 0.962$, $b = B = -1.665, C = 0.925$. By using the ratFilter function, the plot of impulse response from 0 to 6 years is shown below:

4. The plot is shown below:

FT Impulse Response from 0 to 6 years
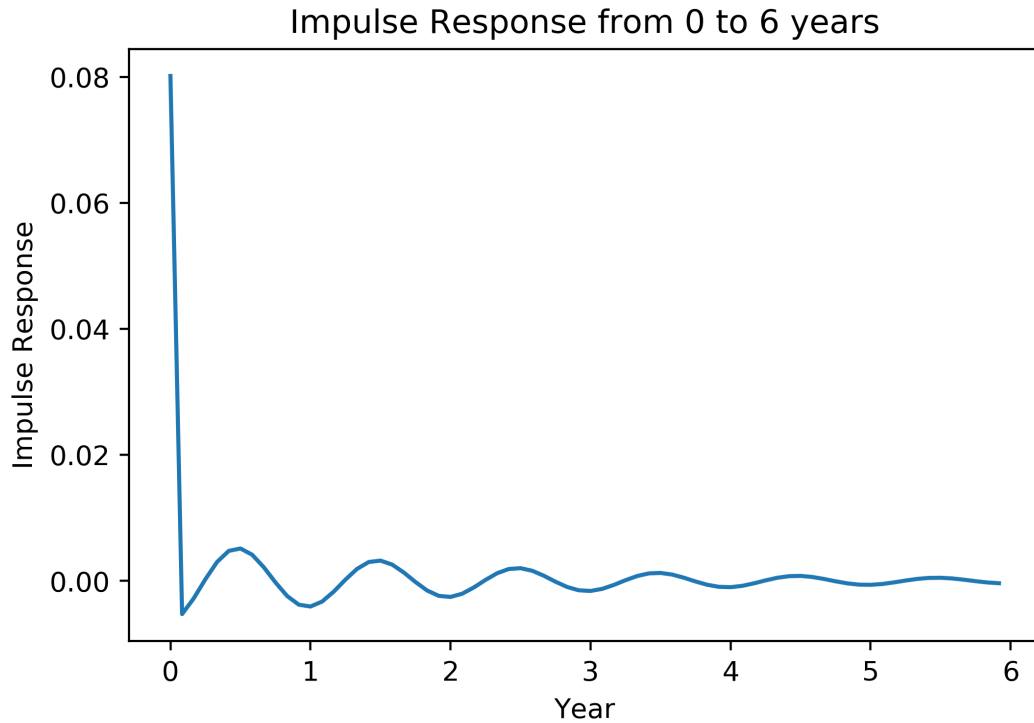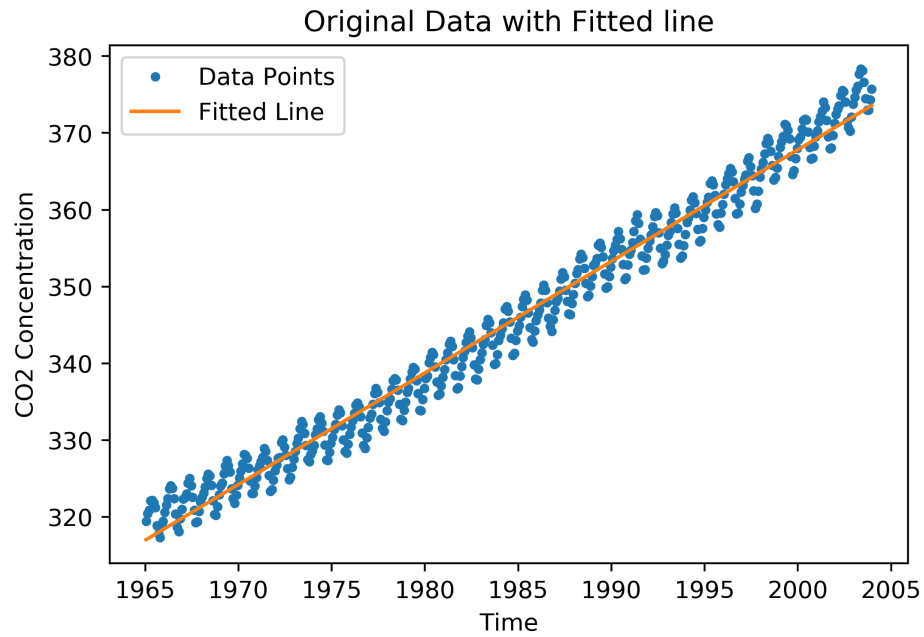
To my understanding, the plot should theoretically resemble the shape of the previous plot, but I have no idea how this failed.
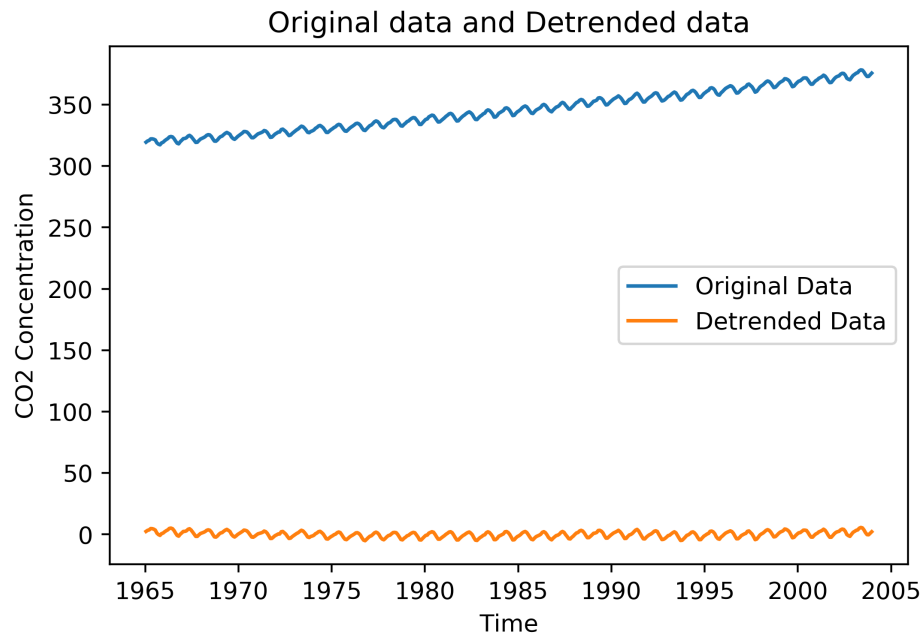
3 The Mauna Loa CO_2 Data

Collaborator: Will Zeng, Shenzhi Wang

1. Using numpy function polyfit, we first fit a straight line through our original data:
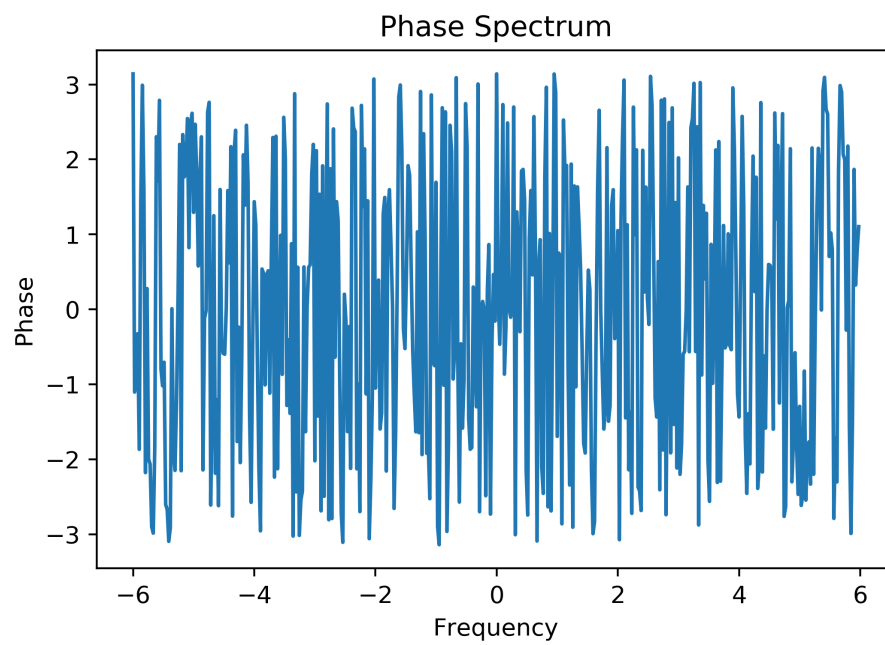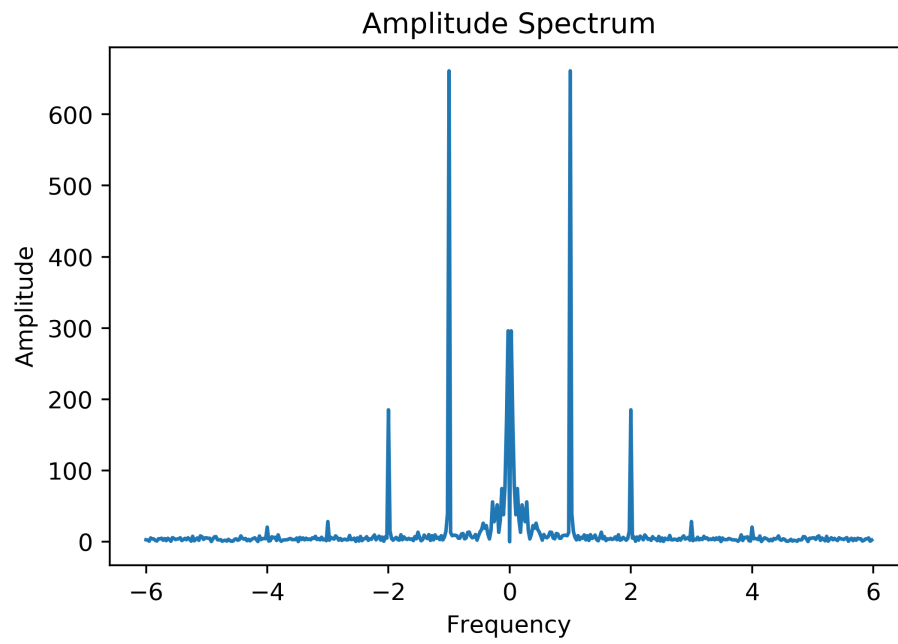
Original Data with Fitted line

Now we remove the straight line and plot both the original data and the detrended data.
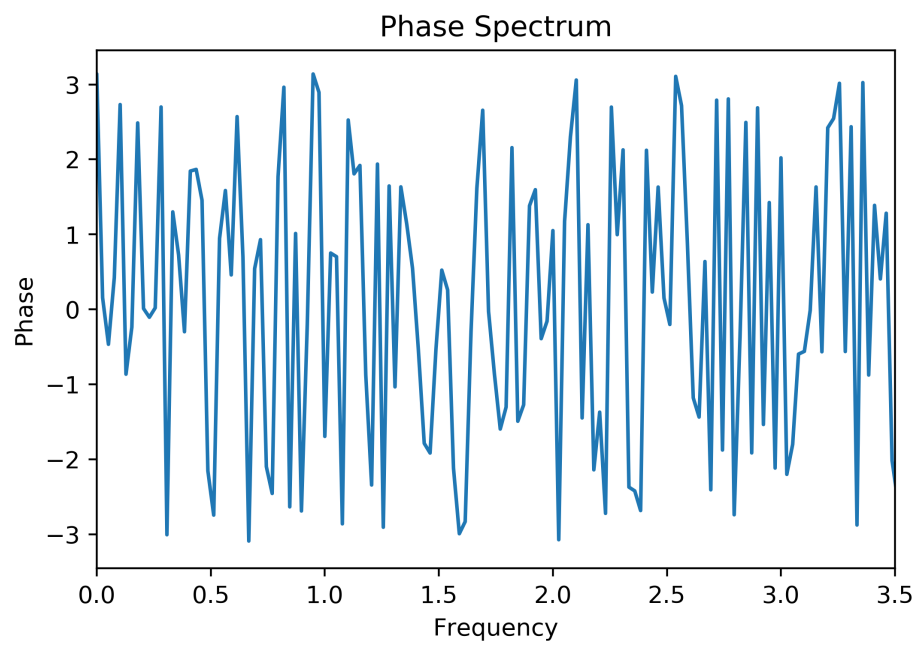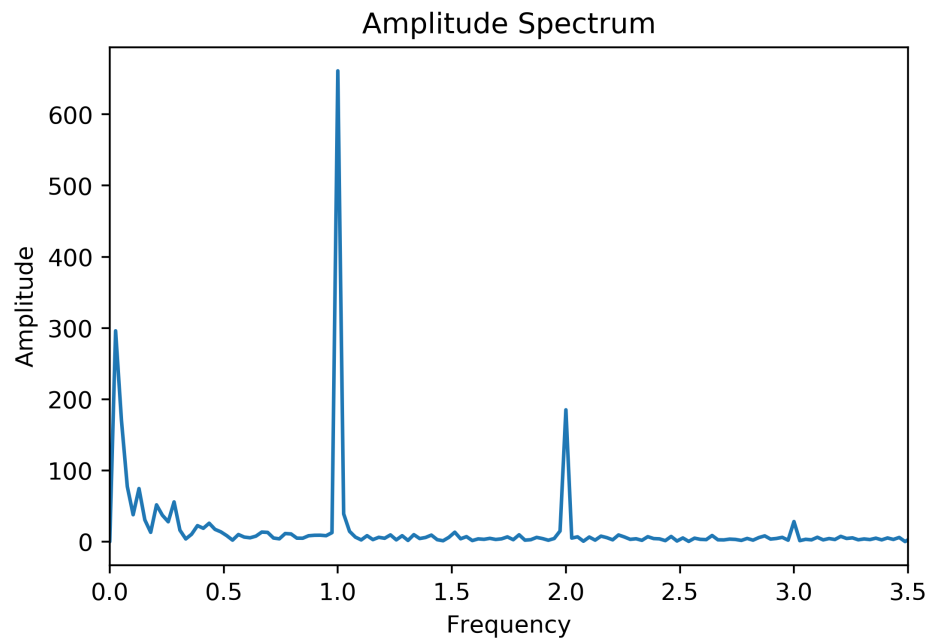

Original data and Detrended data

2. Code from code file for applying the notch filter and adding back the trend:

```
filtered_co2 = ratFilter(N,D, detrend_co2)
filtered_co2 = filtered_co2 + trend
```
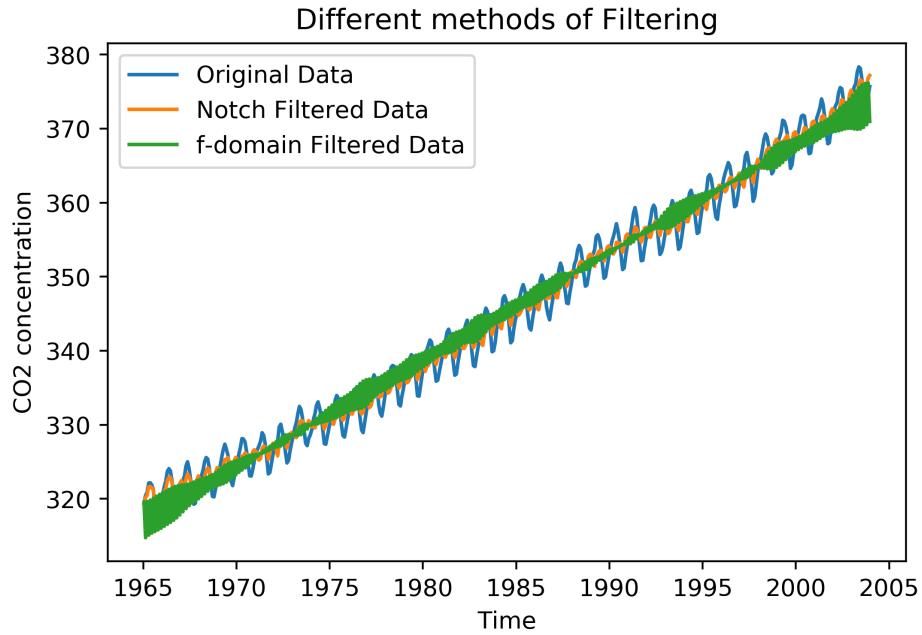
3. The amplitude and phase spectrum plots are shown below:

Amplitude Spectrum

Phase Spectrum

Now we zoom in at $f = [0,3.5]$:

Amplitude Spectrum

Phase Spectrum

4. The plot is shown below:

Different methods of Filtering

We can see that the f-domain filtering method is better in terms of similarity to the original data, which means that it captures the trend of the original time series better. It also removes the fluctuations. The disadvantage of using this method would be instability, it does not deal with noises as well as the Notch filter. One other thing that is worth noting, implementing a notch filter for real world problem might become difficult because we have to pre-configure all the parameters, this is a potential disadvantage.

5. If we do not detrend before applying our filters, i.e using the original result for Part 2 and Part 3 and Part 4, then the result is terrible:

Applying Filters Without Detrending
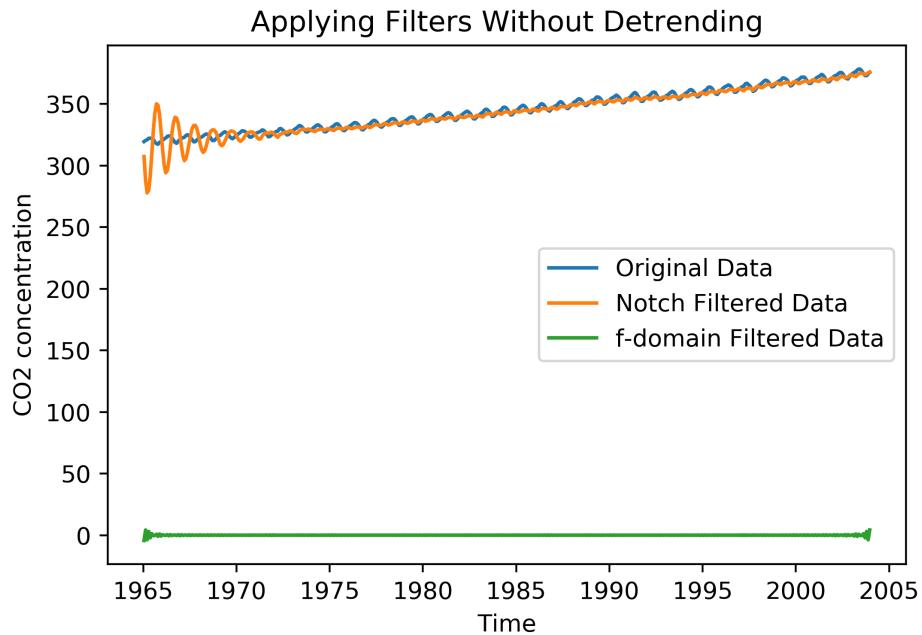
The notch filtered data is not capturing the trend of the original time series as well as before, and the Fourier transform method is shifted drastically. Thus, we can see from purely from visual effect, just how important it is to detrend before applying filters. It tells us that not detrending the data will result in serious problems for capturing trend of the data.