1.2 Methods of Enumeration

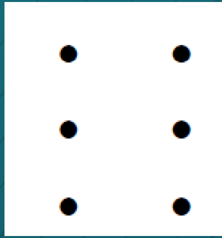
Notes

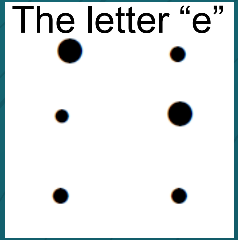
Multiplication Principle

Multiplication Principle: Suppose that an experiment (or procedure) E_1 has n_1 outcomes and, for each of these possible outcomes, an experiment (procedure) E_2 has n_2 possible outcomes. Then the composite experiment (procedure) E_1E_2 that consists of performing first E_1 and then E_2 has n_1n_2 possible outcomes.

Example

- In 1824, Louis Braille invented a system where each written character could be expressed as a six-dot matrix, where certain dots were raised:





How many different characters can be enciphered in Braille?

$2^6 = 64$

Definition 1.2-1

Each of the $n!$ arrangements (in a row) of n different objects is called a **permutation** of the n objects.

- The number of permutations of the four letters:

a, b, c, d is $4! = 24$

Definition 1.2-2

Each of the ${}_nP_r$ arrangements is called a **permutation of n objects taken r at a time.**

$$ {}_nP_r = \frac{n(n-1)\cdots(n-r+1)(n-r)\cdots(3)(2)(1)}{(n-r)\cdots(3)(2)(1)} = \frac{n!}{(n-r)!}. $$

Permutation example:

- The number of possible four letter code words that can be formed where all four letter are different:

$$ {}_{26}P_4 = \frac{26!}{(26-4)!} = \frac{26!}{22!} = 358{,}800 $$

In this case, we have **sampling without replacement**.

Sampling with replacement

- The number of possible four letter code words that can be formed when the letter do not necessarily have to be different:
- 26 x 26 x 26 x 26 = 456,976

Combinations

- Sometimes, the order of selection is not important and we are only interested in the number of subsets of size r that we can make from n objects.
- We can say that the number of ways in which r objects can be selected **without replacement** from n objects is ${}_nC_r$, or "n choose r".

$$ {}_nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}. $$

- The number of possible 5-card poker hands from a deck of 52 playing cards:

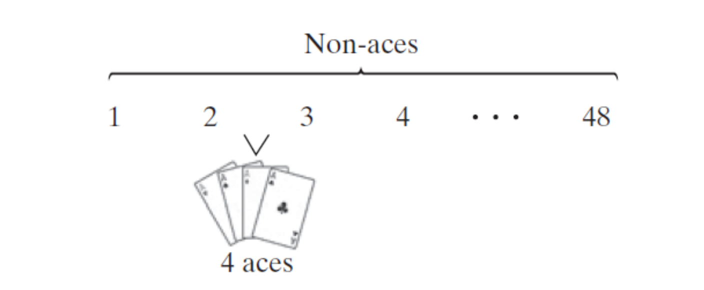
$$_{52}C_5 = \frac{52!}{5!47!} = 2{,}598{,}960$$

- The number of possible 3-person subcommittees from a group of 10 people:

$$_{10}C_3 = \frac{10!}{3!7!} = 120$$

Example

- A deck of 52 cards is shuffled randomly and dealt face up. How many different arrangements are there for the 4 aces to be adjacent?



- $49 \cdot 4! \cdot 48!$

For more examples, please look at the old problems and solutions!