Stat 400 [BL1, CL1, DL1]: Homework 1

Spring 2020

Due: *Mon. Feb 3 – 3:00pm*

Exercise 1

The probability that Chloe is happy given that she has had dinner by 6pm is 0.9. The probability that Chloe is not happy given that she has not had dinner by 6pm is 0.8. Assume that there is a 60% chance that Chloe eats dinner by 6pm on a given day.

a) If Chloe is not happy at 6pm, find the probability that she has not had dinner yet.

Solution:

We want to find the probability that Chloe has not had dinner yet given that she is not happy at 6pm.

$$P(no\ \ dinner \mid not\ \ happy)$$

$$= \frac{P(no\ dinner)P(not\ happy \mid no\ dinner)}{P(no\ dinner)P(not\ happy \mid no\ dinner) + P(dinner)P(not\ happy \mid dinner)}$$

$$= \frac{0.4 \cdot 0.8}{0.4 \cdot 0.8 + 0.6 \cdot 0.1} = \boxed{\frac{0.32}{0.38}} = \boxed{0.8421}$$

b) If Chloe is happy at 6pm, find the probability that she has already had dinner.

Solution:

We want to find the probability that Chloe has had dinner given that she is happy at 6pm.

$$P(dinner \mid happy) = \frac{P(dinner)P(happy \mid dinner)}{P(dinner)P(happy \mid dinner) + P(no\ dinner)P(happy \mid no\ dinner)}$$

$$= \frac{0.6 \cdot 0.9}{0.6 \cdot 0.9 + 0.4 \cdot 0.2} = \boxed{\frac{0.54}{0.62}} = \boxed{0.8709}$$

Alternatively:

For parts (a) and (b) you can use a probability table.

	happy	not happy	total
dinner	0.54	0.06	0.6
no dinner	0.08	0.32	0.4
total	0.62	0.38	1.0

Exercise 2

Find a value **c** such that

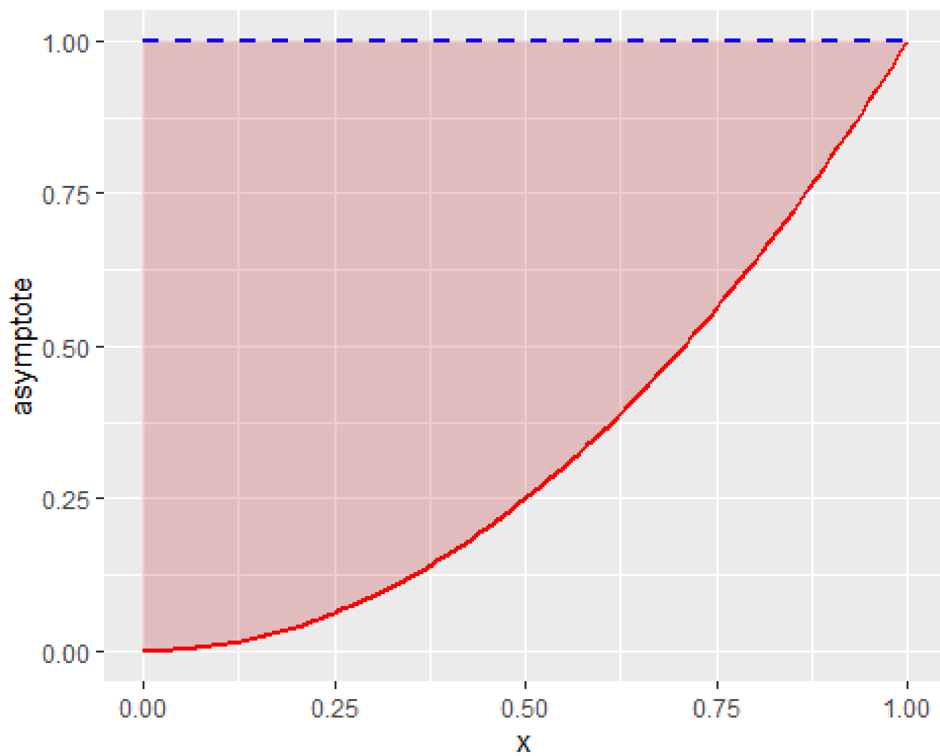
$$\iint_A c\, x^3 y^2\, dx\, dy$$

,

where $A = \{(x, y): 0 < x < \sqrt{y}, 0 < y < 1\}$. Do not use a calculator or computer, except to check your work.

Solution:

Integral Region:



First,

$$\iint_A c\, x^3 y^2\, dx\, dy = 1 = \int_0^1 \left(\int_0^{\sqrt{y}} c\, x^3 y^2\, dx\right) dy = \int_0^1 \frac{c}{4} y^4\, dy = \frac{c}{20}$$

Then,

$$\frac{c}{20} = 1 \Rightarrow c = \boxed{20}$$

Exercise 3

Suppose $S = \{2,3,4,5,\ldots\}$ and

$$P[k] = c\frac{3^k}{k!}$$

Find the value of c that makes this a valid probability distribution.

Solution:

First, note that

$$\sum_{all\,x}^{\infty} P[x] = \sum_{k=2}^{\infty} c\,\frac{3^k}{k!}$$

$$= c(\sum_{k=0}^{\infty}\frac{3^k}{k!} - \frac{3^0}{0!} - \frac{3^1}{1!})$$

$$= c(e^3 - 1 - 3)$$

$$= c(e^3 - 4)$$

Then, since

$$\sum_{all\,x}^{\infty} P[x] = 1,$$

We obtain

$$c(e^3 - 4) = 1 \Rightarrow c = \boxed{\frac{1}{e^3 - 4}} \approx \boxed{0.06217}$$

Exercise 4

Suppose $S = \{0,1,2,3,\ldots\}$ and

$$P[k] = \frac{1/3}{(3/2)^k}$$

Find P[Outcome is *greater* than 2].

Solution:

$$P[Outcome\ is\ greater\ than\ 2] = P[3] + P[4] + P[5] + \ldots$$

$$= \sum_{k=3}^{\infty} \frac{1/3}{(3/2)^k} = \frac{first\ term}{1 - ratio} = \frac{\frac{1/3}{(3/2)^3}}{1 - \frac{2}{3}}$$

$$= \boxed{\frac{8}{27}} \approx \boxed{0.2963}$$

Or alternatively,

$$P[Outcome\ is\ greater\ than\ 2] = 1 - P[0] - P[1] - P[2]$$

$$= 1 - \frac{1/3}{(3/2)^0} - \frac{1/3}{(3/2)^1} - \frac{1/3}{(3/2)^2}$$

$$= \boxed{\frac{8}{27}} \approx \boxed{0.2963}$$

Exercise 5

Suppose P[A] = 0.5, P[B'] = 0.3, and P[A ∩ B] = 0.2

Solution:

Venn Diagram (chart) for P[A] = 0.5, P[B'] = 0.3, and P[A ∩ B] = 0.2

	B	B'	total
A	0.2	0.3	0.5
A'	0.5	0	0.5
total	0.7	0.3	1.0

a) Find P[A ∪ B]

Solution:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.7 - 0.2 = \boxed{1}$$

$$P(A \cup B) = 1 - P(A' \cap B') = 1 - 0 = \boxed{1}$$

$$P(A \cup B) = P(A \cap B) + P(A' \cap B) + P(A \cap B') = 0.2 + 0.5 + 0.3 = \boxed{1}$$

b) Find P[B | A]

Solution:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \boxed{\frac{0.2}{0.5}} = \boxed{0.4}$$

c) Find P[B'| A']

Solution:

$$P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \boxed{\frac{0}{0.5}} = \boxed{0}$$

d) Find P[A | B]

Solution:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \boxed{\frac{0.2}{0.7}} = \boxed{0.28571}$$

$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(B)} = \frac{0.5 * 0.4}{0.7} = \boxed{\frac{0.2}{0.7}} = \boxed{0.28571}$$