1.1

1. $$SE(3) = \left\{ A \;\middle|\; A = \left[\begin{array}{c:c} R & r \\ \hdashline 0_{1\times 3} & 1 \end{array}\right],\ R \in \mathbb{R}^{3\times 3},\ r \in \mathbb{R}^3,\ R^T R = R R^T = I,\ |R| = 1 \right\}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 30 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_{3,3} = 0 \Rightarrow A \text{ invalid}$$

$\uparrow$ (bottom-right)

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad R_B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix},$$

$$R_B R_B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = R_B^T R_B = I, \quad |R_B| = 1 \Rightarrow B \text{ valid}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}, \quad R_C^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix},$$

$$R_C R_C^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = R_C^T R_C, \quad |R_C| = \underline{-1} \Rightarrow C \text{ invalid}$$

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad R_D^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$R_D R_D^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = R_D^T R_D, \quad |R_D| = 1 \Rightarrow D \text{ valid}$$

② $T^{-1} = \begin{bmatrix} R & d \\ 0_n & 1 \end{bmatrix}^{-1} = \begin{bmatrix} R^T & -R^T d \\ 0_n & 1 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $R_B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$, $-R_B^T d_B = \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -5 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ #

$BB^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = B^{-1}B = I$ #

$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $R_D^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $-R_D^T d_D = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}$, $D^{-1} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ #

$DD^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = D^{-1}D = I$ #

1.2

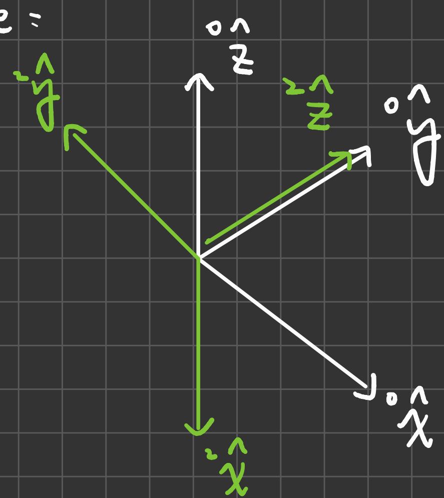
① ${}^2p = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$, $\begin{bmatrix} {}^1p \\ 1 \end{bmatrix} = {}^1T_2 \begin{bmatrix} {}^2p \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \\ -4 \\ 1 \end{bmatrix}$

$\Rightarrow {}^1p = [4 \ -5 \ -4]^T$ #

② ${}^0T_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $(= {}^0T_1 {}^1T_2)$, ${}^0R_2 = \begin{array}{c} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \\ \begin{matrix} \uparrow & \uparrow & \uparrow \\ {}^2\hat{x} & {}^2\hat{y} & {}^2\hat{z} \end{matrix} \end{array}$

(x, y, z axes' unit vector of frame 2 with respect to frame 0)

visualize :



the motion from frame 0 to frame 2 :

* extrinsic convention
(assign frame 0 as global frame)

1. Rotate -90° about ${}^0\hat{x}$
2. Rotate 90° about ${}^0\hat{y}$

(no translation) #

1.3

1.3.1

(in homogeneous coordinate frame)

① $^{w}P_m = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ ② $^{e}P_m = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$ ③ $^{e}T_w = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

$$^{e}T_w \, ^{w}P_m = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} = {}^{e}P_m$$

1.3.2

(homogeneous coordinate frame)

① $^{w}P_m = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ ② $^{e}P_m = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ ③ $^{e}T_w = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 5 \\ \sin 90^\circ & \cos 90^\circ & -4 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix} \quad {}^{e}T_w$$

$$, \quad ^{e}T_w \, ^{w}P_m = \begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = {}^{e}P_m$$

④ Eve's pose in Wall-E's coord. frame : $^{w}T_e$

$$^{w}T_e = \begin{bmatrix} \cos -90^\circ & -\sin -90^\circ & 4 \\ \sin -90^\circ & \cos -90^\circ & 5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

1.3.3 (homogeneous coordinate frame)

① $${}^{w}P_m = \begin{bmatrix} 6 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

② $${}^{w}P_e \begin{bmatrix} 6 \\ 10 \\ 4 \\ 1 \end{bmatrix}$$

③ $${}^{e}T_w = \begin{bmatrix} \cos -90^\circ & -\sin -90^\circ & 0 & -10 \\ \sin -90^\circ & \cos -90^\circ & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

recall:

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

④ $${}^{e}T_m = \begin{bmatrix} \cos 90^\circ & 0 & \sin 90^\circ & -10 \\ 0 & 1 & 0 & 0 \\ -\sin 90^\circ & 0 & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -10 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

${}^{m}T_w$:

$${}^{m}R_w = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(extrinsic) 2. y_m by -90° 1. z_m by -90°

$$\Rightarrow {}^{m}T_w = \begin{bmatrix} 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(5) $${}^{e}T_{m}\,{}^{m}T_{w} = \begin{bmatrix} 0 & 0 & 1 & -10 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{e}T_{w}$$