Answers to questions in

Lab 1: Filtering operations

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Program: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Instructions**: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

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**Question 1**: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9),

(17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers: Waveforms of different frequencies and directions

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**Question 2**: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers: The Fourier transform generates a waveform in 2D, where the frequency is q in the x-direction and p in the y-direction, assuming that p and q starts with 0, not 1 as in Matlab.

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**Question 3**: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers: The amplitude is 1/N=1/128 in theory, but 1/(128\*128) in Matlab.

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**Question 4**: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers: The magnitude of (p,q) determines the frequency of the waveform, whereas the relation between p and q determines its direction. The length is the inverse of the frequency scaled by the number of pixels, N/sqrt(p^2 + q^2).

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**Question 5**: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers: The frequencies are so high that the angular step between two points is more than half the period. It looks as if the frequencies start to decrease and the waveforms move in the opposite direction.

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**Question 6**: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers: It shifts the frequencies so that you get (0,0) in the center. Due to the periodicy of the Fourier transform this doesn’t change anything.

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**Question 7**: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers: The signal consists of two components. For one component there is only variations x-wise and in the other only y-wise. Thus you have information in Fourier space only for frequencies where either p=0 or q=0.

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**Question 8**: Why is the logarithm function applied?

Answers: To make small variations in the Fourier transform visible to the human eye.

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**Question 9**: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers: A Fourier transform is linear, since F(a+b)=F(a)+F(b).

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**Question 10**: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers: You can compute the Fourier transform of F and G separately and convolve the outputs. For this to work in Matlab you have to use fftshift to get the right quadrant of the output that is 2N\*2N in size after the convolusion and then rescale with 1/(N\*N).

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**Question 11**: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers: If you scale up and image, the image content will be larger and the frequencies smaller. The Fourier transform will look similar, but be compressed. The opposite if true if you scale down the image.

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**Question 12**: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers: A rotation is just a change in coordinate systems. Thus the Fourier transform rotates similarly. However, for many angles you get aliasing that is visible in the higher frequencies, due to the limited resolution of images.

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**Question 13**: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers: The magnitude contains the amplitudes of the waveforms that the image consists of, whereas the phase tells where each waveform starts. With many waveforms combined the phase will tell where edges are located and the magnitude what the intensities are on either side of the edge. The phase is thus more important for the interpretation of the image.

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**Question 14**: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and

100.0?

Answers: You get considerable errors for t = 0.1 and some errors for t = 0.3, but not for t = 1.0 and t = 10.0.

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**Question 15**: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers: The computed variances should be the same as the theoretical ones. For t = 0.1 and t = 0.3 though the waveforms are so small that they only contain some pixels and you get discretization errors. A-question: For really large kernels you get errors because the Gaussian bell doesn’t fit the size of the image.

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**Question 16**: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers: An image that gets blurrier and blurrier.

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**Question 17**: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers: Ideal low-pass filters are ok for Gaussian noise, as long as you don’t use cut-off frequencies and have edges so that ringing effects becomes apparent. Gaussian filters are fine for Gaussian noise, but smear out salt-and-pepper noise. Median filters are good for salt-and-pepper noise, but for Gaussian noise it leads to painting-like images.

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**Question 18**: What conclusions can you draw from comparing the results of the respective methods?

Answers: Use Gaussian filter for Gaussian noise and median filter for salt-and-pepper noise.

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**Question 19**: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers: The results are blocky when you don’t smooth, but somewhat blurry when you smooth the images.

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**Question 20**: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers: Smoothing prevents aliasing when subsampling and usually lead to better looking images. A-question: Smoothing removes higher frequencies that cannot be represented when the resolution is decreased. As a result these frequencies will not distort the lower frequencies and cause aliasing.

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