# Matter Non-conservation in the Universe and Dynamical Dark Energy

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_Abstract_.  In an expanding universe the vacuum energy density $\rho_\Lambda$ is expected to be a dynamical quantity. In quantum field theory in curved space-time $\rho_\Lambda$ should exhibit a slow evolution, determined by the expansion rate of the universe $H$. Recent measurements on the time variation of the fine structure constant and of the proton-electron mass ratio suggest that basic quantities of the Standard Model, such as the QCD scale parameter $\Lambda_{\rm QCD}$, may not be conserved in the course of the cosmological evolution. The masses of the nucleons $m_N$ and of the atomic nuclei would also be affected. Matter is not conserved in such a universe. These measurements can be interpreted as a leakage of matter into vacuum or vice versa. We point out that the amount of leakage necessary to explain the measured value of $\dot{m}_N/m_N$ could be of the same order of magnitude as the observationally allowed value of $\dot{\rho}_\Lambda/\rho_\Lambda$, with a possible contribution from the dark matter particles. The dark energy in our universe could be the dynamical vacuum energy in interaction with ordinary baryonic matter as well as with dark matter.

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# 1 Introduction

The Standard Model (SM) of the strong and electroweak (EW) interactions contains 27 independent fundamental constants: the QED fine structure constant $\alpha_{\rm em} = e^2/4\pi$, the $SU(2)_L$ gauge coupling $g$ of the EW interactions, the gauge coupling constant of the strong interactions $g_s$, the mass $M_W$ of the weak gauge boson $W$, the mass $M_{\mathcal{H}}$ of the Higgs boson $\mathcal{H}$, the 12 masses of the quarks and leptons, the 3 mixing angles of the quark mass matrix, a CP-violating phase, the 3 mixing angles in the lepton sector, a CP-violating phase and two additional phases, if the neutrino masses are Majorana masses. One of the parameters in the list, the mass of the Higgs boson $M_{\mathcal{H}}$, has not been measured thus far, despite some recent hints [1].

If we include the Einstein-Hilbert (EH) Lagrangian of gravity, there are two more fundamental constants, both of them dimensionful: Newton's gravitational coupling $G_N$ and the cosmological constant $\Lambda$ (also denoted as the CC term). The gravity constant has the dimension of an inverse mass squared (in natural units): $G = 1/M_P^2$, where $M_P \simeq 1.22 \times 10^{19}$ GeV is the Planck mass, the largest mass scale in the universe. The cosmological constant has the dimension of mass squared, the mass being of order $H_0 \sim 10^{-33}$ eV, i.e. essentially the value of the Hubble parameter at present (the smallest mass scale in the universe).

Until recently the observational data on $|\Lambda|$ could only place an upper bound, but now cosmological observations give a value, which is tiny, but non-vanishing (in particle physics standards) and positive [2, 3]. It can be expressed as an energy density: $\rho_\Lambda^0 = \Lambda/(8\pi\,G_N) \sim 10^{-47}$ GeV$^4$ – the so-called vacuum energy density. We can define the mass scale associated to the CC term as follows: $m_\Lambda \equiv \left(\rho_\Lambda^0\right)^{1/4} \sim 10^{-3}$ eV. The scale $m_\Lambda$ is the geometric mean of the two extreme mass scales in the universe: $m_\Lambda \sim (H_0\,M_P)^{1/2}$. In the $\Lambda$CDM model (i.e. the standard model of cosmology) this scale associated to the vacuum is assumed to be constant. This is a big puzzle within the $\Lambda$CDM model.

The dark energy (DE) problem was originally presented as the cosmological constant (CC) problem [4, 5, 6]. This is one the basic problems of physics, ever since it was first formulated 45 years ago [7] [1].

In this paper we suggest the possibility that some of the cosmological constant problems might be related to basic parameters of the Standard Model. The nucleon mass and the QCD scale $\Lambda_{\rm QCD}$ might not have remained constant throughout the history of the cosmological evolution [9, 10, 11, 12]. This is related to the time variation of the fine structure constant. Constraints on the ratio $\dot{\alpha}_{\rm em}/\alpha_{\rm em}$ can be derived from limits on the position of nuclear resonances in natural fission reactors which have been working for the last few billions years – the so-called "Oklo phenomenon" [13, 14, 15].

There could also be a cosmic time variation of the strong coupling constant, $\alpha_s$, related to the variation of the fundamental QCD scale $\Lambda_{\rm QCD}$. One expects that $\dot{\Lambda}_{\rm QCD}/\Lambda_{\rm QCD}$ should be larger than $\dot{\alpha}_{\rm em}/\alpha_{\rm em}$. Recent high precision experiments performed both in the laboratory with atomic clocks [16, 17, 18] and in astrophysics using data from quasars [19] support these ideas.

It has been suggested that the parameters of the EH action, $G_N$ and $\Lambda$, may be varying with

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[1] For a recent detailed account of the old fine tuning CC problem, see e.g. sect. 2 and Appendix B of [8].

time due to the interaction of the vacuum with the matter [2]. This time evolution might be linked to the time variation of the QCD scale and to the time shift of all the particle masses, including the dark matter ones. All the atomic masses of the chemical elements would be affected.

Here is the outline of this paper. In section 2 we review the models with time evolving cosmological parameters. In section 3 we specialize to a class of these models, where the time evolution is viewed as a renormalization group evolution. In section 4 we describe some experiments, providing evidence of the time variation of masses and couplings, and suggests a link of this variation with that of the cosmological parameters. In section 5 we propose the existence of a leakage of matter into the vacuum as a possible source of dynamical dark energy and compute the variation of the particle masses and the QCD scale with the Hubble rate. The last section contains our conclusions.

## 2   Cosmological models with time evolving parameters

We discuss the possibility that the cosmic time variations of the constants of particle physics and of cosmology are related. Scenarios, in which $G$ could be variable, have been previously discussed in the literature. Dirac suggested in the thirties (through his "large number hypothesis" [21]), that the gravitational constant $G$ could be varying with time in correlation with other fundamental constants. We also mention the ideas on time varying fundamental constants by Milne and Jordan at about the same time [22]. Later the time variation of $G$ was tied to the existence of a dynamical scalar field coupled to the curvature - the original Jordan and Brans-Dicke proposals [23].

Consider the General Relativity field equations in the presence of the cosmological term:

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = 8\pi G\,T_{\mu\nu}\,. \tag{2.1}$$

Here $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor, and $T_{\mu\nu}$ is the energy-momentum tensor of the isotropic matter and radiation in the universe. Without violating the Cosmological Principle within the context of the FLRW (Friedmann-Lemaître-Robertson-Walker) cosmology, nothing prevents the parameters $G = G(t)$ and $\Lambda = \Lambda(t)$ to be functions of the cosmic time, as it is the case with the scale factor itself $a = a(t)$. The possibility of a variable CC term has been considered by many authors from different points of view [24, 25], including the more recent quintessence approach – cf. [5] and references therein.

The contribution from the $\Lambda$ term, originally on the *l.h.s.* of Einstein's equations, can be absorbed on the *r.h.s.* after introducing the quantity $\rho_\Lambda = \Lambda/(8\pi G_N)$, which represents the vacuum energy density associated to the cosmological term. Einstein's equations can then be rewritten formally the same way as in (2.1), but replacing the ordinary energy-momentum tensor of matter by the total energy-momentum tensor of matter and the vacuum energy:

$$T_{\mu\nu} \to \tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + g_{\mu\nu}\,\rho_\Lambda = (\rho_\Lambda - p_m)\,g_{\mu\nu} + (\rho_m + p_m\,)\,U_\mu\,U_\nu\,. \tag{2.2}$$

Here $\rho_m$ and $p_m$ are the proper density and pressure of the isotropic matter, and $U_\mu$ is the 4-velocity of the cosmic fluid.

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[2]For a recent review, see e.g. [20] and references therein.

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The corresponding equation of state (EoS) $\omega_m = p_m/\rho_m$ reads: $\omega_m = 1/3$ and $\omega_m = 0$, for relativistic and non-relativistic matter respectively. The redefinition of the energy-momentum tensor can be done in the same way as in Eq. (2.2), whether $\rho_\Lambda$ is strictly constant or time varying. In both cases it enters with the equation of state $p_\Lambda = -\rho_\Lambda$, i.e. $\omega_\Lambda = -1$. This is in distinction to the general DE fluids, whose EoS take the generic form $p_D = \omega_D \rho_D$ (with $\omega_D < -1/3$) [5, 6].

We discuss now some possible scenarios for variable cosmological parameters that appear when we solve Einstein's equations (2.1) in the spatially flat FLRW metric, $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$, where $a(t)$ is the time-evolving scale factor. We restrict ourselves to the spatially flat case, since this seems to be the most plausible possibility in view of the present observational data [2] and the natural expectation from the inflationary universe. We consider Friedmann's equation with non-vanishing $\rho_\Lambda$, which provides Hubble's expansion rate $H = \dot{a}/a$ ($\dot{a} \equiv da/dt$) as a function of the matter and vacuum energy densities:

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda).\tag{2.3}$$

As stated, we assume that $\rho_\Lambda = \rho_\Lambda(t)$ and $G = G(t)$ can be functions of the cosmic time $t$. We will denote the current value of the Hubble rate by $H_0 \equiv 100\, h\, Km/s/Mpc$. The observations give $h \simeq 0.70$. The dynamical equation for the acceleration of the universe is:

$$\frac{\ddot{a}}{a} = -\frac{4\pi\, G}{3}(\rho_m + 3p_m - 2\rho_\Lambda) = -\frac{4\pi\, G}{3}(1 + 3\omega_m)\,\rho_m + \frac{8\pi\, G}{3}\,\rho_\Lambda.\tag{2.4}$$

In the late universe ($\rho_m \to 0$) the vacuum energy density $\rho_\Lambda$ dominates. It accelerates the cosmos for $\rho_\Lambda > 0$. This may occur either, because $\rho_\Lambda$ is constant, and for a sufficiently old universe one finally has $\rho_m(t) < 2\,\rho_\Lambda$, or because $\rho_\Lambda(t)$ evolves with time, and the situation $\rho_\Lambda(t) > \rho_m(t)/2$ is eventually reached sooner or later than expected. The general Bianchi identity $\bigtriangledown^\mu G_{\mu\nu} = 0$, involving the Einstein tensor on the *l.h.s.* of Eq. (2.1), leads to the following relation for the full source tensor on its *r.h.s.* (after we include the CC term):

$$\bigtriangledown^\mu \left(G\,\tilde{T}_{\mu\nu}\right) = \bigtriangledown^\mu \left[G\left(T_{\mu\nu} + g_{\mu\nu}\,\rho_\Lambda\right)\right] = 0.\tag{2.5}$$

The last equation provides the following "mixed" local conservation law:

$$\frac{d}{dt}\left[G(\rho_m + \rho_\Lambda)\right] + 3\,G\,H\,(\rho_m + p_m) = 0,\tag{2.6}$$

where $G$ and/or $\rho_\Lambda$ may be functions of the cosmic time. Although the previous equation is not independent of (2.3) and (2.4), it is useful to understand the possible transfer of energy between the vacuum and matter, with or without the participation of a time-evolving gravitational coupling. For instance, if $\dot{\rho}_\Lambda \neq 0$, matter is not generally conserved, since the vacuum could decay into matter, or matter could disappear into vacuum energy (including a possible contribution from a variable $G$, if $\dot{G} \neq 0$). The local conservation law (2.6) mixes the matter-radiation energy density with the vacuum energy $\rho_\Lambda$.

We mention the following possibilities:

- **Model I**: $G =$const. and $\rho_\Lambda =$const.:

If there are no other components in the cosmic fluid, this is the standard case of $\Lambda$CDM cosmology, implying the local covariant conservation law of matter-radiation:

$$\dot{\rho}_m + 3\,H\,(\rho_m + p_m) = 0. \tag{2.7}$$

- **Model II**: $G =$const and $\dot{\rho}_\Lambda \neq 0$:

  Here Eq.(2.6) leads to the mixed conservation law:

  $$\dot{\rho}_\Lambda + \dot{\rho}_m + 3\,H\,(\rho_m + p_m) = 0\,. \tag{2.8}$$

  An exchange of energy between the matter and the vacuum takes place.

- **Model III**: $\dot{G} \neq 0$ and $\rho_\Lambda =$const.:

  $$\dot{G}(\rho_m + \rho_\Lambda) + G[\dot{\rho}_m + 3H(\rho_m + p_m)] = 0\,. \tag{2.9}$$

  Since $G$ does not stay constant here, this equation implies a non-conservation of matter. It could be solved e.g. for $G$, if $\rho_m$ and $\rho_\Lambda$ would be given by some non-conservation ansatz.

- **Model IV**: $\dot{G} \neq 0$ and $\dot{\rho}_\Lambda \neq 0$:

  There are many possibilities here. We consider the simplest one by assuming the standard local covariant conservation of matter-radiation, i.e Eq. (2.7). Eq. (2.6) leads to:

  $$(\rho_m + \rho_\Lambda)\dot{G} + G\dot{\rho}_\Lambda = 0\,. \tag{2.10}$$

  This situation is complementary to the previous one. Here the dynamical interplay is between $G$ and $\rho_\Lambda$, whereas $\rho_m$ is also time evolving, but decoupled from the feedback between $G$ and $\rho_\Lambda$.

- **Model V**: Another possibility with $\dot{G} \neq 0$ and $\dot{\rho}_\Lambda \neq 0$ is the case that there is no matter in the universe: $\rho_m = 0$. Then Eq. (2.6) implies $G\,\rho_\Lambda =$const. This does not exclude that both parameters can be time evolving while the product remains constant. This situation could only be of interest in the early universe, when matter still did not exist and only the vacuum energy was present.

Only in the class of Models I and IV matter is covariantly self-conserved, i.e. matter evolves according to Eq. (2.7). In terms of the scale factor we find:

$$\rho'_m(a) + \frac{3}{a}(1 + \omega_m)\,\rho_m(a) = 0\,. \tag{2.11}$$

The prime indicates $d/da$. Its solution can be expressed as follows:

$$\rho_m(a) = \rho_m^0\,a^{-3(1+\omega_m)} = \rho_m^0\,(1 + z)^{3(1+\omega_m)}\,. \tag{2.12}$$

We have expressed the result (2.12) in terms of the scale factor $a = a(t)$ and the cosmological redshift $z = (1 - a)/a$.

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We shall focus on Models II, III and IV. Each of these models stands for a whole class of possible scenarios. One has to introduce more specifications before being able to perform concrete calculations. The variation of the "fundamental constants" (e.g. $\rho_\Lambda$, $G$) could emerge as an effective description of some deeper dynamics associated to QFT in curved space-time, e.g. in quantum gravity or in string theory. This should provide definite time/redshift-evolution laws $\rho_\Lambda = \rho_\Lambda(z)$, $G = G(z)$. Examples will be discussed in the next sections.

Other fundamental parameters could also be variable. The fine structure constant might change in time/redshift – see e.g. [26, 27]. However positive evidences [28] are questioned [29]. The possibility that the fundamental QCD scale parameter $\Lambda_{QCD}$ of the strong interactions could also be time-evolving (hence redshift dependent) is of special interest (see sections 4 and 5 for details). This could lead to the non-conservation of matter in the universe. In this paper we discuss the possibility that this non-conservation of matter might be related to the cosmological matter non-conservation. This would lead to a departure from the standard cosmological scenario.

## 3  Running vacuum energy and the coupling of gravity

The running of the vacuum energy and/or the gravitational coupling is expected in QFT in curved space-time [30, 31], see also [20] and references therein. Running couplings in flat QFT provide a useful theoretical tool to investigate theories as QED or QCD. Here the corresponding gauge coupling constants run with the typical energy of the process.

In the universe we expect that the running of $\rho_\Lambda$ and $G$ is associated with the typical energy of the classical gravitational external field linked to the FLRW metric. Here the Hubble rate $H$ will set the scale, since it is related to the non-trivial structure of the FLRW background. The universe in an accelerated expansion ($H \neq 0$, $\dot{H} \neq 0$) is a space-time with dynamical intrinsic curvature:

$$R = -6 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) = -12\,H^2 - 6\,\dot{H}\,. \tag{3.1}$$

In the effective action of QFT in curved space-time [32] $\rho_\Lambda$ and $G$ should be effective couplings depending on a mass scale $\mu$. This scale parameterizes the various quantum effects from the matter fields. In some cases the vacuum energy and the gravitational coupling can be represented as a power series of $\mu$. The rates of change are given by:

$$\frac{d\rho_\Lambda(\mu)}{d\ln\mu^2} = \sum_{k=0,1,2,...} A_{2k}\,\mu^{2k} = A_0 + A_2\,\mu^2 + A_4\,\mu^4 + ...\,, \tag{3.2}$$

$$\frac{d}{d\ln\mu^2}\left(\frac{1}{G(\mu)}\right) = \sum_{k=0,1,2,...} B_{2k}\,\mu^{2k} = B_0 + B_2\,\mu^2 + B_4\,\mu^4 + ....  \tag{3.3}$$

Such a "running" of $\rho_\Lambda$ and $G$ with $\mu$ reflects the dependence of the leading quantum effects on a cosmological quantity $\xi$ associated with $\mu$, hence $\rho_\Lambda = \rho_\Lambda(\xi)$ and $G = G(\xi)$. In cosmology we expect that the physical scale $\xi$ could be the Hubble rate $H(t)$, or the scale factor $a(t)$ [30], which in most of the cosmological past also maps out the evolution of the energy densities with $H$. We will concentrate here on the setting $\mu = H$, which naturally points to the non-trivial curvature of the background – Eq (3.1) – and also to the typical energy of the FLRW "gravitons" attached to the

quantum matter loops contributing to the running of $\rho_\Lambda$ and $G^{-1}$ in a semi-classical description of gravity. The coefficients $A_{2k}, B_{2k}$ receive contributions from boson and fermion matter fields of different masses $M_i$. The series (3.2) becomes an expansion in powers of the small quantities $H/M_i$ (see Eq. (3.4) below). Only even powers of $H$ are involved, due to the general covariance of the effective action [30, 31] [3]. These expansions converge very fast for $\mu = H$, since $H/M_i \ll 1$ for any ordinary particle mass. No other $H^{2n}$-terms beyond $H^2$ (not even $H^4$) can contribute significantly on the *r.h.s.* of equation (3.2) at any stage of the cosmological history below the GUT scale $M_X \lesssim M_P$. We find:

$$\frac{d\rho_\Lambda(\mu)}{d\ln \mu^2} = \frac{1}{(4\pi)^2} \left[ \sum_i c_i M_i^2\, \mu^2 + \sum_i c_i'\, \mu^4 + \sum_i \frac{c_i''}{M_i^2}\, \mu^6 + ... \right] \equiv n_2\, \mu^2 + \mathcal{O}(\mu^4)\,. \qquad (3.4)$$

We have omitted the $A_0$ term - it would be of order $M_i^4$. This would produce a too fast running of $\rho_\Lambda$. This can also be derived from the fact that all known particles satisfy $\mu < M_i$ (for $\mu = H$). None of them is an active degree of freedom for the running of $\rho_\Lambda$, and only the subleading terms are available. Approximately we obtain a simple expression:

$$\rho_\Lambda(H) = n_0 + n_2 H^2\,. \qquad (3.5)$$

In view of the boundary condition $\rho_\Lambda(H_0) = \rho_\Lambda^0$ it is convenient to rewrite the coefficients of (3.5):

$$n_0 = \rho_\Lambda^0 - \frac{3\nu}{8\pi} M_P^2\, H_0^2\,, \qquad n_2 = \frac{3\nu}{8\pi} M_P^2\,. \qquad (3.6)$$

We have defined the dimensionless parameter

$$\nu = \frac{1}{6\pi} \sum_{i=f,b} c_i \frac{M_i^2}{M_P^2}\,. \qquad (3.7)$$

The sum runs over fermions ($f$) and bosons ($b$) contributing to the loop. The parameter $\nu$ provides the main coefficient of the one-loop $\beta$-function for the running of the vacuum energy. The generic expression (3.7) adopts a concrete form with coefficients $c_i$, depending on the effective action of the underlying QFT (see e.g. [30]). The parameter $\nu$ can have any sign $\sigma = \pm$, depending on whether bosons or fermions dominate.

It is convenient to write (3.7) as follows:

$$\nu = \frac{\sigma}{6\pi} \frac{M^2}{M_P^2}\,. \qquad (3.8)$$

Here $M^2 = |\sum_{i=f,b} c_i M_i^2|$ is an effective mass squared representing all the particles contributing to the running after counting their multiplicities. For $M = M_P$ we have $|\nu| = \mathcal{O}(10^{-2})$. In general we expect that the set of $M_i$ includes masses of some GUT theory with a mass scale $M_X \sim 10^{16}$ GeV ($M \simeq M_X < M_P$). A natural estimate is in the range $\nu = 10^{-6} - 10^{-3}$ [30].

If we would instead take the string scale as the characteristic GUT scale [34, 35], then $M/M_P \sim 10^{-2}$, and $|\nu|$ could move to the upper range $10^{-3}$. For $\nu = 0$ we have $n_2 = 0$ in Eq. (3.5). In

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[3]In practice, if one tries to fit the data with a time dependent CC term which is linear in the expansion rate, i.e. of the form $\Lambda \propto H$, the results deviate significantly from the standard $\Lambda$CDM predictions [33].

this case the vacuum energy remains strictly constant at all times: $\rho_\Lambda = \rho_\Lambda^0$, and we recover the standard situacion of the $\Lambda$CDM model. For non-vanishing $\nu$ the evolution law (3.5 leads to:

$$\rho_\Lambda(H) = \rho_\Lambda^0 + \frac{3\nu}{8\pi} M_P^2 \left(H^2 - H_0^2\right). \tag{3.9}$$

The expansions (3.2)-(3.3) are correlated by the Bianchi identity (2.6). If $\mu = \mu(t)$ is a well defined invertible function, $d\mu/dt \neq 0$ – as it is in the case with $\mu = H(t)$ – we must have

$$\frac{dG}{d\mu} \left(\rho_m + \rho_\Lambda\right) + G \frac{d\rho_\Lambda}{d\mu} + G \left[\frac{d\rho_m}{d\mu} + \frac{3}{a} \left(\rho_m + p_m\right) \frac{da}{d\mu}\right] = 0. \tag{3.10}$$

This expression shows, that the dynamical dependence of $\rho_\Lambda$ and $G$ may not be in the cosmic time $t$ (as in many phenomenological models in the literature [24]), but in $\mu$. There is a possible connection of the evolution of $\rho_\Lambda$ with the quantum effects of QFT in a curved background, i.e. with the running $\rho_\Lambda(\mu)$ in an expanding universe [20]. Since the quantum effects on $G$ and $\rho_\Lambda$ must satisfy the above differential constraint, they must be correlated. If we assume that $\rho_\Lambda$ evolves as indicated in (3.9), the corresponding running of $G$ must fulfill (3.10). But this is still not enough to determine $G = G(H)$ explicitly, since it depends on whether matter is conserved or not. Then one has to have a specific ansatz for the matter non-conservation equation.

We consider two possibilities. We assume that matter is conserved, as in Model IV of the previous section. The term in brackets on Eq. (3.10) vanishes – see (2.11). Using Friedmann's equation (2.3) and (3.9), we are left with:

$$\frac{3 H^2}{8\pi G} \frac{dG}{dH} + G \frac{3\nu}{4\pi} M_P^2 H = 0. \tag{3.11}$$

After integration we obtain:

$$G(H) = \frac{G_0}{1 + \nu \ln\left(H^2/H_0^2\right)}. \tag{3.12}$$

Here we have defined $G_0 = 1/M_P^2$, the current value of $G$, i.e. $G_0 = G(H_0)$. From (3.12) we find:

$$\frac{d}{d\ln H^2} \frac{1}{G} = \nu M_P^2. \tag{3.13}$$

Thus (3.12) is the solution of (3.3), when we take only the leading term in the expansion, which does not depend on $\mu = H$. This is consistent, since $1/G$ is a large quantity and must be dominated by this term. The quantity $\rho_\Lambda$, which in contrast is a much smaller quantity, can not be dominated by $A_0 \sim M_i^4$, but rather by the next-to-leading term, which is proportional to $H^2$. The leading term in each case dominates the corresponding running equation. Higher order corrections (involving more powers of $H$) are possible, but they are negligible in view of the current value of $H$.

We mention another simple case, where matter is not conserved. We write $dG/d\mu = G'(a) \, d\mu/da$, $d\rho_\Lambda/d\mu = \rho_\Lambda'(a) \, d\mu/da$, and $d\rho_m/d\mu = \rho_m'(a) \, d\mu/da$. Assuming that $\mu = \mu(a)$ is a well defined invertible function (which is indeed the case, when $\mu = H$) we have $d\mu/da \neq 0$. If $G$ is constant, Eq. (3.10) simplifies again:

$$\rho_\Lambda'(a) + \rho_m'(a) + \frac{3}{a}(1 + \omega_m) \rho_m(a) = 0. \tag{3.14}$$

This result is consistent with (2.8). The running of the vacuum energy is due to the non-conservation of matter. The solution is well-known (see [20] and references therein). The corresponding matter non-conservation law is:

$$\rho_m(a) = \rho_m^0 \, a^{-3(1+\omega_m)(1-\nu)} \, . \tag{3.15}$$

The associated running of the vacuum energy density as a function of the scale factor is given by:

$$\rho_\Lambda(a) = \rho_\Lambda^0 + \frac{\nu \, \rho_m^0}{1-\nu} \left[ a^{-3(1+\omega_m)(1-\nu)} - 1 \right] \, . \tag{3.16}$$

The equations (3.15) and (3.16) do satisfy (3.14) and the boundary conditions $\rho_m(a=1) = \rho_m^0$ and $\rho_\Lambda(a=1) = \rho_\Lambda^0$ are fulfilled for the present universe. The running vacuum law (3.16) is a consequence of the original equation (3.9). The consistency of these two formulae implies that the invertible function $\mu = \mu(a)$ (i.e. $H = H(a)$) is given by:

$$H^2(a) = \frac{8\pi \, G}{3 \, (1-\nu)} \left[ \rho_\Lambda^0 - \nu \, \rho_c^0 + \rho_m^0 \, a^{-3(1+\omega_m)(1-\nu)} \right] \, . \tag{3.17}$$

Here $\rho_c^0$ is the present value of the critical density. For $\nu = 0$ we obtain a dilution law for the matter density of the form (2.12), i.e. $\sim a^{-3}$ for non-relativistic and $\sim a^{-4}$ for relativistic matter; also a constant $\rho_\Lambda = \rho_\Lambda^0$, and the canonical form for the Hubble expansion rate $H = H(a)$.

These models are compatible with the observational data, both on the Hubble expansion (e.g. from SNIa+BAO) as well as on structure formation (power spectrum, growth factor and CMB) for values of the relevant parameter (3.7) up to $|\nu| \sim 10^{-3}$ – see [33, 36] for details, and [37] for some astrophysical applications. We shall come back to this cosmological input in sect. 5.

We note the coincidence of this order of magnitude estimate for $\nu$ with its theoretical expectations for being a $\beta$-function coefficient of $\rho_\Lambda$. The generalizations of these running vacuum models are possible at a similar level of phenomenological success, see e.g. [38]. As also shown in this reference, alternative dynamical models (such as the so-called entropic-force cosmologies) are not successful, although they have many elements in common. There exist time evolving vacuum models, which can help to cure the old cosmological constant problem and the coincidence problem [8, 39].

Thus there exists an interesting class of cosmological models with time evolving vacuum energy which are phenomenologically acceptable, but not every phenomenological model can be successfully tested (in this respect we have also mentioned the unsuccessful cosmologies with vacuum energy linear in $H$ – see [33] and references therein).

# 4 Time evolving masses in the Standard Model of particle physics

In this section we discuss experiments on the time variation of the fundamental constants of Nature. We suggest that they could be related to matter non-conservation.

## 4.1 The Oklo phenomenon

There are experiments which suggest that the fine structure constant $\alpha_{\rm em}$ has not remained constant throughout the cosmic evolution. There are many independent observations suggesting this

possibility [26, 27]. We also mention the "Oklo phenomenon" [13, 14, 15]. It is related to the natural fission reactor (the Oklo uranium mine) in Gabon (West Africa), first discovered in 1972 by the French Commissariat à l'Énergie Atomique. This natural reactor operated nearly 2 billion years ago for a period of some two hundred thousand years at a power of $\sim$ 100 Kw. The data correspond to a process that occurred at the redshift $z \simeq 0.16$ (for the typical values $h \simeq 0.70$, $\Omega_M^0 \simeq 0.27$, $\Omega_\Lambda^0 \simeq 0.73$ of the cosmological parameters). This is the redshift at which we may be sensitive to variations of the fundamental constants.

The fraction of $^{235}U$ in the Oklo site has decreased since then from 3.68% to 0.72%. This depletion with respect to the current standard value is a proof of the past existence of a spontaneous chain reaction. Water from river Oklo provided the moderator for the neutrons. One of the nuclear fission products is the Samarium's isotope $^{149}Sm_{62}$ which upon neutron capture becomes the excited isotope of the same element $^{150}Sm_{62}$:

$$^{149}Sm_{62} + n \rightarrow\ ^{150}Sm_{62} + \gamma\,. \tag{4.1.1}$$

The sustained fission chain at the Oklo mine leads to the process (4.1.1). The relatively light isotope $^{149}Sm_{62}$ is not a fission product of the $^{235}U$, so the reaction (4.1.1) took place in the natural ores of Oklo. It was observed that the ratio of isotopes $^{149}Sm_{62}/^{147}Sm_{62}$ in samples of Samarium in these ores is 0.02, while in normal Samarium is 0.9. The depletion shows that the reaction (4.1.1) took place for a long time in the Oklo reactor.

The cross section of the neutron capture (4.1.1) depends on the energy of a resonance at $E_r = 97.3$ meV and is well described by the Breit-Wigner formula:

$$\sigma(E) = \frac{g\pi\hbar^2}{2m_n E} \frac{\Gamma_n \Gamma_\gamma}{(E - E_r)^2 + \Gamma^2/4}\,. \tag{4.1.2}$$

Here $g = 9/16$ is a spin-dependent statistical factor, $\Gamma$ is the total width, i.e. the sum of the neutron partial width ($\Gamma_n = 0.533$ meV) and of the radiative partial width ($\Gamma_\gamma = 60.5$ meV). In order to estimate the cross-section in a more realistic way, one has to thermal average the above Breit-Wigner formula, using the geophysical conditions at the Oklo site. From here one can infer the uncertainty in the resonance energy, $\delta E_r$, which is set equal to $E^{\text{Oklo}} - E_r^0$, where $E^{\text{Oklo}}$ is the value of the resonance during the Oklo phenomenon and $E_r^0$ is the possibly different value taken today. From the mass formula of heavy nuclei the change in resonance energy is related to $\alpha_{\text{em}}$ through the Coulomb energy contribution:

$$\delta E_r = -1.1 \frac{\delta\alpha_{\text{em}}}{\alpha_{\text{em}}} \text{ MeV}\,. \tag{4.1.3}$$

From the estimates on $\delta E_r$ (ranging from a dozen meV to a hundred MeV [14, 15]) one infers from (4.1.3) a tight bound on the time variation of the fine structure constant of order $\dot{\alpha}_{\text{em}}/\alpha_{\text{em}} \sim 10^{-17}\,\text{yr}^{-1}$. This is comparable to the best bounds from atomic clocks [26, 27]. But the debate continues on the reliability of the data obtained in the Oklo mine. Even if the corresponding bound, obtained on the time variation of the electromagnetic coupling, is eventually validated, the Oklo phenomenon cannot easily provide information on the time variation of the strength of the nuclear interaction, since it is sensitive only to dimensionless ratios of nuclear quantities. It cannot

be used to extract a possible variation of the QCD scale parameter $\Lambda_{\mathrm{QCD}}$. This is essential to establish a link between the time variation of fundamental nuclear and particle physics constants with the the corresponding variation of the vacuum energy density in the the cosmic expansion.

## 4.2 Time variation of the fundamental QCD constant: implications for the nucleon mass and the nuclear masses in the universe

It has been argued that the fundamental QCD scale parameter $\Lambda_{\mathrm{QCD}}$ could vary much faster than $\alpha_{\mathrm{em}}$ [10, 11, 12]. This change would be related to a corresponding change of the nucleon mass. Within the context of QCD the nucleon mass and the other hadronic masses are determined by the value of the QCD scale parameter $\Lambda_{\mathrm{QCD}}$. The leading contribution to the nucleon mass can be expressed as $m_N \simeq c_{\mathrm{QCD}}\Lambda_{\mathrm{QCD}}$, where $c_{\mathrm{QCD}}$ is a non-perturbative coefficient. The masses of the light quarks $m_u$, $m_d$ and $m_s$ also contribute to the the proton mass, although by less than 10% only. There is also a small contribution from electromagnetism. Let us take for instance the proton mass $m_p \simeq 938$ MeV. It can be computed from the QCD scale parameter $\Lambda_{\mathrm{QCD}}$, the quarks masses and the electromagnetic contribution:

$$
\begin{aligned}
m_p &= c_{\mathrm{QCD}}\Lambda_{\mathrm{QCD}} + c_u\, m_u + c_d\, m_d + c_s\, m_s + c_{\mathrm{em}}\Lambda_{\mathrm{QCD}} \\
&= (860 + 21 + 19 + 36 + 2)\ \mathrm{MeV}\,.
\end{aligned}
\tag{4.2.1}
$$

The QCD scale parameter is related to the strong coupling constant $\alpha_s = g_s^2/(4\pi)$. To lowest (1-loop) order one finds:

$$
\alpha_s(\mu_R) = \frac{1}{\beta_0 \ln\left(\Lambda_{\mathrm{QCD}}^2/\mu_R^2\right)} = \frac{4\pi}{(11 - 2\,n_f/3)\ln\left(\mu_R^2/\Lambda_{\mathrm{QCD}}^2\right)}\,,
\tag{4.2.2}
$$

where $\mu_R$ is the renormalization point and $\beta_0 \equiv -b_0 = -(33 - 2\,n_f)/(12\,\pi)$ ($n_f$ being the number of quark flavors) is the lowest order coefficient of the $\beta$-function.

The QCD scale parameter $\Lambda_{\mathrm{QCD}}$ has been measured: $\Lambda_{\mathrm{QCD}} = 217 \pm 25$ MeV. When we embed QCD in the FLRW expanding background, the value of $\Lambda_{\mathrm{QCD}}$ need not remain rigid anymore. The value of $\Lambda_{\mathrm{QCD}}$ could change with $H$, and this would mean a change in the cosmic time. If $\Lambda_{\mathrm{QCD}} = \Lambda_{\mathrm{QCD}}(H)$ is a function of $H$, the coupling constant $\alpha_s = \alpha_s(\mu_R; H)$ is also a function of $H$ (apart from a function of $\mu_R$). The relative cosmic variations of the two QCD quantities are related (at one-loop) by:

$$
\frac{1}{\alpha_s}\frac{d\alpha_s(\mu_R; H)}{dH} = \frac{1}{\ln(\mu_R/\Lambda_{\mathrm{QCD}})}\left[\frac{1}{\Lambda_{\mathrm{QCD}}}\frac{d\Lambda_{\mathrm{QCD}}(H)}{dH}\right]\,.
\tag{4.2.3}
$$

If the QCD coupling constant $\alpha_s$ or the QCD scale parameter $\Lambda_{\mathrm{QCD}}$ undergo a small cosmological time shift, the nucleon mass and the masses of the atomic nuclei would also change in proportion to $\Lambda_{\mathrm{QCD}}$.

The cosmic dependence of the strong coupling $\alpha_s(\mu_R; H)$ can be generalized to the other couplings $\alpha_i = \alpha_i(\mu_R; H)$ [11]. In a grand unified theory these couplings converge at the unification point. Let $d\alpha_i$ be the cosmic variation of $\alpha_i$ with $H$. Each of the $\alpha_i$ is a function of $\mu_R$, but the

expression $\alpha_i^{-1}(d\alpha_i/\alpha_i)$ is independent of $\mu_R$. One can show that the running of $\alpha_{\text{em}}$ is related to the corresponding cosmic running of $\Lambda_{\text{QCD}}$ as follows:

$$\frac{1}{\alpha_{\text{em}}}\frac{d\alpha_{\text{em}}(\mu_R; H)}{dH} = \frac{8}{3}\frac{\alpha_{\text{em}}(\mu_R; H)/\alpha_s(\mu_R; H)}{\ln(\mu_R/\Lambda_{\text{QCD}})}\left[\frac{1}{\Lambda_{\text{QCD}}}\frac{d\Lambda_{\text{QCD}}(H)}{dH}\right]. \qquad (4.2.4)$$

At the renormalization point $\mu_R = M_Z$, where both $\alpha_{\text{em}}$ and $\alpha_s$ are well-known, one finds:

$$\frac{1}{\alpha_{\text{em}}}\frac{d\alpha_{\text{em}}(\mu_R; H)}{dH} \simeq 0.03\left[\frac{1}{\Lambda_{\text{QCD}}}\frac{d\Lambda_{\text{QCD}}(H)}{dH}\right]. \qquad (4.2.5)$$

Thus the electromagnetic fine structure constant runs more than 30 times slower with the cosmic expansion than $\Lambda_{\text{QCD}}$. Searching for a cosmic evolution of $\Lambda_{\text{QCD}}$ is much easier than searching for the time variation of $\alpha_{\text{em}}$.

## 4.3   Time evolution of the proton - electron mass ratio

We consider the mass ratio:

$$\mu_{\text{pe}} \equiv \frac{m_p}{m_e}. \qquad (4.3.1)$$

This ratio is known with high accuracy: $\mu_{\text{pe}} = 1836.15267247(80)$ [40]. Since a change of $\Lambda_{\text{QCD}}$ would not affect the electron mass, the mass ratio (4.3.1) would change during the cosmological evolution.

First we consider astrophysical tests. The spectrum of $H_2$ provides a direct operational handle to test possible variations of (4.3.1). Particularly significant is the study of Ref.[19], based on comparing the $H_2$ spectral Lyman and Werner lines, observed in the Q 0347-383 and Q 0405-443 quasar absorption systems, with the laboratory measurements.

The result indicates, that $\mu_{\text{pe}}$ could have decreased in the past 12 Gyr, corresponding to a relative time variation of

$$\frac{\dot{\mu}_{pe}}{\mu_{\text{pe}}} = (-2.16 \pm 0.52) \times 10^{-15}\,\text{yr}^{-1}. \qquad (4.3.2)$$

It has been pointed more recently by other authors [41] that this measurement may suffer from spectral wavelength calibration uncertainties, and the reanalysis of the time variation would show a significance at the $1\,\sigma$ level only.

Now we consider laboratory tests, using atomic clocks. According to our estimate (4.2.5), the largest effect is expected to be a cosmological redshift (hence time variation) of the nucleon mass, which can be observed by monitoring molecular frequencies. These are precise experiments in quantum optics, e.g. obtained by comparing a cesium clock with 1S-2S hydrogen transitions. In a cesium clock the time is measured by using a hyperfine transition [4]. Since the frequency of the clock depends on the magnetic moment of the cesium nucleus, a possible variation of the latter is proportional to a possible variation of $\Lambda_{\text{QCD}}$. A hyperfine splitting is a function of $Z\,\alpha_{\text{em}}$ ($Z$ being the atomic number) and is proportional to $Z\,\alpha_{\text{em}}^2(\mu_N/\mu_B)(m_e/m_p)\,R_\infty$, where $R_\infty$ is the Rydberg

---

[4]Recall that the cesium hyperfine clock provides the modern definition of time. In SI units, the second is defined to be the duration of $9.192631770 \times 10^9$ periods of the transition between the two hyperfine levels of the ground state of the $^{133}$Cs atom

constant, $\mu_N$ is the nuclear magnetic moment and $\mu_B = e\hbar/2m_p c$ is the nuclear magneton. We have $\dot\mu_N/\mu_N \propto -\dot\Lambda_{\mathrm{QCD}}/\Lambda_{\mathrm{QCD}}$. The hydrogen transitions are only dependent on the electron mass, which we assume to be constant. The comparison over a period of time between the cesium clock with hydrogen transitions provides an atomic laboratory measurement of the ratio (4.3.1). The most recent atomic clock experiment at the MPQ (Max-Planck-Institut für Quantenoptik) at Garching near Munich gives a limit [16]:

$$\left| \frac{\dot\Lambda_{\mathrm{QCD}}}{\Lambda_{\mathrm{QCD}}} \right| < 10^{-14}\,\mathrm{yr}^{-1}\,. \tag{4.3.3}$$

Since the proton mass is given essentially by $\Lambda_{\mathrm{QCD}}$, as indicated by Eq. (4.2.1), we have $\dot m_p \simeq c_{\Lambda_{\mathrm{QCD}}} \dot\Lambda_{\mathrm{QCD}}$. The corresponding time variation of the ratio (4.3.1) would be:

$$\left| \frac{\dot\mu_{pe}}{\mu_{\mathrm{pe}}} \right| = \left| \frac{\dot m_p}{m_p} \right| \simeq \left| \frac{\dot\Lambda_{\mathrm{QCD}}}{\Lambda_{\mathrm{QCD}}} \right| < 10^{-14}\,\mathrm{yr}^{-1}\,. \tag{4.3.4}$$

Thus the atomic clock result (4.3.3) would indicate a time variation of the ratio $\mu_{\mathrm{pe}}$, which is consistent (in absolute value) with the astrophysical measurement (4.3.2). The result above implies also a bound for a possible time variation of the light quark masses:

$$\left| \frac{\dot m_q}{m_q} \right| \lesssim 10^{-14}\,\mathrm{yr}^{-1}\,. \tag{4.3.5}$$

# 5   Dynamical dark energy and a cosmic link with nuclear and particle physics

The time evolution of the fundamental "constants" $\rho_\Lambda$ and $G$ of gravity could be related to the time variation of the fundamental "constants" in nuclear and particle physics. In some models one can have matter conservation even though $\rho_\Lambda$ is running, but at the expense of having a running $G$ as well – confer Model IV of sect. 2 and Eq. (2.10). In an alternative class of models, $G$ runs thanks to the non-conservation of matter, as in Model III of sect. 2, but then $\rho_\Lambda$ stays fixed. If $G$ stays fixed and $\rho_\Lambda$ is evolving, there is a transfer of energy from matter into the vacuum, or vice versa – cf. the Model II class of sect. 2 and Eq. (2.8). The various classes of cosmological scenarios are interesting, but the last two could help us to understand the potential cosmic time variation of the fundamental "constants" of nuclear and particle physics, such as the QCD scale, the nucleon mass and the masses of nuclei.

## 5.1   Non-conservation of matter at fixed $G$

First we consider the class of scenarios denoted as Model II. Let $\rho_M^0$ be the total matter density of the present universe, which is essentially non-relativistic ($\omega_m \simeq 0$). The corresponding normalized density is $\Omega_M^0 = \rho_M^0/\rho_c^0 \simeq 0.27$, where $\rho_c^0$ is the current critical density. Similarly, $\Omega_\Lambda^0 = \rho_\Lambda^0/\rho_c^0 \simeq 0.73$ is the current normalized vacuum energy density, for flat space. If $\rho_\Lambda$ evolves with the Hubble rate in the form indicated in Eq. (3.9), the non-relativistic matter density and vacuum energy

density evolve with the scale factor, given in (3.15) and (3.16). Expressing the result in terms of the cosmological redshift $z = (1 - a)/a$, we find:

$$\rho_M(z; \nu) = \rho_M^0 \left(1 + z\right)^{3(1-\nu)},$$ (5.1.1)

and

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\nu \, \rho_M^0}{1 - \nu} \left[(1 + z)^{3(1-\nu)} - 1\right].$$ (5.1.2)

The crucial parameter is $\nu$, which we have introduced in sect. 3. It is responsible for the time evolution of the vacuum energy. From Eq. (5.1.1) we confirm, that it accounts also for the non-conservation of matter, since it leads to the exact local covariant conservation law (2.11). For non-relativistic matter we find:

$$\rho_M(z) = \rho_M^0 \left(1 + z\right)^3.$$ (5.1.3)

$\delta\rho_M \equiv \rho_M(z; \nu) - \rho_M(z)$ is the net amount of non-conservation of matter per unit volume at a given redshift. This expression must be proportional to $\nu$, since we subtract the conserved part. At this order we have $\delta\rho_M = -3 \nu \, \rho_M^0 (1+z)^3 \ln(1+z)$. We differentiate it with respect to time and expand in $\nu$, and finally divide the final result by $\rho_M$. This provides the relative time variation:

$$\frac{\delta\dot\rho_M}{\rho_M} = 3\nu \left(1 + 3\ln(1 + z)\right) H + \mathcal{O}(\nu^2).$$ (5.1.4)

Here we have used $\dot z = (dz/da)\dot a = (dz/da)aH = -(1 + z)H$. Assuming relatively small values of the redshift, we may neglect the log term and are left with:

$$\frac{\delta\dot\rho_M}{\rho_M} \simeq 3\nu \, H.$$ (5.1.5)

From (5.1.2) we find:

$$\frac{\dot\rho_\Lambda}{\rho_\Lambda} \simeq -3\nu \frac{\Omega_M^0}{\Omega_\Lambda^0} (1 + z)^3 H + \mathcal{O}(\nu^2).$$ (5.1.6)

It is of the same order of magnitude as (5.1.5) and has the opposite sign. Let us compare the theoretical expression (5.1.5) with the experimental results (4.3.2) and (4.3.4), described in the previous section. Taking the current value of the Hubble parameter as a reference, $H_0 = 1.0227 \, h \times 10^{-10} \, \text{yr}^{-1}$, where $h \simeq 0.70$, we obtain $|\nu| \lesssim \mathcal{O}(10^{-4})$ for the most conservative case. It is a rather tight bound, in accordance with the QFT expectations in sect. 3.

What is the role played by the running vacuum energy (5.1.2)? Its evolution in combination with the non-conservation of matter affects many relevant cosmological observables, which have currently been measured with high precision. From a detailed analysis of the combined data on type Ia supernovae, the Cosmic Microwave Background (CMB), the Baryonic Acoustic Oscillations (BAO) and the structure formation data a direct cosmological bound on $\nu$ has been obtained in the literature [33, 36]:

$$|\nu|^{\text{cosm.}} \lesssim \mathcal{O}(10^{-3}), \qquad \text{(Model II sect. 2)}.$$ (5.1.7)

It is consistent with the theoretical expectations. In the next section we analyze another model which can also accommodate matter non-conservation in the form (5.1.1), but at the expense of a time varying $G$. We compare it with a similar model, where matter is conserved.

## 5.2 Non-conservation of matter at fixed $\rho_\Lambda$

Within the class of scenarios indicated as Model III of sect. 3 the parameter $\rho_\Lambda$ remains constant ($\rho_\Lambda = \rho_\Lambda^0$) and $G$ is variable. This is possible due to the presence of the *non* self-conserved matter density (5.1.1). Trading the time variable by the scale factor, we can rewrite Eq. (2.9) as follows:

$$G'(a)\left[\rho_M(a) + \rho_\Lambda^0\right] + G(a)\left[\rho_M'(a) + \frac{3}{a}\,\rho_M(a)\right] = 0\,. \tag{5.2.1}$$

The primes indicate differentiation with respect to the scale factor. We insert equation (5.1.1) in (5.2.1), integrate the resulting differential equation for $G(a)$ and express the final result in terms of the redshift:

$$G(z) = G_0\left[\Omega_M^0\,(1+z)^{3(1-\nu)} + \Omega_\Lambda^0\right]^{\nu/(1-\nu)}\,. \tag{5.2.2}$$

Here $G_0 = 1/M_P^2$ is the current value of the gravitational coupling. The previous equation is correctly normalized: $G(z=0) = G_0$, due to the cosmic sum rule in flat space: $\Omega_M^0 + \Omega_\Lambda^0 = 1$. For $\nu = 0$ the gravitational coupling $G$ remains constant: $G = G_0$. Since $\rho_\Lambda$ is constant in the current scenario, the small variation of $G$ is entirely due to the non-vanishing value of the $\nu$-parameter in the matter non-conservation law (5.1.1). This leads to the dynamical feedback of $G$ with matter [5]. For the present model Friedmann's equation (2.3) becomes:

$$H^2(z) = \frac{8\pi G(z)}{3}\left[\rho_M^0\,(1+z)^{3(1-\nu)} + \rho_\Lambda^0\right] = H_0^2\frac{G(z)}{G_0}\left[\Omega_M^0\,(1+z)^{3(1-\nu)} + \Omega_\Lambda^0\right]\,. \tag{5.2.3}$$

Combining (5.2.2) and (5.2.3), we find the Hubble function of this model in terms of $z$:

$$H^2(z) = H_0^2\left[\Omega_M^0\,(1+z)^{3(1-\nu)} + \Omega_\Lambda^0\right]^{1/(1-\nu)}\,, \tag{5.2.4}$$

and we obtain:

$$\frac{G(z)}{G_0} = \left[\frac{H^2(z)}{H_0^2}\right]^\nu\,. \tag{5.2.5}$$

Since $\nu$ is presumably small in absolute value (as in the previous section), we can expand (5.2.5) in this parameter:

$$G(H) \simeq G_0\left(1 + \nu\,\ln\frac{H^2}{H_0^2} + \mathcal{O}(\nu^2)\right)\,. \tag{5.2.6}$$

At leading order in $\nu$ this expression for the variation of $G$ is identical to the one found for Model IV of sect. 2, see Eq. (3.12), except for the sign of $\nu$. The equation (5.2.6) allows us to estimate the value of the parameter $\nu$ by confronting the model with the experimental data on the time variation of $G$. Differentiating (5.2.6) with respect to the cosmic time, we find in leading order in $\nu$:

$$\frac{\dot{G}}{G} = 2\nu\,\frac{\dot{H}}{H} = -2\,(1+q)\,\nu\,H\,, \tag{5.2.7}$$

where we have used the relation $\dot{H} = -(1+q)H^2$, in which $q = -\ddot{a}/(aH^2)$ is the deceleration parameter. From the known data on the relative time variation of $G$ the bounds indicate that

---

[5]This feedback can also be conceived in the context of gravitation holography [42] if one also takes as a starting point the matter non-conservation law (5.1.1). This law was first suggested and analyzed in [43] and later on in [44].

$|\dot{G}/G| \lesssim 10^{-12} \,\mathrm{yr}^{-1}$ [26, 27]. If we take the present value of the deceleration parameter, we have $q_0 = 3\Omega_M^0/2 - 1 = -0.595 \simeq -0.6$ for a flat universe with $\Omega_M^0 = 0.27$. It follows:

$$\left| \frac{\dot{G}}{G_0} \right| \lesssim 0.8 |\nu| \, H \,. \tag{5.2.8}$$

Taking the current value of the Hubble parameter: $H_0 \simeq 7 \times 10^{-11} \,\mathrm{yr}^{-1}$ (for $h \simeq 0.70$), we obtain $|\nu| \lesssim 10^{-2}$. The real value of $|\nu|$ can be smaller, but to compare the upper bound that we have obtained with observations makes sense in view of the usual interpretation of $\nu$ in sect. 3 and the theoretical estimates indicated there. The constraints from Big Bang nucleosynthesis (BBN) for the time variation of $G$ are more stringent and lead to the improved bound:

$$|\nu|^{\mathrm{BBN}} \lesssim 10^{-3} \,, \qquad (\text{Model III sect. 2}) \,. \tag{5.2.9}$$

This bound can be obtained by adapting the study of Ref. [45], which was made for Model IV of sect. 2. Since Models III and IV share a similar kind of running law for the gravitational coupling (except for the sign of $\nu$) – confer equations (3.12) and (5.2.6) —, we can extract the same bound for $|\nu|$ in the two models following the method of sect. 5.2 of Ref. [45] and references therein, particularly [46]. The final result is Eq. (5.2.9). The cosmological data from different sources furnish about the same upper bound on $|\nu|$ for the two running models where matter is non-conserved, i.e. Models II and III of sect. 2. In both cases the upper bound on $|\nu|$ is $\sim 10^{-3}$, as shown by equations (5.1.7) and (5.2.9).

The previous bounds on $|\nu|$ for Models II and III are completely general (meaning that they apply to all forms of matter), since they are obtained from cosmological data tracing the possible evolution of $\rho_\Lambda$ and $G$, respectively. But these cosmological bounds are weaker than those that follow, if we interpret $\nu$ as a matter non-conservation parameter. Since matter is indeed non-conserved in both of these models, Eq. (5.1.5) and the lab bound (4.3.3) do apply in the present case, but *only* if the non-conserved matter is of nuclear nature. In this case we obtain the stronger constraint

$$|\nu|^{\mathrm{lab.}} \lesssim \mathcal{O}(10^{-4}) \,, \qquad (\text{Models II and III  sect. 2}) \,. \tag{5.2.10}$$

But if the non-conserved matter is dark matter, then only the weaker (purely cosmological) bound (5.2.9) is valid (see the next section for a detailed discussion on the distinct contributions from nuclear matter and dark matter).

Despite $|G|$ varies with time in a comparable way in Models III and IV, the stronger bound (5.2.10) does *not* apply for Model IV, since matter is conserved in it and hence Eq. (5.1.1) does not hold for this model. Only the pure BBN cosmological bound (5.2.9) is applicable in this case. This primordial nucleosynthesis bound on Model IV coincides with an independent bound obtained for this model from type Ia supernovae, the Cosmic Microwave Background, the Baryonic Acoustic Oscillations and the structure formation data (cf. [36] for details). For Model IV two independent cosmological bounds (BBN plus the current cosmological data) converge to the same result:

$$|\nu|^{\mathrm{BBN+cosm.}} \lesssim \mathcal{O}(10^{-3}) \,, \qquad (\text{Model IV sect. 2}) \,. \tag{5.2.11}$$

Although the order of magnitude of the bounds on $|\nu|$ are sometimes coincident for different models, they are different. For example, Model IV cannot – in contrast to Models II and III – be used to explain the possible time variation of the fundamental constants of the strong interactions and the particle masses. It can only be used to explain the time variation of the cosmological parameters $\rho_\Lambda$ and $G$ in a way which is independent from the microphysical phenomena in particle physics and nuclear physics.

Finally, we note that the above cosmic changes in the values of the proton to electron mass ratio and $G$ or $\rho_\Lambda$ can be written in terms of dimensionless quantities (in natural units). For example, for Model II (where $G$ is fixed and $\rho_\Lambda$ is variable) we can define the dimensionless quantity $\lambda \equiv \Lambda/m_p^2 = 8\pi G \rho_\Lambda/m_p^2$. Then,

$$\frac{1}{\lambda}\frac{d\lambda}{dt} = \frac{\dot{\rho}_\Lambda}{\rho_\Lambda} - 2\frac{\dot{m}_p}{m_p} \propto \nu H \,, \tag{5.2.12}$$

because both terms on the *r.h.s.* are proportional to $\nu$ (cf. sect. 5.1). Similarly, for Model III (where $\rho_\Lambda$ is fixed and $G$ is variable) we can construct the dimensionless quantity $G m_p^2$. Its relative variation is also proportional to $\nu$:

$$\frac{1}{G m_p^2}\frac{d(G m_p^2)}{dt} = \frac{\dot{G}}{G} + 2\frac{\dot{m}_p}{m_p} \propto \nu H \,. \tag{5.2.13}$$

## 5.3 Non-conservation of baryonic matter versus dark matter and the cosmic evolution of $\Lambda_{\mathrm{QCD}}$

Here we focus on the impact of the cosmological Models II and III of sect. 2 on the non-conservation of matter in the universe. In the previous section we have considered bounds on the "leakage parameter" $\nu$ within the class of these models based on the non-conservation matter density law (5.1.1). We must be careful in interpreting such a non-conservation law. For example, if we take the baryonic density in the universe, which is essentially the mass density of protons, we can write $\rho_M^B = n_p m_p$, where $n_p$ is the number density of protons and $m_p^0 = 938.272013(23)$ MeV is the current proton mass. If this mass density is non-conserved, either $n_p$ does not exactly follow the normal dilution law with the expansion, i.e. $n_p \sim a^{-3} = (1 + z)^3$, but the anomalous law:

$$n_p(z) = n_p^0 (1 + z)^{3(1-\nu)} \quad \text{(at fixed proton mass } m_p = m_p^0\text{)}, \tag{5.3.1}$$

and/or the proton mass $m_p$ does not stay constant with time and redshifts with the cosmic evolution:

$$m_p(z) = m_p^0 (1 + z)^{-3\nu} \quad \text{(with normal dilution } n_p(z) = n_p^0 (1 + z)^3\text{)}. \tag{5.3.2}$$

In all cases it is assumed that the vacuum absorbs the difference (i.e. $\rho_\Lambda = \rho_\Lambda(z)$ "runs with the expansion"). The first possibility implies that during the expansion a certain number of particles (protons in this case) are lost into the vacuum (if $\nu < 0$; or ejected from it, if $\nu > 0$), whereas in the second case the number of particles is strictly conserved. The number density follows the normal dilution law with the expansion, but the mass of each particle slightly changes (decreases for $\nu < 0$, or increases for $\nu > 0$) with the cosmic evolution.

Here we adopt the second point of view, i.e. Eq. (5.3.2). We can interpret the tight bounds from the laboratory and cosmological observations summarized in sect. 4 as direct bounds on the cosmic time evolution of $\Lambda_{\rm QCD}$ (hence on $m_p$ and on the nuclei in the universe). Since the contribution of the quark masses $m_u, m_d$ and $m_s$ to the proton mass is small – cf. Eq. (4.2.1) – we can approximate the proton mass by $m_p \simeq c_{\rm QCD}\,\Lambda_{\rm QCD}$. It will be sufficient to take into account the leading effects of the time variation of $m_p$ through the corresponding effects in $\Lambda_{\rm QCD}$.

Since the matter content of the universe is dominated by the dark matter (DM), we cannot exclude that it also varies with cosmic time. Let us denote the mass of the dominant DM particle $m_X$, and let $\rho_X$ and $n_X$ be its mass density and number density, respectively. The overall matter density of the universe can be written as follows:

$$
\begin{aligned}
\rho_M &= \rho_B + \rho_L + \rho_R + \rho_X = (n_p\,m_p + n_n\,m_n) + n_e\,m_e + \rho_R + n_X\,m_X \\
&\simeq n_p\,m_p + n_n\,m_n + n_X\,m_X .
\end{aligned}
\tag{5.3.3}
$$

Here $n_p, n_n, n_e, n_X$ $(m_p, m_n, m_e, m_X)$ are the number densities (and masses) of protons, neutrons, electrons and DM particles. The baryonic and leptonic parts are $\rho_B = n_p\,m_p + n_n\,m_n$ and $\rho_L = n_e\,m_e$ respectively. The small ratio $m_e/m_p \simeq 5 \times 10^{-4}$ implies that the leptonic contribution to the total mass density is negligible: $\rho_L \ll \rho_B$. We have also neglected the relativistic component $\rho_R$ (photons and neutrinos).

If we assume that the mass change through the cosmic evolution is due to the time change of $m_p$, $m_n$ and $m_X$, we can compute the mass density anomaly per unit time, i.e. the deficit or surplus with respect to the conservation law, by differentiating (5.3.3) with respect to time and subtracting the ordinary (i.e. fixed mass) time dilution of the number densities. The result is:

$$
\delta\dot\rho_M = n_p\,\dot m_p + n_n\,\dot m_n + n_X\,\dot m_X .
\tag{5.3.4}
$$

The relative time variation of the mass density anomaly can be estimated as follows:

$$
\frac{\delta\dot\rho_M}{\rho_M} = \frac{n_p\,\dot m_p + n_n\,\dot m_n + n_X\,\dot m_X}{n_p\,m_p + n_p\,m_p + n_X\,m_X} \simeq \frac{n_p\,\dot m_p + n_n\,\dot m_n + n_X\,\dot m_X}{n_X\,m_X}\left(1 - \frac{n_p\,m_p + n_n\,m_n}{n_X\,m_X}\right) .
\tag{5.3.5}
$$

The current normalized DM density $\Omega^0_{\rm DM} = \rho_X/\rho_c \simeq 0.23$ is significantly larger than the corresponding normalized baryon density $\Omega^0_B = \rho_B/\rho_c \simeq 0.04$. Therefore $n_X\,m_X$ is larger than $n_p\,m_p + n_n\,m_n$ by the same amount. If we assume $\dot m_n = \dot m_p$, we find approximately:

$$
\frac{\delta\dot\rho_M}{\rho_M} = \frac{n_p\,\dot m_p}{n_X\,m_X}\left(1 + \frac{n_n}{n_p} - \frac{\Omega_B}{\Omega_{\rm DM}}\right) + \frac{\dot m_X}{m_X}\left(1 - \frac{\Omega_B}{\Omega_{\rm DM}}\right) .
\tag{5.3.6}
$$

In the approximation $m_n = m_p$ we can rewrite the prefactor on the *r.h.s* of Eq. (5.3.6) as follows:

$$
\frac{n_p\,\dot m_p}{n_X\,m_X} = \frac{\Omega_B}{\Omega_{\rm DM}}\frac{\dot m_p}{m_p}\left(1 - \frac{n_n/n_p}{1 + n_n/n_p}\right) \simeq \frac{\Omega_B}{\Omega_{\rm DM}}\frac{\dot m_p}{m_p}\left(1 - \frac{n_n}{n_p}\right) .
\tag{5.3.7}
$$

The ratio $n_n/n_p$ is of order 10% after the primordial nucleosynthesis. Since $\Omega_B/\Omega_{\rm DM}$ is also of order 10%, we can neglect the product of this term with $n_n/n_p$. When we insert the previous

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equation into (5.3.6), the two $n_n/n_p$ contributions cancel each other. The expression $1 - \Omega_B/\Omega_{\rm DM}$ factorizes in the two terms on the *r.h.s* of Eq. (5.3.6). The final result is:

$$\left(1 - \frac{\Omega_B}{\Omega_{\rm DM}}\right)^{-1} \frac{\delta\dot\rho_M}{\rho_M} = \frac{\Omega_B}{\Omega_{\rm DM}} \frac{\dot m_p}{m_p} + \frac{\dot m_X}{m_X} = \frac{\Omega_B}{\Omega_{\rm DM}} \frac{\dot\Lambda_{\rm QCD}}{\Lambda_{\rm QCD}} + \frac{\dot m_X}{m_X} . \tag{5.3.8}$$

We have used $m_p \simeq c_{\rm QCD}\,\Lambda_{\rm QCD}$, the latter being accurate up to $10\%$ corrections at most – see (4.2.1). Equation (5.3.8) should be a good approximation (at most $10\%$ corrections).

The expression $\delta\dot\rho_M/\rho_M$ in Eq. (5.3.8) must be the same as the one we have computed in (5.1.4), if we consider the models based on the generic matter non-conservation law (5.1.1). Therefore the two expressions should be equal, and we obtain approximately:

$$3\nu_{\rm eff}\,H = \frac{\Omega_B}{\Omega_{\rm DM}} \frac{\dot\Lambda_{\rm QCD}}{\Lambda_{\rm QCD}} + \frac{\dot m_X}{m_X} , \tag{5.3.9}$$

where we have defined

$$\nu_{\rm eff} = \frac{\nu}{1 - \Omega_B/\Omega_{\rm DM}} . \tag{5.3.10}$$

We have $\nu_{\rm eff} \simeq 1.2\,\nu$. The differential equation (5.3.9) describes approximately the connection between the matter non-conservation law (5.1.1), the evolution of the vacuum energy density $\rho_\Lambda$ (and/or $G$) and the time variation of the nuclear and particle physics quantities. Even if the DM does not change with the cosmic expansion, it is necessary to include it as a part of the total energy density of the universe.

We assume that the dark matter particles do not vary with time, i.e. $\dot m_X = 0$, and only the cosmic evolution of $\Lambda_{\rm QCD}$ accounts for the non-conservation of matter. Trading the cosmic time for the scale factor through $\dot\Lambda_{\rm QCD} = (d\Lambda_{\rm QCD}/da)\,a\,H$ and integrating the resulting equation, we can express the final result in terms of the redshift:

$$\Lambda_{\rm QCD}(z) = \Lambda_{\rm QCD}^0\,(1+z)^{-3\,(\Omega_{\rm DM}^0/\Omega_B^0)\,\nu_{\rm eff}} . \tag{5.3.11}$$

For the protons we obtain:

$$m_p(z) = m_p^0\,(1+z)^{-3\,(\Omega_{\rm DM}^0/\Omega_B^0)\,\nu_{\rm eff}} . \tag{5.3.12}$$

Here $\Lambda_{\rm QCD}^0$ and $m_p^0$ are the QCD scale and proton mass at present ($z=0$). $\Omega_{\rm DM}^0$ and $\Omega_B^0$ are the current values of these cosmological parameters.

The presence of the factor $\Omega_B^0/\Omega_{\rm DM}^0$ in the power law makes eq. (5.3.12) more realistic than eq. (5.3.2). In the case $\nu = 0$ the QCD scale and the proton mass would not vary with the expansion of the universe, but for non-vanishing $\nu$ it describes the cosmic running of $\Lambda_{\rm QCD} = \Lambda_{\rm QCD}(z)$ and $m_p = m_p(z)$. For $\nu > 0$ ($\nu < 0$) the QCD scale and proton mass decrease (increase) with the redshift. This is consistent, since for $\nu > 0$ ($\nu < 0$) the vacuum energy density is increasing (decreasing) with the redshift – cf. Eq. (5.1.2) –, and it is smaller (larger) now than in the past.

We can write down the variation of the QCD scale in terms of the Hubble rate $H$. With the help of Eq. (3.17) equation (5.3.11) can be turned into an expression for $\Lambda_{\rm QCD}$ given explicitly in terms of the primary cosmic variable $H$:

$$\Lambda_{\rm QCD}(H) = \Lambda_{\rm QCD}^0 \left[\frac{1-\nu}{\Omega_M^0} \frac{H^2}{H_0^2} - \frac{\Omega_\Lambda^0 - \nu}{\Omega_M^0}\right]^{-(\Omega_{\rm DM}^0/\Omega_B^0)\,\nu_{\rm eff}/(1-\nu)} , \tag{5.3.13}$$

with $\Omega_M^0 = \Omega_B^0 + \Omega_{DM}^0$. $\nu$ and $\nu_{eff}$ are involved in (5.3.13), since they come from different sources. This equation satisfies the normalization condition $\Lambda_{QCD}(H_0) = \Lambda_{QCD}^0$ due to the cosmic sum rule for flat space: $\Omega_M^0 + \Omega_\Lambda^0 = 1$.

Obviously the cosmic time variation of the $\Lambda_{QCD}$ scale is very small in our framework. This can be more easily assessed if we use Eqs. (5.3.11) and (5.3.13) to compute the relative time variation of the QCD scale with respect to the present value. Since $\nu$ is small it it easy to show that

$$\frac{\Lambda_{QCD}(z) - \Lambda_{QCD}^0}{\Lambda_{QCD}^0} = -\frac{\Omega_{DM}^0}{\Omega_B^0} \frac{\nu_{eff}}{1-\nu} \ln\left[\frac{1-\nu}{\Omega_M^0} \frac{H^2(z)}{H_0^2} - \frac{\Omega_\Lambda^0 - \nu}{\Omega_M^0}\right]. \tag{5.3.14}$$

As a concrete example, let us consider the studies made in Ref. [19] on comparing the $H_2$ spectral Lyman and Werner lines observed in the Q 0347-383 and Q 0405-443 quasar absorption systems. The comparison with the corresponding spectral lines at present may be sensitive to a possible evolution of these lines in the last twelve billion years and involves redshifts in the range $z \simeq 2.6 - 3.0$. A positive result could be interpreted as a small variation of the proton to electron mass ratio (4.3.1) between two widely separated epochs of the cosmological evolution [19] . Assuming that $|\nu| = \mathcal{O}(10^{-3})$, as suggested by Eq. (5.1.7), it follows from the previous formulae that the relative variation of $\Lambda_{QCD}$ in this lengthy time interval is only at the few percent level with respect to its present day value. From Eq. (3.16) we can then easily check that the corresponding variation of $\rho_\Lambda(z)$ with respect to the current value $\rho_\Lambda^0$ is also of a few percent. As expected, the two scales undergo tiny variations over very long periods of time, in fact cosmological periods, and therefore the large hierarchy between them at present – namely $\Lambda_{QCD} = \mathcal{O}(100)$ MeV$= \mathcal{O}(10^8)$ eV and $\rho_\Lambda^{1/4} = \mathcal{O}(10^{-3})$ eV – is essentially preserved over the cosmological evolution. However, even this small crosstalk between these two widely separated scales could be sufficient for being eventually detected by the aforementioned high precision experiments aiming at measuring very tiny variation of the proton to electron mass ratio. This is suggested by the fact that the expected range of values of $\nu$ is within the scope of the precision of these experiments.

Using the above equations and Eq. (4.2.2), we can obtain the corresponding evolution of the strong coupling constant $\alpha_s$ with the redshift and the Hubble rate, i.e. $\alpha_s(\mu_R; z)$ and $\alpha_s(\mu_R; H)$:

$$\frac{1}{\alpha_s(\mu_R; z)} = \frac{1}{\alpha_s(\mu_R; 0)} + 6\, b_0\, \frac{\Omega_{DM}^0}{\Omega_B^0} \nu_{eff} \ln(1+z). \tag{5.3.15}$$

Here $\alpha_s(\mu_R; 0)$ is the value of $\alpha_s(\mu_R; z)$ today $(z = 0)$. Since $b_0 > 0$ (cf. sect. 4.2), we observe that for $\nu > 0$ $(\nu < 0)$ the strong interaction $\alpha_s(\mu_R; z)$ decreases (increases) with $z$, i.e. with the cosmic evolution. We also find[6]:

$$\frac{1}{\alpha_s(\mu_R; H)} = \frac{1}{\alpha_s(\mu_R; H_0)} + 2\, b_0\, \frac{\Omega_{DM}^0}{\Omega_B^0} \frac{\nu_{eff}}{1-\nu} \ln\left[\frac{1-\nu}{\Omega_M^0} \frac{H^2}{H_0^2} - \frac{\Omega_\Lambda^0 - \nu}{\Omega_M^0}\right]. \tag{5.3.16}$$

Here $\alpha_s(\mu_R; H_0)$ is the current value of $\alpha_s(\mu_R; H)$.

Above we have determined the strong coupling as a function of *two* running scales: one is the ordinary QCD running scale $\mu_R$, the other is the cosmic scale defined by the Hubble rate $H$, which

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[6]It is interesting to note that a similar running of $\alpha_s$ with the cosmic expansion was pointed out in a different context by J.D. Bjorken in [47].

has dimension of energy in natural units. The second term on the *r.h.s.* depends on the product of the two $\beta$-function coefficients, the one for the ordinary QCD running ($b_0$) and the one for the cosmic running ($\nu \propto \nu_{\rm eff}$).

We find:

i) for $\nu = 0$ there is no cosmic running of the strong interaction,

ii) for $\nu > 0$ the strong coupling $\alpha_s(\mu_R; H)$ is "doubly asymptotically free". It decreases for large $\mu_R$ and also for large $H$, whereas for $\nu < 0$ the cosmic evolution drives the running of $\alpha_s$ opposite to the normal QCD running,

iii) the velocity of the two runnings is very different, because $H$ is slowly varying with time and $|\nu| \ll 1$ and $|\nu| \ll b_0 \lesssim 1$. The cosmic running only operates in the cosmic history and is weighed with a very small $\beta$-function. But it may soon be measured in the experiments with atomic clocks and through astrophysical observations.

The previous equations describe not only the leading cosmic evolution of the QCD scale and the proton mass with the redshift and the expansion rate $H$ of the universe, but they can account for the redshift evolution of the nuclear masses. For the neutron we can write approximately: $m_n \simeq c_{\rm QCD} \Lambda_{\rm QCD}$. For an atomic nucleus of current mass $M_A$ and atomic number $A$ we have $M_A = Z\, m_p + (A - Z)\, m_n - B_A$, where $Z$ is the number of protons and $A - Z$ the number of neutrons, and $B_A$ is the binding energy. Although $B_A$ may also change with the cosmic evolution, the shift should be less significant, since at leading order the binding energy relies on pion exchange among the nucleons. The pion mass has a softer dependence on $\Lambda_{\rm QCD}$: $m_\pi \sim \sqrt{m_q \Lambda_{\rm QCD}}$, due to the chiral symmetry.

In the previous approximations we have neglected the light quark masses $m_q$. We can assume that the binding energy has a negligible cosmic shift as compared to the masses of the nucleons. In the limit where we neglect the proton-neutron mass difference and assume a common nucleon mass $m_N^0$ at present, the corresponding mass of the atomic nucleus at redshift $z$ is given at leading order by:

$$M_A(z) \simeq A\, m_N^0\, (1 + z)^{-3\,(\Omega_{\rm DM}^0/\Omega_B^0)\,\nu_{\rm eff}} - B_A\,. \tag{5.3.17}$$

Although the chemical elements redshift their masses, a disappearance or overproduction of nuclear mass (depending on the sign of $\nu$) is compensated by a running of the vacumm energy $\rho_\Lambda$, which is of opposite in sign, see (5.1.6).

Above we have described a simplified case, in which the nuclear matter evolves with the cosmic evolution as a result of the evolution of the fundamental QCD scale. In this scenario the light quark masses are neglected, and the DM does not participate in the cosmic time evolution.

Alternatively we can assume that the nuclear matter does not vary with time, i.e. $\dot{\Lambda}_{\rm QCD} = 0$, and only the DM particles account for the non-conservation of matter. In general we expect a mixed situation, in which the temporal rates of change for nuclear matter and for DM particles are different:

$$\frac{\dot{\Lambda}_{\rm QCD}}{\Lambda_{\rm QCD}} = 3\,\nu_{\rm QCD}\, H\,, \qquad \frac{\dot{m}_X}{m_X} = 3\,\nu_X\, H\,. \tag{5.3.18}$$

We have defined the QCD time variation index, which is characteristic of the redshift rate of the QCD scale, while $\nu_X$ is the corresponding one for the DM. In this more general case we find:

$$\Lambda_{\mathrm{QCD}}(z) = \Lambda_{\mathrm{QCD}}^0 \, (1+z)^{-3\,\nu_{\mathrm{QCD}}} \,, \qquad m_X(z) = m_X^0 \, (1+z)^{-3\,\nu_X} \,. \tag{5.3.19}$$

We introduce the effective baryonic redshift index $\nu_B$:

$$\nu_B = \frac{\Omega_B}{\Omega_{\mathrm{DM}}} \, \nu_{\mathrm{QCD}} \,. \tag{5.3.20}$$

The equations (5.3.19) satisfy the relation (5.3.9), provided the coefficients $\nu_B$ and $\nu_X$ are related by

$$\nu_{\mathrm{eff}} = \nu_B + \nu_X \,. \tag{5.3.21}$$

$\nu_{\mathrm{QCD}}$ is the intrisic cosmic rate of variation of the strongly interacting particles. The effective index $\nu_B$ weighs the redshift rate of these particles taking into account their relative abundance with respect to the DM particles. Even if the intrinsic cosmic rate of variation of $\Lambda_{\mathrm{QCD}}$ would be similar to the DM index (i.e. if $\nu_{\mathrm{QCD}} \gtrsim \nu_X$), the baryonic index (5.3.20) would still be suppressed with respect to $\nu_X$, because the total amount of baryon matter in the universe is much smaller than the total amount of DM.

In this mixed scenario the mass redshift of the dark matter particles follows a similar law as in the case of protons (5.3.12), except now we have $\nu_{\mathrm{eff}} \to \nu_B$. The proton would have the index $\nu_{\mathrm{QCD}}$ characteristic of the free (and bound) stable strongly interacting matter:

$$m_p(z) = m_p^0 \, (1+z)^{-3\,(\Omega_{\mathrm{DM}}/\Omega_B)\,\nu_B} = m_p^0 \, (1+z)^{-3\nu_{\mathrm{QCD}}} \,. \tag{5.3.22}$$

The DM particles have another independent index $\nu_X$. The sum (5.3.21) must reproduce the original index $\nu_{\mathrm{eff}} \propto \nu$, which we associated with the non-conservation of matter.

Finally we consider the possible quantitative contribution to the matter density anomaly from the dark matter. The global mass defect (or surplus) is regulated by the value of the $\nu$ parameter, but the contribution of each part (baryonic matter and DM) depends on the values of the individual components $\nu_B$ and $\nu_X$. We can obtain a numerical estimate of these parameters by setting the expression (5.3.8) equal to (5.1.5). The latter refers to the time variation of the matter density $\rho_M$ without tracking the particular way in which the cosmic evolution can generate an anomaly in the matter conservation. The former does assume that this anomaly is entirely due to a cosmic shift in the masses of the stable particles. Taking the absolute values, we obtain:

$$3|\nu_{\mathrm{eff}}|\,H \simeq \left| \frac{4}{23} \frac{\dot{\Lambda}_{\mathrm{QCD}}}{\Lambda_{\mathrm{QCD}}} + \frac{\dot{m}_X}{m_X} \right| < \frac{4}{23} \times 10^{-14}\,\mathrm{yr}^{-1} + \left| \frac{\dot{m}_X}{m_X} \right| \,. \tag{5.3.23}$$

Here we have used the experimental bound (4.3.3) on the time variation of $\Lambda_{\mathrm{QCD}}$.

Several cases can be considered, depending on the relation between the intrinsic cosmic rates of variation of the strongly interacting particles and DM particles, $\nu_{\mathrm{QCD}}$ and $\nu_X$. Since these indices can have either sign, we shall compare their absolute values:

- 1) $|\nu_X| \ll |\nu_B|$:

This condition implies $|\nu_X| \ll |\nu_{\mathrm{QCD}}|$. By demanding the stronger condition $|\nu_X| \ll |\nu_B|$, we insure that the intrinsic QCD cosmic rate $|\nu_{\mathrm{QCD}}|$ is much larger than the corresponding DM rate $|\nu_X|$. We can neglect the $\dot{m}_X/m_X$ term on the *r.h.s.* of (5.3.23), and we recover the equations (5.3.11)-(5.3.16) with $\nu_{\mathrm{eff}} \simeq \nu_B$. Using $H_0 \simeq 7 \times 10^{-11}\,\mathrm{yr}^{-1}$, we find:

$$|\nu_X| \simeq 0, \qquad |\nu_{\mathrm{eff}}| \simeq |\nu_B| < 10^{-5}, \qquad |\nu_{\mathrm{QCD}}| < 5 \times 10^{-5}. \qquad (5.3.24)$$

The bound on $\nu_B \simeq \nu_{\mathrm{eff}}$ that we have obtained above can be compared with (5.2.10). The former (which is more stringent) is more realistic than the latter because here we have taken into account explicitly the suppression factor $\Omega_B/\Omega_{\mathrm{DM}}$ of baryonic matter versus dark matter – and also the (small) difference between $\nu$ and $\nu_{\mathrm{eff}}$.

- 2) $|\nu_X| \simeq |\nu_B|$:

  Here we still have $|\nu_X|$ smaller than $|\nu_{\mathrm{QCD}}|$, but the requirement is weaker. It follows: $|\nu_{\mathrm{eff}}| \simeq 2|\nu_X| \simeq 2|\nu_B| = 2(\Omega_B/\Omega_{\mathrm{DM}})|\nu_{\mathrm{QCD}}|$, and we find

$$|\nu_{\mathrm{eff}}| < 2 \times 10^{-5}, \qquad |\nu_X| \simeq |\nu_B| < 10^{-5}, \qquad |\nu_{\mathrm{QCD}}| < 5 \times 10^{-5}. \qquad (5.3.25)$$

- 3) $|\nu_X| \simeq |\nu_{\mathrm{QCD}}|$:

  The two intrinsic cosmic rates for strongly interacting and DM particles are similar, i.e. $\dot{\Lambda}_{\mathrm{QCD}}/\Lambda_{\mathrm{QCD}}$ and $\dot{m}_X/m_X$ do not differ significantly. In this case Eq. (5.3.23) leads to

$$3|\nu_{\mathrm{eff}}|\,H < \left(\frac{4}{23} + 1\right) \times 10^{-14}\,\mathrm{yr}^{-1}. \qquad (5.3.26)$$

  There are two sign possibilities ($\nu_{\mathrm{QCD}} = \pm \nu_X$), and we take the absolute value:

$$|\nu_{\mathrm{eff}}| \lesssim \left(\frac{\Omega_B}{\Omega_{\mathrm{DM}}} + 1\right)|\nu_{\mathrm{QCD}}| \simeq |\nu_{\mathrm{QCD}}|. \qquad (5.3.27)$$

  We find:

$$|\nu_{\mathrm{eff}}| \lesssim |\nu_{\mathrm{QCD}}| \simeq |\nu_X| < 5 \times 10^{-5}. \qquad (5.3.28)$$

- 4) $|\nu_{\mathrm{QCD}}| \ll |\nu_X|$:

  Here the nuclear part is frozen. The non-conservation of matter is entirely due to the time variation of the DM particles. Eq. (5.3.23) gives:

$$3\nu\,H \simeq \frac{\dot{m}_X}{m_X}\left(1 - \frac{\Omega_B}{\Omega_{\mathrm{DM}}}\right). \qquad (5.3.29)$$

 We have written this expression directly in terms of the original $\nu$ parameter. In this case we cannot get information from any laboratory experiment on $\dot{m}_X/m_X$, but we do have independent experimental information on the original $\nu$ value (irrespective of the particular contributions form the nuclear and DM components). It comes from the cosmological data on type Ia supernovae, BAO, CMB and structure formation. The analysis of this data [33, 36] leads to the bound (5.1.7), which applies to all models, in which matter follows the generic

| | $\|\nu\|^{\mathrm{cosm}}$ | $\|\nu\|^{\mathrm{lab}} = \|\nu_B\|$ | $\|\nu_X\|^{\mathrm{cosm}}$ |
|---|---|---|---|
| Model II | $10^{-3}$ (SNIa+BAO+CMB) | $10^{-5}$ (Atomic clocks+Astrophys.) | $10^{-3}$ |
| Model III | $10^{-3}$ (BBN) | $10^{-5}$ (Atomic clocks+Astrophys.) | $10^{-3}$ |
| Model IV | $10^{-3}$ (SNIa+BAO+CMB)+BBN | $0$ | $0$ |

Table 1: Upper bounds on the running index $|\nu|$ for the various models defined in sect. 2. Only for Models II and III a non-vanishing value of $|\nu|$ is related to non-conservation of matter and a corresponding time evolution of $\rho_\Lambda$ and $G$, respectively. For these models, a part of $\nu$ (viz. $\nu_B$) is accessible to lab experiments, whereas the DM contribution ($\nu_X$) can only be bound indirectly from cosmological observations (same cosmological bound as for the overall $\nu$). For Model IV matter is conserved, and a non-vanishing value of $|\nu|$ (only accessible from pure cosmological observations) is associated to a simultaneous time evolution of $\rho_\Lambda$ and $G$ – with no microphysical implications.

non-conservation law (5.1.1) and the running vacuum law (3.9) — or the same matter non-conservation law and the running gravitational coupling law (5.2.6), as shown in Eq. (5.2.9). Since it depends on the cosmological effects from all forms of matter, it applies to the DM particles in particular. We find:

$$|\nu_X|^{\mathrm{cosm}} \lesssim 10^{-3} \,. \tag{5.3.30}$$

This bound is significantly weaker than any of the bounds found for the previous scenarios in which the nuclear matter participated of the cosmic time variation. It cannot be excluded that the matter non-conservation and corresponding running of the vacuum energy in the universe is mainly caused by the general redshift of the DM particles. In this case only cosmological experiments could be used to check this possibility. If the nuclear matter also participates in a significant way, it could be analyzed with the help of experiments in the laboratory. For a summary of the bounds, see Table 1.

If in the future we could obtain a tight cosmological bound on the effective $\nu_{\mathrm{eff}}$-parameter (5.3.21), using the astrophysical data, and an accurate laboratory (and/or astrophysical) bound on the baryonic matter part $\nu_B$, we could compare them and derive the value of the DM component $\nu_X$. If $\nu_{\mathrm{eff}}$ and $\nu_B$ would be about equal, we should conclude that the DM particles do not appreciably shift their masses with the cosmic evolution, or that they do not exist. If, in contrast, the fractional difference $|(\nu_{\mathrm{eff}} - \nu_B)/\nu_{\mathrm{eff}}|$ would be significant, the DM particles should exist to compensate for it.

# 6    Conclusions

In this paper we have described theoretical models based on the assumption that the basic constants of nature are slowly varying functions of the cosmic expansion, as suggested by numerous experiments. We have connected the variation of the nuclear and particle masses, fundamental scales and particle physics couplings (e.g. the fine structure and the strong coupling "constant")

to the possible cosmic evolution of the two parameters $\rho_\Lambda$ and $G$ of Einstein's gravity theory, i.e. the vacuum energy density (or cosmological "constant") and the gravity constant. The non-conservation of matter, associated to a time variation of the parameters in nuclear and particle physics, must be compensated by the corresponding evolution of the vacuum energy density and/or gravitational coupling $G$. This resulting picture of the cosmic evolution is compatible with the cosmological principle, but $\rho_\Lambda$ and/or $G$ evolve with the cosmic time in combination with the fundamental "constants" [48].

We have represented the possible time evolution of the physical quantities in terms of the effective (dimensionless) parameter $\nu_{\rm eff}$, proportional to the original $\nu$. If the experiments would detect a mass density anomaly in the microphysics world, e.g. through a (red)shift in the value of the proton to electron mass ratio, it would lead to a non-vanishing value of $\nu_B$ (which is the baryonic part of $\nu_{\rm eff}$). This anomaly would be correlated with the corresponding shift of the dimensionless quantities $\Lambda/m_p^2$ and $G\,m_p^2$ (for the class of Models II and III respectively). A shift in the value of these dimensionless quantities would determine $\nu \propto \nu_{\rm eff}$, and the corresponding value of $\nu_{\rm eff}$ could be confronted with a possible mass anomaly $\nu_B$ of the nuclear matter. From the difference with $\nu_{\rm eff}$ we could infer an indirect effect from dark matter, which is controlled by the dimensionless index $\nu_X$.

We have described the cosmic evolution of the various quantities through the Hubble rate $H$ as the basic scale, which can parameterize the running of the masses and couplings as well as the vacuum energy and/or Newton's constant $G$. The running of $\rho_\Lambda$ and $G$ is related to the quantum effects of the particles on the effective action of QFT in curved space-time. The vacuum energy density is written as a function of $H$: $\rho_\Lambda = \rho_\Lambda(H)$.

Matter is non-conserved. We have attributed the non-conservation to a cosmic redshift (hence a cosmic time variation) of the masses of the nucleons, due to the corresponding change of the QCD parameter $\Lambda_{\rm QCD}$. All atoms would be affected as well. One may expect that the redshift should affect the masses of all the fundamental particles (quarks and leptons), including the dark matter particles. We have explicitly proposed a connection of the cosmic time evolution of the $\Lambda_{\rm QCD}$ scale and of the elementary particle masses to the corresponding running of $\rho_\Lambda$ and/or $G$.

The present bounds, obtained for the time variation of the fundamental constants of nuclear matter, point to a rate of change of the nucleon mass and the $\Lambda_{\rm QCD}$ parameter, which is compatible with the corresponding bounds on the cosmic evolution of $\rho_\Lambda$ and $G$. The relevant dimensionless parameter, which controls the running of these quantities, must be of order $|\nu| \lesssim 10^{-3}$ or less. The current time variation of the vacuum energy can be of order $|\dot{\rho}_\Lambda/\rho_\Lambda| \sim 10^{-3}\,H_0 \sim 10^{-14}\,{\rm yr}^{-1}$. This can be compared with the current measured rate of change of the $\Lambda_{\rm QCD}$ scale in astrophysical and in atomic clock experiments, which provide bounds of the same order of magnitude. However the laboratory bounds affect only the nuclear matter contribution to $\nu$. The remaining contribution, as indicated before, should come from dark matter particles. This approach could eventually provide an indirect evidence, that dark matter particles exist.

Let us clarify that in our framework we cannot provide at this stage an explanation for the value of the cosmological constant nor of the QCD scale. The mass scale associated to the vacuum energy or DE is $m_\Lambda \equiv \left(\rho_\Lambda^0\right)^{1/4} \sim 10^{-3}$ eV, which is roughly eleven orders of magnitude smaller

than the value of the QCD scale, $\Lambda_{\mathrm{QCD}} = O(100)$ MeV. To explain the former from first principles would be so much as to provide a solution of the old cosmological constant problem, whereas to explain the latter would be tantamount to explain quark confinement. We do not provide here a clue for the solution of any of these problems, but we suggest that there may be a crosstalk between the scale of the measured vacuum energy in cosmology and the scale of the strong interactions (and in general with the particle masses). Despite we do not understand at this point the absolute value of these scales, the possible interaction between them could smoothly shift their values with the cosmic time; and this cosmological evolution could be measured both at the astrophysical level and in the laboratory within the next generation of atomic clocks [16].

The models of the cosmic evolution, discussed in this paper, offer an interesting perspective to unify the microphysical and the macrophysical laws of nature. The dark energy is the dynamical vacuum energy in interaction with matter. If the dark matter would participate in the cosmic redshift, affecting the baryonic matter, there would be an intimate connection between the evolution of the dark matter and of the dark energy.

The ideas presented here can be tested by different kind of experiments. They could help to understand the structure and cosmic behavior of ordinary matter as well as to uncover the mysteries of dark matter and dark energy. The small cosmic variation of the physical "constants" may signal a connection between the large scale structure of the universe and the quantum phenomena in the microcosmos.

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