$$aRb \iff \boxed{a^2 - b^2 = 3(b-a)}$$

$$\iff a^2 - b^2 = 3b - 3a$$

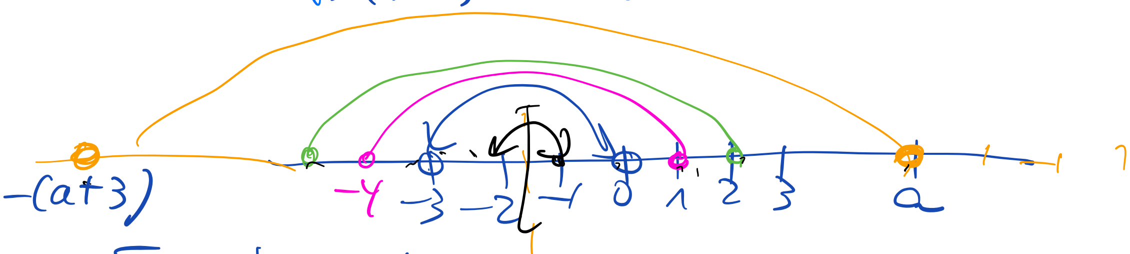
$$\iff \boxed{a^2 + 3a = b^2 + 3b}$$

$$bRa \iff b^2 + 3b = a^2 + 3a$$

$$bRa \iff b^2 - a^2 = 3(a-b)$$

$$\Updownarrow$$

$$3(b-a) = a^2 - b^2$$



$$\overline{0} = \{0, -3\}$$

$$\overline{1} = \{b : 4 = b^2 + 3b\} = \{1, -4\}$$

$$b^2 + 3b - 4 = 0$$

$$b = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm 5}{2} = \begin{cases} 1 \\ -4 \end{cases}$$

$$\overline{a} = \{\underline{b} : a^2 - b^2 = 3(b-a)$$

$$b^2 + 3b - 3a - a^2 = 0$$

$$b = \frac{-3 \pm \sqrt{9 + 4 \cdot (3a + a^2)}}{2}$$

$$= \frac{-3 \pm \sqrt{4a^2+12a+9}}{2} =$$

$$= \frac{-3 \pm \sqrt{(2a+3)^2}}{2}$$

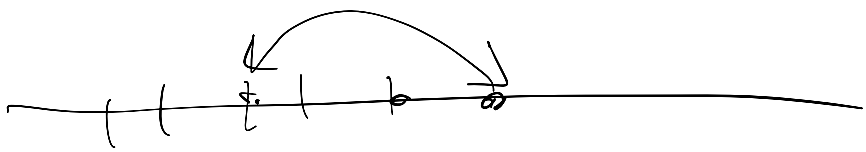
$$= \frac{-3 \pm (2a+3)}{2} = \begin{cases} \frac{2a}{2} = a \\ \frac{-2a-6}{2} = -a-3 \end{cases}$$

$$\overline{a} = \{a, -(a+3)\}$$

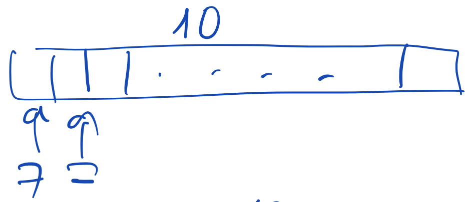
$$\boxed{\mathbb{Z}/_R = \{\overline{a} : a \in \mathbb{Z}\}, \text{ on } \overline{a} = \{a, -(a+3)\}}$$

$$= \{\overline{a} : a \in \mathbb{Z}, a \geq -1\}$$

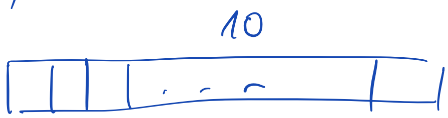
$$= \underbrace{\{\overline{-1}, \overline{0}, \overline{1}, \overline{2}, \ldots\}}_{7}$$



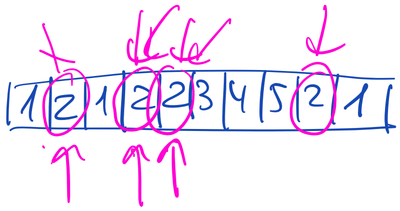
E5.

a) 

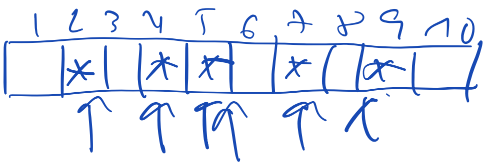
$\boxed{7^{10}}$

b) 

almenys 3 vegades el 2



2a, 4a, 5a
4a, 2a, 5a
2a, 4a, 8a
⋮



$\underbrace{\binom{10}{3}}\cdot 9^7$

(Σ) exactament $\underline{3}$ daus: $\binom{10}{3}\cdot \underbrace{8^7}_{\text{no el 2}}$,

" 4 " $\binom{10}{4}\cdot 8^6$

⋮

o bé: totes − (cap 2 + exact un 2 + exact 2 daus)

$$9^{10} - \left[8^{10} + 10 \cdot 8^9 + \binom{10}{2} \cdot 8^8\right]$$

c) $\underbrace{7777888999}_{\binom{10}{4,3,3}}$

binomial

$$\binom{n}{k} = \binom{n}{k, n-k}$$

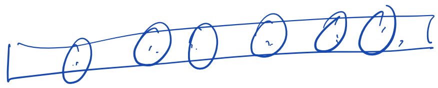
multinomial:

$$\binom{n}{n_1, n_2, \dots, n_r}$$

$$\boxed{n_1 + \dots + n_r = n}$$

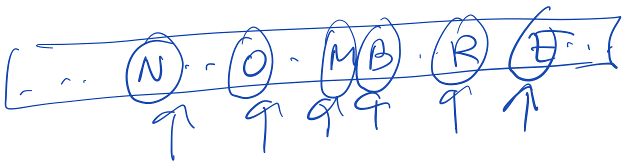
E3

12

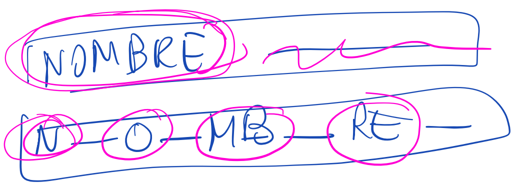


hi ha nombre

$\binom{12}{6}$



$$\underbrace{\binom{12}{6}}\cdot\underbrace{6!}$$



totes − (nombre ∪ fix)

totes − [nombre + fix − nombre fix]

$$\binom{12}{6}\cdot\binom{6}{3}\cdot 3!$$

$$(a_1 + \dots + a_r)^m =$$

$$= \underbrace{(a_1 + \dots + a_r)}\underbrace{(a_1 + \dots + a_r)} \dots \underbrace{(a_1 + \dots a_r)}$$

$$(a_1 + a_2)^n = \sum_{k=0}^{n} \binom{n}{k} a_1^{k} a_2^{n-k}$$

$$k \to a \qquad n-k \to b$$

$$= \sum_{a+b=n} \binom{n}{a,b} a_1^a a_2^b$$

$$\underbrace{\binom{n}{k}}_{\frac{n!}{k!\,(n-k)!}} = \binom{n}{k,\, n-k}$$

$x_1 + x_2 + \ldots + x_n = k$

$\boxed{\#\text{mult. } 1^{x_1}, 2^{x_2}, \ldots, n^{x_n}, \ldots, \quad x_1 + \ldots + x_n = k}$

8 mult. de $\{\underbrace{1, 2, 3}_{n}\}$

k

$111\ 22222 \qquad 1^3, 2^5$

$1\ 222\ 3333 \qquad 1^1, 2^3, 3^4$

---

4 mult. de $\{1, 2, 3, 4, 5, 6, 7, 8\}$

$1\ 2\ 3\ 4$

1^4

$1^2\ 3^2$

$x + y + z = 13$

$\underbrace{(x-3)}_{=} + \underbrace{(y-5)}_{=} + z = 13 - 3 - 5$

$$\boxed{\underbrace{x'}_{\geq 0} + \underbrace{y'}_{\geq 0} + z = 5}$$

$$A \Leftarrow B$$

$$\underbrace{A \Rightarrow B}_{\eta} \quad i \qquad B \Rightarrow A$$

$$\underbrace{\left([A \cap (B \cup C)] \cap \overline{C} \right) \setminus B}$$

$$= \left[\underbrace{(A \cap (B \cup C))} \underbrace{\cap \overline{C}} \right] \underbrace{\cap \overline{B}}$$

$$= [A \cap (B \cup C)] \cap \underbrace{(\overline{B} \cap \overline{C})}$$

$$= A \cap \underbrace{(B \cup C) \cap (\overline{B \cup C})}_{\emptyset}$$

$$= \emptyset$$