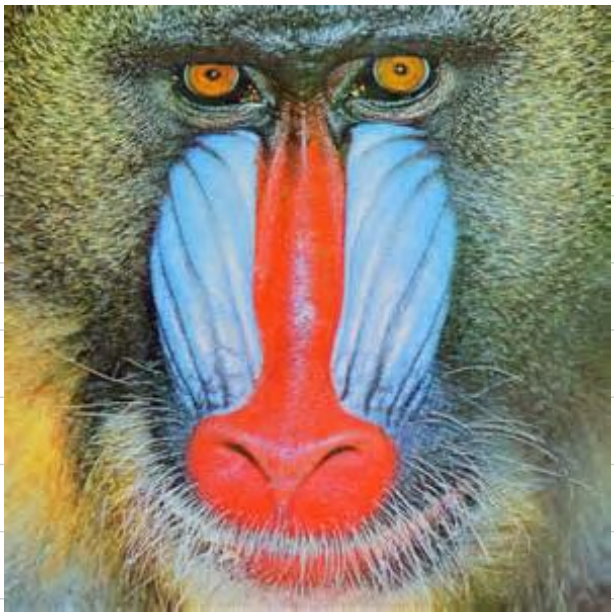
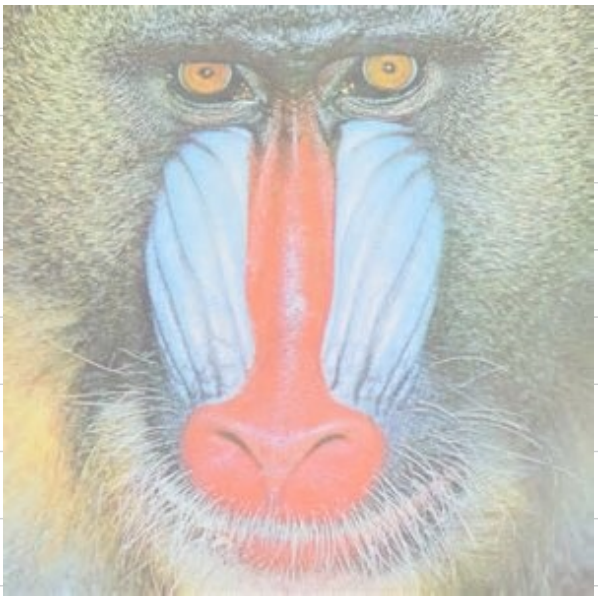
1.





SSIM = 0.748

2.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0.9511 & 0.5878 & 0 & -0.5878 & -0.9511 \\ 0.8090 & -0.309 & -1 & -0.309 & 0.8090 \\ 0.5878 & -0.9511 & 0 & 0.9511 & -0.5878 \\ 0.3090 & -0.8090 & 1 & -0.8090 & 0.3090 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ a & b & 0 & -b & -a \\ c & d & -1 & d & c \\ b & -a & 0 & a & -b \\ -d & -c & 1 & -c & -d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \underline{X}_1 \\ \underline{X}_2 \\ \underline{X}_3 \\ \underline{X}_4 \\ \underline{X}_5 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x_1 - x_5 \\ x_2 - x_4 \\ x_1 + x_5 \\ x_2 + x_4 \end{bmatrix}$$

$\underline{X}_1 = x_1 + x_2 + x_3 + x_4 + x_5 \rightarrow$ 0 MUL

$$\begin{bmatrix} \underline{X}_2 \\ \underline{X}_4 \end{bmatrix} = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$= \begin{bmatrix} b & b \\ b & b \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} a-b & 0 \\ 0 & -a-b \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

1 MUL　　2 MUL

→ 3 MUL

$$\begin{bmatrix} Z_3 \\ Z_5 \end{bmatrix} = \begin{bmatrix} c & d \\ -d & -c \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} -X_3 \\ X_3 \end{bmatrix}$$

$$\begin{bmatrix} c & -c \\ c & -c \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 & c+d \\ -d-c & 0 \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix}$$

$c(z_3 - z_4)$ → 1 MUL

$c+d = 0.5$

$-d-c = -0.5$

trivial multiplication → 0 MUL

→ 1 MUL

tatal 4 MUL

3.

$x = a + jb$, $e^{j\theta} = \cos\theta + j\sin\theta$

$x \cdot e^{j\theta} = (a\cos\theta - b\sin\theta) + j(b\cos\theta + a\sin\theta)$

$= z_1 + jz_2$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

if MUL = 2.

1. $\sin\theta = \cos\theta$, $\theta = \frac{\pi}{4} + \frac{\pi}{2}n$, $n \in N$

2. $\sin\theta = -\cos\theta$, $\theta = \frac{3}{4}\pi + \frac{\pi}{2}n$, $n \in N$

3. $\sin\theta_1 = \pm\frac{1}{2^k}$, $\theta_1 = \sin^{-1}\left(\pm\frac{1}{2^k}\right) + 2\pi n$, $n \in N$, $k \in Z$

$= \phi + 2\pi n$

$\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (由 $\sin^{-1}()$ 值域)

4. $\cos\theta_2 = \pm\frac{1}{2^k}$, $\theta_2 = \cos^{-1}\left(\pm\frac{1}{2^k}\right) + 2\pi n$, $n \in N$, $k \in Z$

$= \phi + 2\pi n$

$\phi \in [0, \pi]$ (由 $\cos^{-1}()$ 值域)

5. 由 case 3 得，因 sin 在一二象限值相同，
三四 ∴ 相同

$\theta = \pi - \theta_1$

6. 由 case 4 得，因 cos 在一四象限值相同
=三 ∴ 相同

$\theta = -\theta_2$

4.

(a) 220 points

$220 = 11 \times 20$

$\Rightarrow 20MUL_{11} + 11MUL_{20} = 20 \times 40 + 11 \times 40 = 1240$

(b) 231 points

$231 = 11 \times 21$

$\Rightarrow 21MUL_{11} + 11MUL_{21} = 21 \times 40 + 11 \times 62$

$= 1522$

(c) 245 points

$245 = 5 \times 49$

$\Rightarrow 49MUL_5 + 5MUL_{49}$

$= 49 \times 10 + 5(7MUL_7 + 7MUL_7 + 3 \times 6 \times 6)$

$= 490 + 5(7 \times 16 + 7 \times 16 + 108)$

$= 2150$

5.

1. 運算複雜度降為 O(N)

2. P固定，硬體架構固定

6.

$$y[n] = x[n] * h[n]$$

$$= 0.03x[n+3] + 0.06x[n+2] + 0.24x[n+1] + 0.34x[n]$$

$$+ 0.03x[n-3] + 0.06x[n-2] + 0.24x[n-1]$$

$$= 0.03\left(x[n+3] + x[n-3]\right) + 0.06\left(x[n+2] + x[n-2]\right)$$

$$+ 0.24\left(x[n+1] + x[n-1]\right) + 0.34x[n]$$

$$= 0.03\Big\{x[n+3] + x[n-3] - 2x[n] + 2\Big[x[n+2] + x[n-2] - 2x[n] +$$

$$4\left(x[n+1] + x[n-1] - 2x[n]\right)\Big]\Big\} + x[n]$$

每個 output 只需乘 0.03 的一個 MUL，其他皆為 trivial multiplication

7.

(a) $N=1100$, $M=200$

Direct: $3\times1100\times200=660000$

Section: $L_0=550$, $P_0=550+200-1=749$

① $P=504$, $MUL_p=2300$, $L=P-M+1=305$

$S=\lceil\frac{N}{L}\rceil=4$

$2S\times MUL_p+3SP=24448$

② $P=784$, $MUL_p=4412$, $L=P-M+1=585$

$S=\lceil\frac{N}{L}\rceil=2$

$2S\times MUL_p+3SP=22352$

③ $P=672$, $MUL_p=3496$, $L=P-M+1=473$

$S=\lceil\frac{N}{L}\rceil=3$

$2S\times MUL_p+3SP=27100$

$min=22352$

DFT: $P\geq N+M-1=1299$, $P=1344$

$2MUL_{1344}+3\times1344=20536$

DFT based, 20536 mul #

(b) $N=1100$, $M=20$

Direct : $3MN = 66000$

Sectioned : $L_0 = 105$, $P_0 = 105+20-1 = 124$

① $P = 120$, $MUL_P = 380$, $L = P-M+1 = 101$

$S = \lceil \frac{N}{L} \rceil = 11$, $2S \cdot MUL_P + 3SP = 12320$

② $P = 144$, $MUL_P = 436$, $L = P-M+1 = 125$

$S = \lceil \frac{N}{L} \rceil = 9$, $2SMUL_P + 3SP = 11736$

③ $P = 96$, $MUL_P = 280$, $L = P-M+1 = 77$

$S = \lceil \frac{N}{L} \rceil = 15$, $2SMUL_P + 3SP = 12720$

④ $P = 168$, $MUL_P = 580$, $L = P-M+1 = 149$

$S = \lceil \frac{N}{L} \rceil = 8$, $2SMUL_P + 3SP = 13312$

$min = 11736$

PFT : $P \geq N+M-1 = 1119$, $P = 1152$, $2MUL_{1152} + 3\times 1152$
$= 17632$

sectioned based, 11736 mul #

(c) $M=1100$, $M=7$

Direct : $3MN=23100$

Sectioned : $L_0=25$, $P_0=31$

① $P=32$, $MUL_P=72$, $L=P-M+1=26$, $S=43$

$25\times MUL_P+35P=10320$

② $P=24$, $MUL_P=28$, $L=P-M+1=18$, $S=62$

$25\times MUL_P+35P=7936$

③ $P=36$, $MUL_P=64$, $L=P-M+1=30$, $S=37$

$25\times MUL_P+35P=8732$

④ $P=48$, $MUL_P=92$, $L=P-M+1=42$, $S=27$

$25\times MUL_P+35P=8856$

$min=7936$

DFT : $P\geq N+M-1=1106$, $P=1152$

$2\,MUL_{1152}+3\times 1152=17632$

sectioned based, 7936 mul #

(d) $N=1100$, $M=2$

Direct: $3MN = 6600$

Sectioned: $L_0 = 2$, $P_0 = 2+2-1 = 3$

① $P=4$, $MUL_P = 0$, $L = P-M+1 = 3$

$S = \lceil \frac{N}{L} \rceil = 367$, $2S \times MUL_P + 3SP = 4404$

② $P=8$, $MUL_P = 4$, $L = P-M+1 = 7$, $S = 158$

$2S \times MUL_P + 3SP = 5048$

③ $P=12$, $MUL_P = 8$, $L = P-M+1 = 11$, $S = 101$

$2S \times MUL_P + 3SP = 5200$

$min = 4404$

DFT: $P \geq N+M-1 = 1101$, $P = 1152$,

$2MUL_{1152} + 3 \times 1152 = 17632$

Sectioned based, 4404 muls #

Extra (尾数7)

trivial multiplication 是指 x 若乘 2^k 倍或 x 乘 $j2^k$ 倍, $k \in \mathbb{N}$, 则 x 不需做乘法運算, 只需将 x 做 bit shift