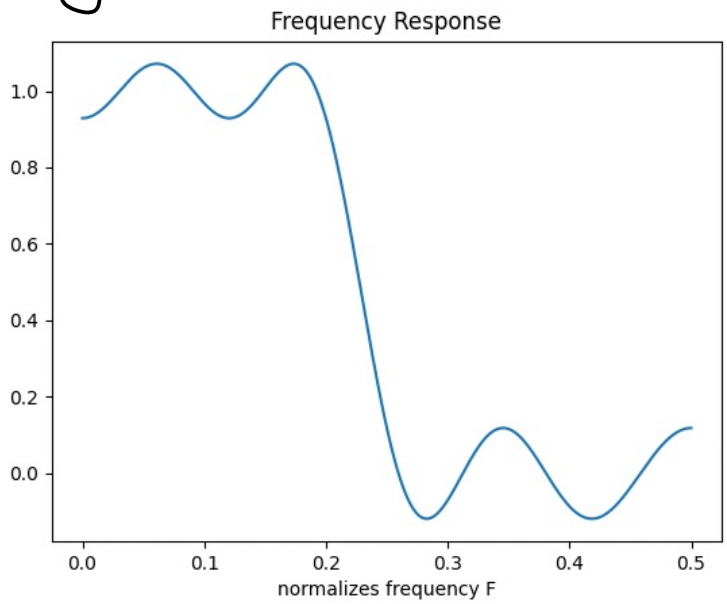
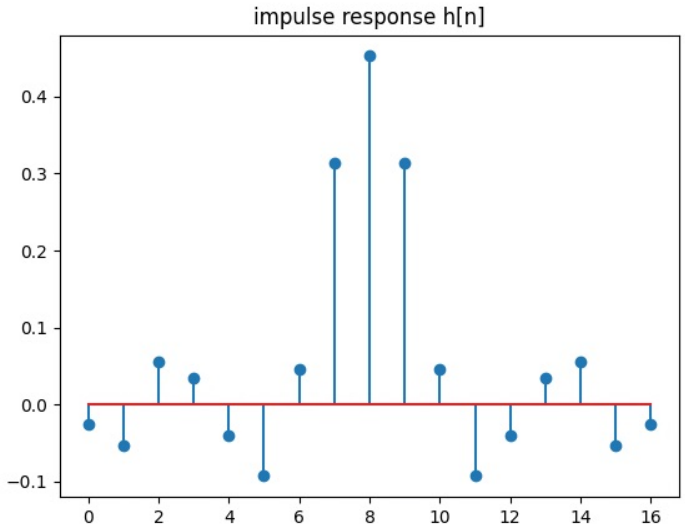
1. Frequency responce



impulse response h[n]



max error

iter	1	2	3	4	5	6
error	0.2466	0.21	0.1073	0.0731	0.0712	0.0712

2.

(a) 以 $y = x * h$ 為例，先做 FT 變成 $Y = XH$ 相乘形式，再兩邊取 log 變成 $\log Y = \log X + \log H$ 相加形式

(b)

1. complex multiplication : one complex MUL = for real MULs

$(a+jb)(c+jd) = ac - bd + j(ad+bc)$

2. irrational numbers

3.

$$y[n] = x[n] * (0.8^n u[n] - 0.6^n u[n])$$

做 Z transform

$$Y(z) = X(z)\left(\frac{1}{1-0.8z^{-1}} - \frac{1}{1-0.6z^{-1}}\right)$$

$$Y(z) = X(z)\frac{z^{-1}}{\frac{12}{5}z^{-2} - 7z^{-1} + 5}$$

$$\frac{12}{5}z^{-2}Y(z) - 7z^{-1}Y(z) + 5Y(z) = X(z)z^{-1}$$

$$\frac{12}{5}y[n-2] - 7y[n-1] + 5y[n] = x[n-1]$$

$$5y[n] = x[n-1] - \frac{12}{5}y[n-2] + 7y[n-1]$$

$$y[n] = \frac{1}{5}x[n-1] - \frac{12}{25}y[n-2] + \frac{7}{5}y[n-1]$$

4.

(a) 在對 step response 做積分時，會在頻域除 $j2\pi f$，增強低頻，降低高頻 response，若 Nyquist criterion 未滿足，aliasing effect 發生，交疊處之高頻已在做積分時降低，故減少了 aliasing effect

(b) 此法將 $f_{old} \in (-\infty, \infty)$ mapping 至 $f_{new} \in (-\frac{f_s}{2}, \frac{f_s}{2})$

故所有信号皆在 $(-\frac{f_s}{2}, \frac{f_s}{2})$，可確保 $B \leq \frac{f_s}{2}$，

$f_s \geq 2B$，滿足 Nyquist criterion，避免 aliasing effect

5.

$$f_s = \frac{1}{\Delta t} = \frac{1}{0.002} = 500$$

$$N = 2000$$

(a) $m = 300$, $m \leq \frac{N}{2}$

$$f = m\frac{f_s}{N} = 300 \times \frac{500}{2000} = 75 \quad (Hz)$$

(b) $m = 1800$, $m > \frac{N}{2}$

$$f = (m-N)\frac{f_s}{N} = (1800-2000)\times\frac{500}{2000} = -50 \ (Hz)$$

6.

定義 $R = \frac{W_{pass}}{W_{stop}}$，$e$ 為 pass band error

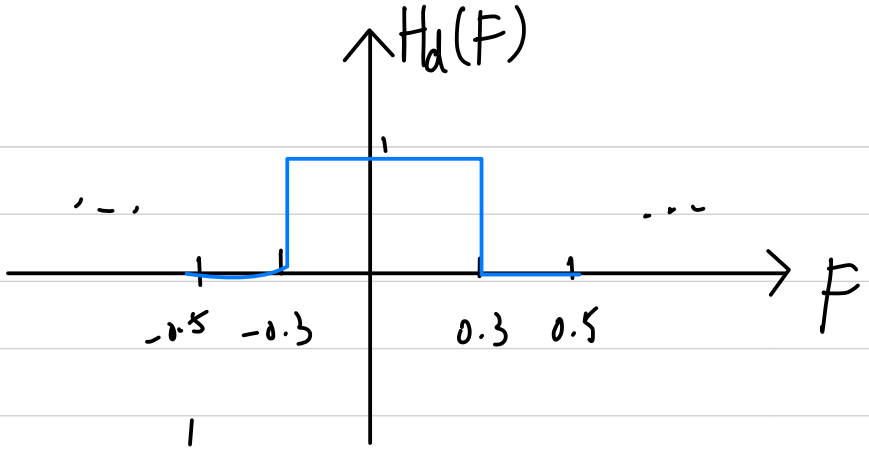
(a)(d) 同 OF，$R_d > R_a$，$e_d < e_a$

(b)(c) 同 OF，$R_c > R_b$，$e_c < e_b$

(c) 的 OF 較 (d) 大，且 $R_c > R_d$，相同情形下，OF 越大，error 越小、

故選 (C)

7.



$$S[0] = \int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)\,dF = 1 \times (0.3 - (-0.3)) = 0.6$$

$$s[n] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF) H_d(F)\,dF$$

$$= 2 \cdot \frac{1}{2\pi n} \cdot \sin(2\pi nF) \Big|_{-0.3}^{0.3}$$

$$= \frac{2}{\pi n} \sin(0.6\pi n)$$

$$N = 5, \quad k = \frac{5-1}{2} = 2$$

$$h[2] = s[0] = 0.6$$

$$h[1] = h[3] = \frac{S[1]}{2} = \frac{1}{\pi} \sin(0.6\pi)$$

$$h[0] = h[4] = \frac{S[2]}{2} = \frac{1}{2\pi} \sin(1.2\pi)$$

$$h[n] = \begin{cases} 0.6, & n = 2 \\ \frac{1}{\pi}\sin(0.6\pi), & n = 1, 3 \\ \frac{1}{2\pi}\sin(1.2\pi), & n = 0, 4 \\ 0, & \text{elsewhere} \end{cases}$$

Extra (尾款7)

在 mini-max 法，$W|R-H_d|=C$，若將 W 在 pass band 和 stop band 同時乘 10，只會將 C 放大 10 倍，不會減少 error，R 的值仍不改，需調 pass band 和 stop band W 的比例才可減少一邊 error