EM_HW3 Solutions

Problem 1

(a)

$g(x) = e^{-\pi x^2/2}(x^3 + x)$

1. $g_1(x) = e^{-\pi x^2} \iff G_1(f) = e^{-\pi f^2}$

2. $g_2(x) = e^{-\pi x^2/2} = g_1(x/\sqrt{2}) = \sqrt{2}G_1(\sqrt{2}f)$

   $\iff G_2(f) = \sqrt{2}e^{-2\pi f^2}$

3. $g_3(x) = x^3 e^{-\pi x^2/2} = x^3 g_2(x)$

   $\iff G_3(f) = (\frac{j}{2\pi})^3 G_2^{(3)}(f) = j\sqrt{2}e^{-2\pi f^2}(8f^3 - \frac{6}{\pi}f)$

4. $g_4(x) = xe^{-\pi x^2/2} = xg_2(x)$

   $\iff G_4(f) = (\frac{j}{2\pi})G_2'(f) = j\sqrt{2}e^{-2\pi f^2}(-2f)$

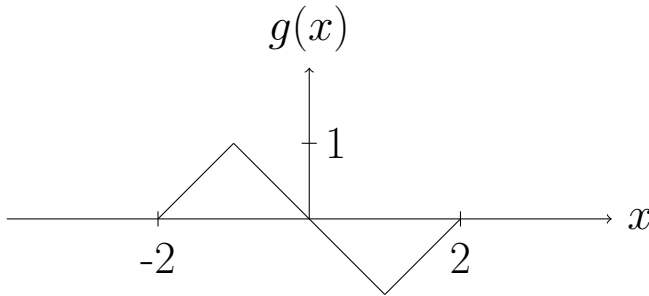5. $g(x) = g_3(x) + g_4(x) \iff G(f) = j\sqrt{2}e^{-2\pi f^2}(8f^3 - \frac{6}{\pi}f - 2f)$

(b)

$g(x) = sin(\pi x/6)\prod(\frac{x-3}{6}) = \frac{1}{2j}e^{j\pi x/6}\prod(\frac{x-3}{6}) - \frac{1}{2j}e^{-j\pi x/6}\prod(\frac{x-3}{6})$

1. $\prod(\frac{x-3}{6}) \iff 6e^{-j6\pi f}sinc(6f)$

2. $e^{j\pi x/6} \prod(\frac{x-3}{6}) = e^{j2\pi(\frac{1}{12})x} \prod(\frac{x-3}{6}) \Longleftarrow 6e^{-j6\pi(f-\frac{1}{12})} sinc(6(f - \frac{1}{12}))$

3. $e^{-j\pi x/6} \prod(\frac{x-3}{6}) = e^{j2\pi(\frac{-1}{12})x} \prod(\frac{x-3}{6}) \Longleftarrow 6e^{-j6\pi(f+\frac{1}{12})} sinc(6(f+\frac{1}{12}))$

4. $G(f) = 3je^{-j6\pi(f+\frac{1}{12})} sinc(6(f+\frac{1}{12})) - 3je^{-j6\pi(f-\frac{1}{12})} sinc(6(f-\frac{1}{12}))$

(c)

$g(x) = \Lambda(x + 1) - \Lambda(x - 1)$
$\Longleftarrow G(f) = e^{j2\pi f} sinc^2(f) - e^{-j2\pi f} sinc^2(f) = 2j sin(2\pi f) sinc^2(f)$

(d)

$g(x) = \delta(sin(x)) = \sum_n \delta(x - n\pi) \Longleftarrow G(f) = \frac{1}{\pi} \sum_n \delta(f - n/\pi)$

Problem 2

$g_1(x, y) = 1$ for $x^2 + y^2 < 1$, $g_1(x, y) = 0$ otherwise
$\Longleftarrow G_1(f, h) = \frac{J_1(2\pi\sqrt{f^2+h^2})}{\sqrt{f^2+h^2}}$

$g(x, y) = g_1(x-1, \frac{y}{2}) \Longleftarrow G(f, h) = 2G_1(f, 2h)e^{-j2\pi f} = \frac{2J_1(2\pi\sqrt{f^2+4h^2})}{\sqrt{f^2+4h^2}}e^{-j2\pi f}$

Problem 3

$g[n] = p_3[n] + p_5[n] - p_{15}[n]$

$$\Longleftarrow G[m] = \frac{30}{3}p_{30/3}[m] + \frac{30}{5}p_{30/5}[m] - \frac{30}{15}p_{30/15}[m]$$
$$= 10p_{10}[m] + 6p_6[m] - 2p_2[m]$$

Problem 4

(a)

$$g(x) = \sin(5\pi x)\cos(3\pi x) * sinc(5x) * sinc(10x)$$

1. $g_1(x) = \sin(5\pi x)\cos(3\pi x) = \frac{1}{2}(\sin(8\pi x) + \sin(2\pi x))$

2. $g_2(x) = sinc(5x) \Longleftarrow G_2(f) = \frac{1}{5}\prod(\frac{f}{5})$

3. $g_3(x) = sinc(10x) \Longleftarrow G_3(f) = \frac{1}{10}\prod(\frac{f}{10})$

Thus $g(x)$ is the result of passing $g_1(x)$ through two lowpass filters $G_2(f)$ and $G_3(f)$. As a result, $g(x) = \frac{1}{100}\sin(2\pi x)$

(b)

$$g(x) = \delta'(x) * \delta(2x) * \delta(x - 3) * \exp(-x^2)$$

1. $\exp(-x^2) * \delta'(x) = -2x\exp(-x^2)$ (differentiate $\exp(-x^2)$ with

   respect to x)

2. $(-2x\exp(-x^2)) * \delta(2x) = -x\exp(-x^2)$ (divided by 2)

3. $(-x\exp(-x^2)) * \delta(x - 3) = (3 - x)\exp(-(3 - x)^2)$ (shift by 3)