(1) $x[n] = [1, 0, 2, 3, -1, 2, 2, 1, 0]$

$h[n] = [2, 2, 1, 1, 0, 0, 0, 0, 0]$

$y[0] = 2 + 1 + 2 = 5$, $y[1] = 0 + 2 + 0 + 1 = 3$

$y[2] = 4 + 0 + 1 + 0 = 5$, $y[3] = 6 + 4 + 0 + 1 = 11$

$y[4] = -2 + 6 + 2 + 0 = 6$, $y[5] = 4 - 2 + 3 + 2 = 7$

$y[6] = 4 + 4 - 1 + 3 = 10$, $y[7] = 2 + 4 + 2 - 1 = 7$

$y[8] = 0 + 2 + 2 + 2 = 6$

$y[n] = [5, 3, 5, 11, 6, 7, 10, 7, 6]$

(2)

(a) $CDF = \int_{-\infty}^{x} f_x(x)\,dx$

$x \leq 0$, $CDF = 0$

$0 < x \leq 2$, $CDF = \int_0^x \frac{x}{2}\,dx = \frac{1}{4}x^2$

$x > 2$, $CDF = \int_0^2 \frac{x}{2}\,dx = 1$

mean: $\int_0^2 x \cdot \frac{x}{2}\,dx = \frac{4}{3}$

$$CDF = \begin{cases} 0, & x \leq 0 \\ \frac{1}{4}x^2, & 0 < x \leq 2 \\ 1, & x > 2 \end{cases}, \quad mean = \frac{4}{3}$$

(b)

var : $\int_0^2 \left(x - \frac{4}{3}\right)^2 \frac{x}{2}\,dx = \underline{\frac{2}{9}}$ #

std : $\sqrt{var} = \underline{\frac{\sqrt{2}}{3}}$ #

skewness : $\dfrac{\int_0^2 \left(x-\frac{4}{3}\right)^3 \frac{x}{2}\,dx}{var^{3/2}} = \dfrac{\frac{-8}{135}}{\frac{2\sqrt{2}}{27}} = \underline{-\frac{2}{5\sqrt{2}}}$ #

3.

(a) $Y = \frac{X}{5} + 5$, $\frac{1}{5}$ is positive , 5 is constant

$corr_{X,Y} = 1$

(b) $Y = \frac{X}{5} + 5$

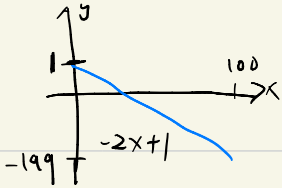
$\mu_Y = E[Y] = E\left[\frac{X}{5} + 5\right] = \frac{1}{5}E[X] + 5 = \underline{\frac{\mu}{5} + 5}$ #

$Var(Y) = E[(Y-\mu_Y)^2] = E\left[\frac{1}{25}(X-\mu)^2\right] = \underline{\frac{1}{25}V}$ #

$S(Y) = \dfrac{E[(Y-\mu_Y)^3]}{Var(Y)^{3/2}} = \dfrac{E\left[\frac{1}{125}(X-\mu)^3\right]}{\left(\frac{1}{25}V\right)^{3/2}} = \underline{S(X) = S}$ #

$K(Y) = \dfrac{E[(Y-\mu_Y)^4]}{Var(Y)^2} = \dfrac{\left(\frac{1}{5}\right)^4 E[(X-\mu)^4]}{\left(\frac{1}{5}\right)^4 V^2} = \underline{k}$ #

4. $f_{X,Y} = \begin{cases} \frac{1}{100}\delta(y+2x-1), & 0 < x < 100 \\ 0, & \text{else} \end{cases}$



$$f_X = \frac{1}{100}\int_{-199}^{1} \delta(y+2x-1)\,dy = \frac{1}{100}$$

$$f_Y = \frac{1}{100}\int_{0}^{100} \delta(y+2x-1)\,dx = \frac{1}{200}$$

$$\mu_X = \frac{1}{100}\int_{0}^{100} x\,dx = 50, \quad \mu_Y = \frac{1}{200}\int_{-199}^{1} y\,dy = -99$$

$$\mathrm{COV}_{X,Y} = \int_{0}^{100}\int_{-199}^{1} (x-50)(y+99)\frac{1}{100}\delta(y+2x-1)\,dy\,dx$$

$$= \frac{1}{100}\int_{0}^{100} (x-50)(-2x+1+99)\,dx = -\frac{5000}{3}$$

$$\sigma_X = \sqrt{\int_{0}^{100} \frac{1}{100}(x-50)^2\,dx} = \frac{50}{\sqrt{3}}, \quad \sigma_Y = \sqrt{\frac{1}{200}\int_{-199}^{1} (y+99)^2\,dy} = \frac{100}{\sqrt{3}}$$

$$\mathrm{Corr}_{X,Y} = \frac{\mathrm{COV}_{X,Y}}{\sigma_X \sigma_Y} = -1$$

5. $$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A \otimes B = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} & a_{13}b_{11} & a_{13}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} & a_{13}b_{21} & a_{13}b_{22} \end{bmatrix}$$

$$B \otimes A = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{11} & a_{13}b_{11} & a_{11}b_{12} & a_{12}b_{12} & a_{13}b_{12} \\ a_{11}b_{21} & a_{12}b_{21} & a_{13}b_{21} & a_{11}b_{22} & a_{12}b_{22} & a_{13}b_{22} \end{bmatrix}$$

$$J_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

6. $\left(\sum |x_i|^p\right)^{1/p}$

$$\|V(\alpha)\|_0 = \begin{cases} 1, & \alpha = 0 \\ N, & \text{otherwise} \end{cases}$$

$$\|V(\alpha)\|_1 = \begin{cases} N, & \alpha = \pm 1 \\ \dfrac{1-|\alpha|^N}{1-|\alpha|}, & \text{otherwise} \end{cases}$$

$$\|V(\alpha)\|_2 = \begin{cases} \sqrt{N}, & \alpha = \pm 1 \\ \sqrt{\dfrac{1-\alpha^{2N}}{1-\alpha^2}}, & \text{otherwise} \end{cases}$$

$$\|V(\alpha)\|_\infty = \begin{cases} 1, & -1 \le \alpha \le 1 \\ |\alpha|^{N-1}, & \text{otherwise} \end{cases}$$

$$\|V \cdot V^T\|_F = \sqrt{tr(VV^T(VV^T)^T)}$$

$$VV^T(VV^T)^T = VV^TVV^T = V(V^TV)V^T$$

$$V^TV = 1 + \alpha^2 + \dots + (\alpha^{N-1})^2 = c$$

$$V(V^TV)V^T = cVV^T$$

$$\sqrt{tr(cVV^T)} = \sqrt{(1+\alpha^2 + \dots + (\alpha^{N-1})^2)^2}$$

if $\alpha = \pm 1$, $\sqrt{tr(cVV^T)} = N$

if $\alpha \neq \pm 1$, $\sqrt{tr(cVV^T)} = \dfrac{1 - \alpha^{2N}}{1 - \alpha^2}$

$$\| V(\alpha) V^T(\alpha) \|_F = \begin{cases} N, & \alpha = \pm 1 \\ \dfrac{1-\alpha^{2N}}{1-\alpha^2}, & \text{elsewhere} \end{cases}$$