1. (a) $g(x) = e^{-\pi\frac{x^2}{2}}(x^3+x)$

令 $h(x) = e^{-\pi/2\, x^2}$, $FT\{h(x)\} = \sqrt{2}\, e^{-2\pi f^2} = H(f)$

$FT\{x h(x)\} = \frac{j}{2\pi} H'(f) = -j2\sqrt{2} f e^{-2\pi f^2}$

$FT\{x^3 h(x)\} = \left(\frac{j}{2\pi}\right)^3 H'''(f)$

$FT\{g(x)\} = j\frac{2\sqrt{2}f}{\pi}\left(-\pi-3+4\pi f^2\right)e^{-2\pi f^2}$

(b) $g(x) = \sin\left(\frac{\pi}{6}x\right)\Pi\left(\frac{x-3}{6}\right) = \frac{1}{2j}\left[e^{j\frac{\pi}{6}x} - e^{-j\frac{\pi}{6}x}\right]\Pi\left(\frac{x-3}{6}\right)$

$FT\left\{\Pi\left(\frac{x-3}{6}\right)\right\} = 6e^{-j6\pi f}\operatorname{sinc}(6f)$

$FT\left\{e^{j\frac{\pi}{6}x}\Pi\left(\frac{x-3}{6}\right)\right\} = 6e^{-j6\pi(f-\frac{1}{12})}\operatorname{sinc}\left(6\left(f-\frac{1}{12}\right)\right)$

$FT\left\{e^{-j\frac{\pi}{6}x}\Pi\left(\frac{x-3}{6}\right)\right\} = 6e^{-j6\pi(f+\frac{1}{12})}\operatorname{sinc}\left(6\left(f+\frac{1}{12}\right)\right)$

$$FT\{g(x)\} = \frac{3}{j}\left[e^{-j6\pi(f-\frac{1}{12})}\operatorname{sinc}\left(6\left(f-\frac{1}{12}\right)\right) - e^{-j6\pi(f+\frac{1}{12})}\operatorname{sinc}\left(6\left(f+\frac{1}{12}\right)\right)\right]$$

(c) $g(x) = \Lambda(x+1) - \Lambda(x-1)$

$FT\{\Lambda(x)\} = \mathrm{sinc}^2 x$

$FT\{g(x)\} = e^{j2\pi f}\,\mathrm{sinc}^2 x - e^{-j2\pi f}\,\mathrm{sinc}^2 x$

(d) $g(x) = \delta(\sin x)$

$$= \sum_{n=-\infty}^{\infty} \delta(x - n\pi)$$

$g(x)$ is impulse train, interval is π

if $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$, $FT\{s(t)\} = \frac{1}{T}\sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T})$

$$FT\{g(x)\} = \frac{1}{\pi}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{\pi}\right)$$

2. If $g(r) = \mathrm{circ}(r)$

$$G(s) = \frac{J_1(2\pi s)}{s} = \frac{J_1\left(2\pi\sqrt{f^2+h^2}\right)}{\sqrt{f^2+h^2}}$$

延 x軸右 shift 1, y軸 scale 2

$$2e^{-j2\pi f}\,\frac{J_1\left(2\pi\sqrt{f^2+4h^2}\right)}{\sqrt{f^2+4h^2}} \quad \#$$

3.

$$X[m] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{mn}{N}}, \quad N = 30$$

$$= \sum_{k=0}^{9} e^{-j2\pi \frac{m(3k)}{30}} + \sum_{k=0}^{5} e^{-j2\pi \frac{m(5k)}{30}} - \sum_{k=0}^{1} e^{-j2\pi \frac{m(15k)}{30}}$$

$$= 10 p_{10}[m] + 6 p_6[m] - 2 p_2[m], \quad 0 \le m < 30$$

4.

(a) $\sin(5\pi x)\cos(3\pi x) * \text{sinc}(5x) * \text{sinc}(10x)$

$$g_1(x) = \sin(5\pi x)\cos(3\pi x) = \frac{1}{2}[\sin(8\pi x) + \sin(2\pi x)]$$

$$g_2(x) = \text{sinc}(5x), \quad g_3(x) = \text{sinc}(10x)$$

$$G_1(f) = FT\{g_1(x)\} = \frac{1}{4j}[\delta(f-4) - \delta(f+4) + \delta(f-1) - \delta(f+1)]$$

$$G_2(f) = FT\{g_2(x)\} = \begin{cases} \frac{1}{5}, & -\frac{5}{2} \le f \le \frac{5}{2} \\ 0, & \text{elsewhere} \end{cases}$$

$$G_3(f) = FT\{g_3(x)\} = \begin{cases} \frac{1}{10}, & -5 \le f \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$G(f) = G_1(f)\,G_2(f)\,G_3(f) = \frac{1}{100} \cdot \frac{1}{2j}[\delta(f-1) - \delta(f+1)]$$

$$g(x) = FT^{-1}\{G(f)\} = \frac{1}{100}\sin(2\pi x)$$

(b) $\delta'(x) * \delta(2x) * \delta(x-3) * e^{-x^2}$

$= -2xe^{-x^2} * \frac{1}{2}\delta(x) * \delta(x-3)$

$= -xe^{-x^2} * \delta(x-3)$

$= -(x-3)e^{-(x-3)^2}$