

1. 應選 large σ , 因 large σ , 在 time domain 解析度高, frequency domain 解析度低, 而音樂信號不同時間音即不同, 欲時域解析度高, 而頻率差一點, 音聽起來差不多, 選頻域解析低

2. $x(t) = e^{-\pi t^2}$, $x(t + \frac{\tau}{2}) = e^{-\pi(t + \frac{\tau}{2})^2}$, $x^*(t - \frac{\tau}{2}) = e^{-\pi(t - \frac{\tau}{2})^2}$
 (a) $x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) = e^{-\pi(2t^2 + \frac{\tau^2}{2})}$

$$e^{-\pi \tau^2} \xrightarrow{FT} e^{-\pi f^2}$$

$$e^{-\pi \frac{\tau^2}{2}} \xrightarrow{FT} \sqrt{2} e^{-\pi \cdot 2f^2}$$

$$e^{-\pi(2t^2)} \cdot e^{-\pi \frac{\tau^2}{2}} \xrightarrow{FT} \sqrt{2} e^{-\pi(2t^2 + 2f^2)} \quad \#$$

$$(b) \quad \delta(2t-1) = \frac{1}{2} \delta\left(t - \frac{1}{2}\right)$$

$$\delta(t) \xrightarrow{\text{WDF}} \delta(t)$$

shift property

$$\delta\left(t - \frac{1}{2}\right) \xrightarrow{\text{WDF}} \delta\left(t - \frac{1}{2}\right)$$

$$\frac{1}{2} \delta\left(t - \frac{1}{2}\right) \xrightarrow{\text{WDF}} \frac{1}{4} \delta\left(t - \frac{1}{2}\right) \#$$

3. (a)(b)

① direct implement: $O(TFQ)$

$$\Delta t < \frac{1}{2\Omega}, \quad \text{bandwidth of } x: \Omega_x$$

$$W = \Omega_w$$

$$\Omega = \Omega_x + \Omega_w$$

② FFT-based: $O(TN \log N)$

$$\frac{1}{\Delta t \Delta f} = N, \quad N \text{ must integer}$$

$$N \geq 2Q + 1 = 2 \frac{B}{\Delta t} + 1, \quad \Delta t < \frac{1}{2(\Omega_x + \Omega_w)}$$

③ Recursive method: $O(TF)$ N must be integer

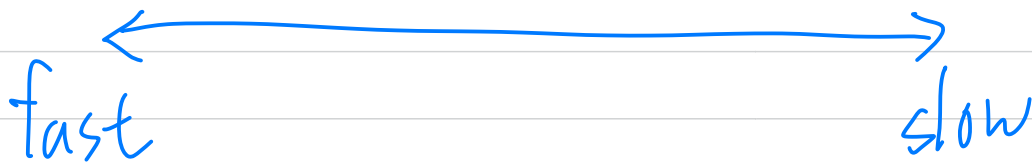
只適用 constant window, $\Delta t < \frac{1}{2(\Omega_x + \Omega_w)}$, $\Delta f = \frac{1}{N}$
 $N \geq 2Q + 1$

④ Chirp Z transform: $O(TN \log N)$

$$\Delta t < \frac{1}{2(\Omega_x + \Omega_w)}$$

Total complexity compare:

recursive \succ fft-based \succ chirp-z \succ direct


fast slow

(c) direct implement, fft-based, chirp-z 可

recursive 大 window 非 constant 無法使用

4.

(a) 若 signal 有兩 component, 利用 $w(z)=0$ for $|z| > B$ 此 mask, 適當的 B , 可使 mask 保留 auto term, 濾掉外圍 cross term, 但此 input 做運算, 需滿足, auto term 在 mask 內, cross term 在外

(b)

auto term 在時頻分佈上中心點處, 當 $(t_d, f_d), (-t_d, -f_d)$ 離原點較遠時, 可用 mask 過濾 cross term

(c) Gabor 無 cross term, 在 WDF 有 cross term 處, Gabor 為零, 故兩式相乘後, 原先 WDF cross term 處, 也為零, 由此消除 cross term

5.

$$C_x(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\tau, \eta) \Phi(\tau, \eta) e^{j2\pi(\eta t - \tau f)} d\eta d\tau$$

$$\text{where } A(\tau, \eta) = \int_{-\infty}^{\infty} x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) e^{-j2\pi t \eta} dt$$

$$C_x^*(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A^*(\tau, \eta) \Phi^*(\tau, \eta) e^{-j2\pi(\eta t - \tau f)} d\eta d\tau$$

$$\tau' = -\tau, \eta' = -\eta$$

$$A^*(\tau', \eta') = \int_{-\infty}^{\infty} x(t + \frac{\tau'}{2}) x^*(t - \frac{\tau'}{2}) e^{-j2\pi t \eta'} dt$$

$$C_x^*(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A^*(\tau', \eta') \Phi^*(\tau', \eta') e^{j2\pi(\eta' t - \tau' f)} d\eta' d\tau'$$

if C_x is real, $C_x = C_x^*$

$$\text{可得 } \Phi(\tau, \eta) = \Phi^*(\tau', \eta') = \Phi^*(-\tau, -\eta)$$

extra: 尾數

AF cross term 很靠近原點

能用 mask 清掉嗎?

若 cross term 和 auto term 重疊, 則

無法用 mask 清除