1. 應選 large σ，因 large σ，在 time domain 解析度高，frequency domain 解析度低，而音樂信號不同時間音即不同，欲時域解析度高，而頻率差一點，音聽起來差不多，選頻域解析低

2. $x(t) = e^{-\pi t^2}$，$x(t+\frac{\tau}{2}) = e^{-\pi(t+\frac{\tau}{2})^2}$，$x^*(t-\frac{\tau}{2}) = e^{-\pi(t-\frac{\tau}{2})^2}$

(a)

$$x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2}) = e^{-\pi(2t^2+\frac{\tau^2}{2})}$$

$$e^{-\pi\tau^2} \xrightarrow{FT} e^{-\pi f^2}$$

$$e^{-\pi\frac{\tau^2}{2}} \xrightarrow{FT} \sqrt{2}\, e^{-\pi\cdot 2f^2}$$

$$e^{-\pi(2t^2)} \cdot e^{-\pi\frac{\tau^2}{2}} \xrightarrow{FT} \underline{\sqrt{2}\, e^{-\pi(2t^2+2f^2)}}\ \#$$

(b) $\delta(2t-1) = \frac{1}{2}\delta(t-\frac{1}{2})$

$$\delta(t) \xrightarrow{WDF} \delta(t)$$

shift property $\delta(t-\frac{1}{2}) \xrightarrow{WDF} \delta(t-\frac{1}{2})$

$$\frac{1}{2}\delta(t-\frac{1}{2}) \xrightarrow{WDF} \underline{\frac{1}{4}\delta(t-\frac{1}{2})}$$ #

3. (a)(b)

① direct implement : $O(TFQ)$

$\Delta t < \frac{1}{2\Omega}$, bandwidth of x : Ω_x

W : Ω_w

$\Omega = \Omega_x + \Omega_w$

② FFT - based : $O(TN\log N)$

$\frac{1}{\Delta t \Delta f} = N$, N must integer

$N \geq 2Q+1 = 2\frac{B}{\Delta t}+1$, $\Delta t < \frac{1}{2(\Omega_x+\Omega_w)}$

③ Recursive method: $O(TF)$ N must integer

只適用 constant window, $\Delta t < \frac{1}{2(\Omega_x+\Omega_w)}$, $\Delta t \Delta f = \frac{1}{N}$

$N \geq 2Q+1$

④ Chirp Z transform: $O(TN\log N)$

$$\Delta t < \frac{1}{2(\Omega_x+\Omega_w)}$$

Total complexity compare:

recursive > fft-based > chirp-Z > direct

fast ⟷ slow

(c) direct implement、fft-based、chirp-Z 可

recursive 因 window 非 constant 無法使用

4.

(a) 若 signal 有兩 component，利用 $w(\tau)=0$ for $|\tau|>B$ 此 mask，適當的 B，可使 mask 保留 auto term，濾掉外圍 cross term，但此 input 做運算，需滿足，auto term 在 mask 內，cross term 在外

(b)

auto term 在時頻分佈上中心點處，當 (t_d, f_d)，$(-t_d, -f_d)$ 離原點較遠時，可用 mask 過濾 cross term

(c) Gabor 無 cross term，在 WDF 有 cross term 處，Gabor 為零，故兩式相乘後，原先 WDF cross term 處，也為零，由此消除 cross term

5.

$$C_x(t,f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(\tau,\eta)\,\Phi(\tau,\eta)\,e^{j2\pi(\eta t-\tau f)}\,d\eta\, d\tau$$

where $A(\tau,\eta)=\int_{-\infty}^{\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-j2\pi t\eta}\,dt$

$$C_x^*(t,f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A^*(\tau,\eta)\,\Phi^*(\tau,\eta)\,e^{-j2\pi(\eta t-\tau f)}\,d\eta\, d\tau$$

$\tau'=-\tau,\ \eta'=-\eta$

$$A^*(\tau',\eta')=\int_{-\infty}^{\infty} x(t+\frac{\tau'}{2})x^*(t-\frac{\tau'}{2})e^{-j2\pi t\eta'}\,dt$$

$$C_x^*(t,f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A^*(\tau',\eta')\,\Phi^*(\tau',\eta')\,e^{j2\pi(\eta' t-\tau' f)}\,d\eta' d\tau'$$

if C_x is real, $C_x=C_x^*$

可得 $\Phi(\tau,\eta)=\Phi^*(\tau',\eta')=\Phi^*(-\tau,-\eta)$

extra：尾數7

AF cross term 很靠近原點

能用 mask 清掉嗎？

若 cross term 和 auto term 重疊，則

無法用 mask 清除