Calculating emission line fluxes

For a Stromgren Sphere:

$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu}\, \mathrm{d}\nu = Q = \int_0^{r_1} 4\pi\, n_e\, n_p\, \alpha_B\, r^2\, \mathrm{d}r$$

The luminosity in a recombination line is an integral over the ionized volume of the nebula. Consider $H\beta$:

$$\begin{aligned} L_{H\beta} = \int_0^{r_1} 4\pi\, j_{H\beta}\, \mathrm{d}V &= \int_0^{r_1} n_e\, n_p\, h\nu_{H\beta}\, \alpha_{H\beta}^{eff}\, \mathrm{d}V \\ &= \int_0^{r_1} 4\pi\, r^2\, n_e\, n_p\, h\nu_{H\beta}\, \alpha_{H\beta}^{eff}\, \mathrm{d}r \end{aligned}$$

Solve $L_{H\beta}$ for $n_e n_p$ and plug into Q.

$$\begin{aligned} Q &= n_e n_p \int_0^{r_1} \alpha_B\, 4\pi r^2\, \mathrm{d}r \\ &= \frac{L_{H\beta} \int_0^{r_1} \alpha_B\, 4\pi r^2\, \mathrm{d}r}{\int_0^{r_1} h\nu_{H\beta}\, \alpha_{H\beta}^{eff}\, 4\pi r^2\, \mathrm{d}r} \end{aligned}$$

The energy and recombination coefficients are independent of radius. And both volumn integrals are over the same volume (ionized volume of nebula). The integrals therefore drop out.

$$Q = \frac{L_{H\beta}\, \alpha_B}{h\nu_{H\beta}\, \alpha_{H\beta}^{eff}}$$

For $n_e = 10^4$ cm^{-3} and $T = 10000$K, $\alpha_{H\beta}^{eff} = 3.04 \times 10^{-14}$ (Osterbrock), and $\alpha_B = 2.59 \times 10^{-13}$ (Hummer & Storey 1987). $\lambda_{H\beta} = 4861$Å, so $h\nu_{H\beta} = 4.1 \times 10^{-12}$ ergs.

$$\begin{aligned} L_{H\beta} &= Q\, h\nu_{H\beta} \frac{\alpha_{H\beta}^{eff}}{\alpha_B} \\ &= 4.8 \times 10^{-13} Q \text{ ergs/s} \end{aligned}$$