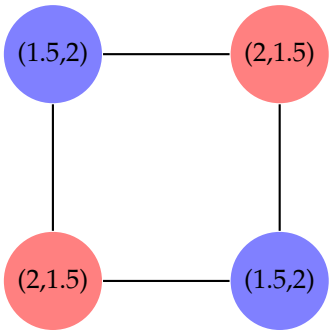
Churn

John Li, Albert Ge, Timothy Chou, Kevin Chang
Rankmaniac Report

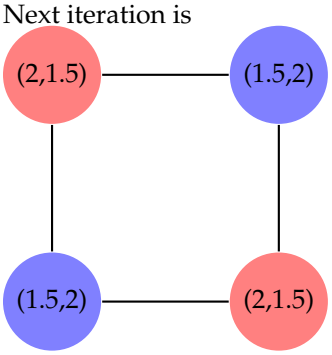
1 Overview

During this Pandemaniac project, our group "churn" implemented a number of algorithms to choose seed nodes in a network to optimize the spread of a simulated epidemic across the graph. The code was written in a Python base.

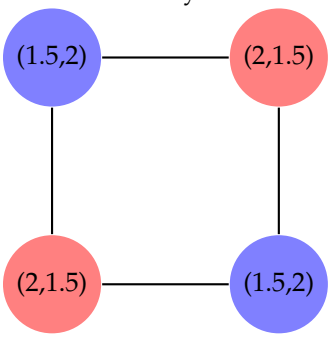
Counterexample



Next iteration is



Which we see cycles back to



Meaning equilibrium will never be reached.

2 Initial Framework

We began this project by first understanding and processing the data from the format of a JSON file. We wrote some initial files to load the data from a JSON file, and use some naive algorithm to pick seed nodes (described below). To present it in a file suitable for submission, a file `submit.py` was created, which took in as its parameter the graph input file, and output results in a file `submit.txt` for upload.

Early Bird Points

To obtain early bird points, we first wrote a short file `deg_centrality.py` which ranks the top X nodes by their degree value. Here X refers to the number of seeds to be picked, as denoted by the name of each graph JSON file.

Beating TA_fewer and TA_degree

The next algorithm we tried was tailored specifically to beat TA_degree written in `deg_centrality2.py`. Given that TA_degree follows the exact same strategy, then all of the initial seed pickings would be cancelled by TA_degree. To combat this, we would pick the top X-1 seeds by degree, and the last seed to just be a neighbor of the highest-degree node. The intuition behind this is that, as the top X-1 nodes would be cancelled out, TA_degree would be left with the Xth-highest degree node, and we would have a neighbor to the highest degree node. Thus, after a single step in a round, we would (ideally) obtain control of the highest degree node.
This algorithm ended up beating both TA_fewer and TA_degree, securing the remaining required points for the programming side of the project.

3 Development and Improvements

Top-K Mix

This algorithm involved finding the degree centrality. closeness centrality, and betweeness centrality, then selecting the top A, B, C respectively from each.

Linear Combinations

The idea was that nodes with high measures of centrality, specifically degree centrality, closeness centrality, and betweenness centrality, would be the best nodes to choose in a 2-player game. This model ignores remote clusters because we assume that since there are only two players and we have so few seed nodes, we need to contest the central area. What is unclear is which measure of centrality is most important, so we assumed the best model would be a linear combination of these three centrality measures. $Score(x) = \alpha * Betweenness - Centrality(x)/0.07 + \beta * Closeness - Centrality(x)/0.54 + \gamma * Degree - Centrality(x)/0.28$
We found maximum Betweenness Centrality, Closeness Centrality, and Degree Centrality of the versus-TA graphs to be approximately 0.07, 0.54, 0.28 respectively. Thus we hard coded these to normalize the contributions due to the three centrality measures.

PageRank

Looking at the graphs, we realized that if a node is near central nodes, it also gains some importance. We therefore implemented a 1-step PageRank-like algorithm that simply increases the score of each node

by the score of each neighbor divided by degree of that neighbor. However, this did not seem to improve performance, so we did not use this in our actual runs.

Monte Carlo

Monte Carlo simulations were done on the various linear combinations and top-K mixes. A number of samples of different linear combinations incrementing α, β, and γ in .05 steps forcing $\alpha+\beta+\gamma = 1$ and every possible selection of A, B, C for a top-K mix was used to represent different strategies we were considering generating different seeds on 16 graphs. In addition, we also hand selected seeds in an attempt to immitate TA_more. We then pitted all the strategies against each other, with one side getting the normal amount of seeds and the player 2 getting 20% more seeds. Doing so involved playing approximately 2 million games, so the simulation was run (`runSim.py`) across multiple computers before the outputs were agreggated using `combineResults.py`. These results were then used for our final attempt against TA_more. The final result was a linear combination with weights $[.70, .25, .5]$ which had won at least 5 out of 16 graphs against every player 2 (which had 20% more seed nodes).

Bridges

In the structure of the graph, we noticed that access to strongly connected components is very important if we want to spread our influence throughout the graph. However, the only way to move from one strongly connected component to the other is through bridge edges. Since an edge only attaches one vertex with another, entrance to another connected component must first begin by going through the gateway node at the other end of the bridge edge. I will refer to these nodes as "bridge nodes" from here on out.

Because of this, we noticed that taking control over some of these bridge nodes would be very valuable, as they secure a place in one of the connected components, it can prevent other players from entering a component, and it gives easier access to an adjacenct connected component. Therefore, this optimization relied on finding bridge nodes, ranking the bridge nodes based on a significance measure (more on that later), and then including these bridge nodes in the final submission.

Now, we first need to find these bridge nodes. The naive approach would be to remove each node, and then call a searching algorithm to determine whether or not the search would return the same number of visited nodes. However, this would require us to call the searching algorithm a total of V times, where V represents the number of vertices. So, this would be polynomial time, and while it is still poly-time, the size of the graph may be so large that we would not produce results within the time constraints. So, what we did instead was to use a linear-time bridge-finding algorithm. In particular, we used Tarjan's Bridge Finding Algorithm[1]. The algorithmic steps are below, followed by brief implementation details that we did:

```
Tarjan's Algorithm:
  1: Find a spanning forest of graph G
      - This was done using DFS until the entire graph is searched.

        If the DFS finishes without the entire graph searched, start
```

```
      another DFS on an unvisited node, since it should be on a
      new connected component.

    - Return value: Adjacency Matrix/Matrices of the directed spanning
      forest.

2: In the spanning tree, order all the nodes in preorder (order which you
   discovered them in your DFS)

    - A map was created mapping each node to its respective preorder.
      This proved very useful for later dynamic programming things.

3: Going in reverse preorder order, we calculate each nodes respective
   lower bound, higher bound, and the number of descendants it has.

   - All three were done in reverse preorder order, so that we could
     use dynamic programming. Dynamic programming table was an array
     with each index of the array representing the preorder value of
     a node 'n'. This is why the map comes in handy.

   - ND(n), for node n, is calculated by adding the number of descendants
     held by n's children. We then add 1 to include 'n' itself into the
     count.

   - L(n) and H(n) were calculated by first examining all edges not in
     the tree, and finding the minimum/maximum preorder value of its
     neighbors. Then, we compare that with the L/H values of
     its children in the spanning tree. We take the minimum/maximum of
     that for L(n)/H(n) values.

4: Going in preorder order, we check each spanning tree edge (u -> v),
   and if:
   1) L(v) = PreOrder(v) AND
   2) H(v) < PreOrder(v) + ND(n)

   then (u, v) is a bridge edge and u, v are bridge nodes.
```

Once we find the bridge edges, that's just the first part. We now have to choose which bridge edges are valuable in the graph, and which ones are not worth choosing. For example, many bridge edges actually end up being edges that connect to an endpoint in the graph (a vertex with a degree of just 1). It is obvious that these bridge nodes are not worth adding, since they do not really capture the strongly connected com-

ponents we were looking for.

Therefore, after finding the bridge nodes, one thing we did was to rank the value of bridge nodes through the following ways:

1. We would rank the bridge nodes solely by degree
2. We would rank the bridge nodes both by degree and the size of its connected component compared with the size of the rest of the graph.

Through trial and error and by using multiple json graphs, we ended up using the following format:

```
|V|/2 = halfGraph
Ranking each bridge node:
  Find the size of its connected component (sizeOfComponent)
  Find the size of the rest of the graph (remainingPart)
  Find its degree (degree)
  Rank the bridge node with the following expression:
    Rank = (sizeOfComponent * (remainingPart + degree) /
            halfGraph^2 - sizeOfComponent * remainingPart + 1)
```

We sorted these rankings where larger ranks equate to more value. While the full ranking hasn't been completely developed, this ranking system is explained below:

NUMERATOR:
We want to maximize this numerator. We want large degrees since this means that this bridge node is not just connected to another component, but also to many vertices in its own component. Further, we wanted the size of the partition to matter as well: we want control of larger partitions rather than small ones. Though both are important, it is more important to secure a spot in a larger component.

DENOMINATOR:
We want to minimize this number, for larger rankings. First, we don't want the size of the partition sizes to be that different. This is to account for nodes involved in "endpoints" or in really really small connected components, which just really aren't that valuable. Even with a larger neighbor partition, it is not worth a node to block its entrance from a smaller connected component. The random "$+1$" is added to ensure that we don't divide by 0 (say, if a bridge node actually has a partition size of half the graph)

Another problem with this system, however, is that this ranking system will take a very long time for large graphs, since finding partition sizes involves searching through each bridge node. Once again, this is, at worst, polynomial time, which can cause a lot of time for large graphs. So, we add one more constraint, where if the number of vertices is greater than 1000, we limit the depth of the search to be only K vertices deep, where K is an integer that we tweak. This transforms the time from polynomial to linear. Although accuracy takes a hit, it allows us to provide ranks for larger graphs on time.

Finally, once the bridge nodes are found and ranked, we output all the bridges in a text file `bridgeNodes.txt`. This is then used for the mixed strategy illustrated below: we do not want to solely pick bridge nodes because we can never be sure that a graph has bridge nodes.

Mixed Strategy

Using a mixed strategy requires some notion picking seed nodes according to some probability distribution. Each strategy will output the rankings of the top X nodes, according to some centrality measure, which provides a quantitative notion of how important a particular seed is relative to others. Thus, our idea was to use the values of centrality measure as a rough probability of how likely it is to be selected as a node - the greater the measure, the more likely it is to be picked. This follows similarly with the idea of computing the probabilities of a mixed strategy - a particular action receives greater probability if its expected payoff is greater.
In a file, `mixed.py`, we outline the procedure for computing a mixed strategy. Assuming that all centrality measures are equally important, we pick one of the measures at random, obtain a list of most important nodes according to that measure, and perform a weighted random choice among those nodes.
Results: Using a mixed strategy for the bridge centrality measure, and degree centrality, we obtained a score of 2 on day3 of the tournament. Though it wasn't much, this was the most points we've scored out of all three days of the tournament, and it seemed to be a promising avenue to explore given more time.

4 Testing

We tested the algorithms for computing seed nodes by running them against each other, using the provided `sim.py`. To facilitate this, a wrapper class `runStrat.py` was created, which takes as arguments a variable number of strategies and a graph input, picks the seeds according to each strategy, and runs the simulation with seeds on the graph.

5 Contributions

There are four members in this group: Kevin Chang, Timothy Chou, Albert Ge, and John Li. Here are each of their contributions:

- KEVIN CHANG: Wrote linear combination strategy generator `create_strats.py`
- TIMOTHY CHOU: Wrote and designed algorithm to beat TA_degree. Wrote monte carlo simulator `runSim.py`, wrote mixed top-K strategy in `create_strats.py` and ran tests to find optimal linear combination.
- ALBERT GE: Wrote most of the initial framework - `deg_centrality.py`, `submit.py`, Wrote the testing script `runStrat.py`. Wrote `mixed.py`, the script for employing mixed strategies according to a probabilistic distribution.
- JOHN LI: Studied Tarjan's linear-time bridge detection algorithm and wrote all of `bridges.py`. Also developed `pagerank.py` before migrating it into another file.

6 Citations

1. Tarjan's Bridge Finding Algorithm:
https://en.wikipedia.org/wiki/Bridge_(graph_theory)#Tarjan.27s_Bridge-finding_algorithm